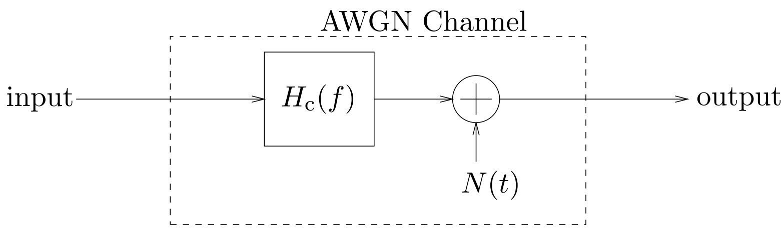
ECE 359 **Spring 2003**

Handout # 14 April 1, 2003

EFFECT OF NOISE ON ANALOG COMMUNICATION SYSTEMS



The additive white Gaussian noise (AWGN) channel model accurately describes many point-to-point communication channels (e.g., the telephone line channel). The channel frequency response is represented by $H_c(f)$, and $N(t)$ is WGN. For mobile communications channels, the channel filter is (randomly) time varying. For most cases of interest in this course, we assume that the filter is LTI and *distortionless*, i.e., $H_c(f)$ that is roughly constant over the frequency range of the input.

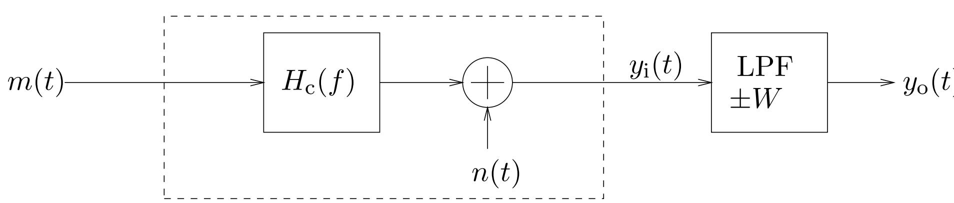
Signal-to-noise Ratio (SNR) A measure of fidelity of a communication system is the signal-to-noise ratio (SNR) that is defined by

$$\text{SNR} = \frac{\text{Useful signal power}}{\text{Noise power}} = \Gamma$$

We will use the symbol Γ to denote SNR.

SNR for Baseband Communications

We begin by studying the SNR for baseband communication of an analog message signal $m(t)$ with bandwidth W Hz on an AWGN channel. The receiver for this baseband communication system (shown below) is simply a LPF with a cut-off frequency of W Hz. The SNR of interest is that at the output of the receiver $y_o(t)$.



Since $m(t)$ has bandwidth W Hz, only the noise $n(t)$ is affected by receiver LPF. Thus

$$y_o(t) = m(t) \star h_c(t) + n_{LP}(t)$$

where $n_{LP}(t)$ is a sample path of the lowpass WGN process $N_{LP}(t)$.

For a distortionless channel $H_c(f) \approx \alpha$ for $|f| \leq W$. Thus

$$y_o(t) = \alpha\, m(t) + n_{LP}(t) \tag{1}$$

Random System Model We now convert the system model of (1) into a *random* system model by replacing all non-deterministic signals by corresponding random processes. It is important to note that the message signal should be considered to be random in this model since deterministic message signals carry no information. Thus we have

$$Y_o(t) = \alpha\, M(t) + N_{LP}(t) \tag{2}$$

We model the random message signal $M(t)$ as a zero-mean WSS process with ACF $R_M(\tau)$ and PSD $S_M(f)$.

The quantity of interest is the SNR at the output of the receiver. To compute this we use the following steps:

Power in $M(t) = P_m = \mathrm{E}[M^2(t)] = R_M(0) = 2 \int_0^W S_M(f)\, df$

Transmitted signal power $= P_{s,t} = P_m$

Signal power at receiver input $= P_{s,i} = 2 \int_0^W |H_c(f)|^2 S_M(f) df = \alpha^2 P_{s,t}$ (for distortionless channel)

Signal power at receiver output $= P_{s,o} = P_{s,i}$

Noise power at receiver output $= P_{N,o} = \mathrm{E}\left[N_{LP}^2(t)\right] = R_{N_{LP}}(0) = N_0 W$

SNR at receiver output $= \Gamma_o = \dfrac{P_{s,o}}{N_0 W} = \dfrac{P_{s,i}}{N_0 W} = \dfrac{\alpha^2 P_{s,t}}{N_0 W}$

Example Suppose the message signal $M(t)$ has PSD that is triangular with peak β and bandwidth 4 kHz. This signal is sent on baseband channel with $H_c(f) \approx 10^{-2}$ for $|f| \leq W$, and $N_0 = 10^{-12}$ Watts/Hz. If Γ_o is to be 20 dB, find the required $P_{s,i}$, $P_{s,t}$ and corresponding β.

As shown in class required $P_{s,i} = 4 \times 10^{-7}$ Watts, $P_{s,t} = 4 \times 10^{-3}$ Watts, and $\beta = 10^{-6}$ Watts/Hz.

Benchmark Baseband SNR For any communication system, we can define a benchmark baseband SNR as

$$\overline{\Gamma} = \frac{\text{Signal power at receiver input}}{\text{Noise power over signal bandwidth}} = \frac{P_{s,i}}{N_0 W}$$

This SNR measure corresponds to the SNR obtained if we used the signal power to send the message directly on the AWGN channel using baseband communications. The actual SNR at the receiver output Γ_o for the given communication system may be equal, smaller or larger than $\overline{\Gamma}$. Clearly for the baseband communication system above $\Gamma_o = \overline{\Gamma}$. We will see that the same is true for DSB-SC and SSB modulation. For conventional AM $\Gamma_o < \overline{\Gamma}$, whereas for wideband PM and FM $\Gamma_o > \overline{\Gamma}$.