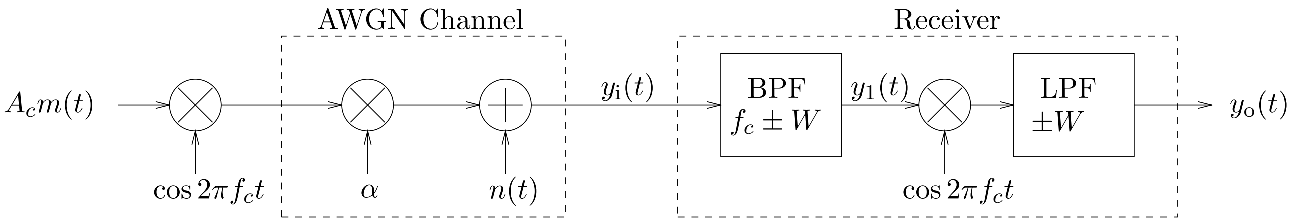
SNR for DSB-SC Communications



Note that we have replaced the channel filter by a multiplication by α to denote a distortionless channel. Also, the BPF at the front-end of the receiver eliminates any unwanted signals/interference and noise before demodulation.

From the figure for DSB-SC communications in noise, we get the following equations:

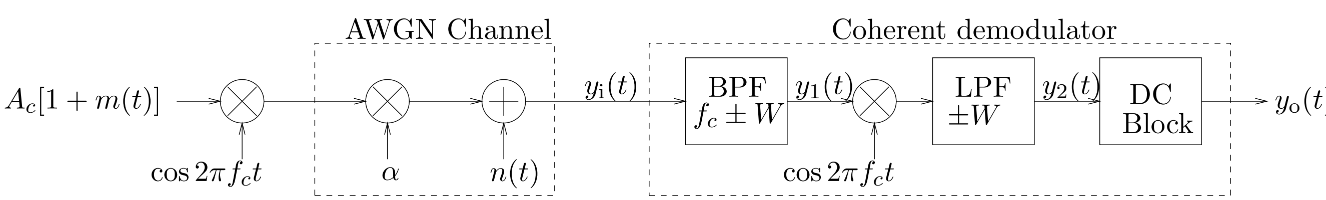
$$\begin{aligned} y_i(t) &= \alpha\, A_c m(t) \cos 2\pi f_c t + n(t) \\ y_1(t) &= [y_i(t)]_{\mathrm{BPF}} = \alpha\, A_c m(t) \cos 2\pi f_c t + n_{\mathrm{BP}}(t) \\ &= [\alpha\, A_c m(t) + n_c(t)] \cos 2\pi f_c t - n_s(t) \sin 2\pi f_c t \\ y_o(t) &= \frac{1}{2}\alpha\, A_c m(t) + \frac{1}{2} n_c(t) \end{aligned}$$

We may compute the output SNR using the following steps:

$$\begin{aligned} P_{s,t} &= \text{Average power in } A_c M(t) \cos 2\pi f_c t = \frac{A_c^2 P_m}{2} \\ P_{s,i} &= \alpha^2 P_{s,t} = \alpha^2\ \frac{A_c^2 P_m}{2} \\ P_{s,o} &= \frac{\alpha^2 A_c^2 P_m}{4} = \frac{P_{s,i}}{2} \\ P_{n,o} &= \frac{1}{4}\ \mathrm{E}\left[N_c^2(t)\right] = \frac{1}{4}\ R_{N_c}(0) = \frac{1}{2}\ N_0 W \\ \Gamma_o &= \frac{P_{s,o}}{P_{n,o}} = \frac{P_{s,i}}{N_0 W} = \overline{\Gamma} \end{aligned}$$

Thus DSB-SC modulation leaves the output SNR unchanged relative to the baseband benchmark.

SNR for Conventional AM with Coherent Demodulation



$$
\begin{aligned}
y_{\rm i}(t) &= \alpha\, A_c[1+m(t)]\cos 2\pi f_c t + n(t) \\
y_1(t) &= [y_{\rm i}(t)]_{\rm BPF} = \alpha\, A_c[1+m(t)]\cos 2\pi f_c t + n_{\rm BP}(t) \\
&= \{\alpha\, A_c[1+m(t)] + n_c(t)\}\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \\
y_2(t) &= \frac{1}{2}\alpha\ A_c[1+m(t)] + \frac{1}{2}n_c(t) \\
y_{\rm o}(t) &= \frac{1}{2}\alpha\, A_c m(t) + \frac{1}{2}n_c(t)
\end{aligned}
$$

We compute the output SNR using the following steps:

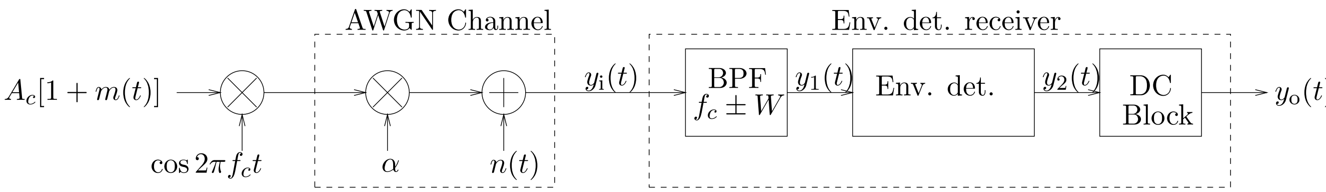
$$
\begin{aligned}
P_{\rm s,t} &= \text{Average power in } A_c[1+m(t)]\cos 2\pi f_c t = \frac{A_c^2[1+P_m]}{2} \\
P_{\rm s,i} &= \alpha^2 P_{\rm s,t} = \alpha^2\ \frac{A_c^2[1+P_m]}{2} \\
P_{\rm s,o} &= \frac{\alpha^2 A_c^2 P_m}{4} = \frac{P_{\rm s,i}}{2}\ \frac{P_m}{1+P_m} \\
P_{\rm n,o} &= \frac{1}{4}\ {\rm E}\left[N_c^2(t)\right] = \frac{1}{4}\ R_{N_c}(0) = \frac{1}{2}\ N_0 W \\
\Gamma_{\rm o} &= \frac{P_{\rm s,o}}{P_{\rm n,o}} = \frac{P_{\rm s,i}}{N_0 W}\ \frac{P_m}{1+P_m} = \overline{\Gamma}\ \frac{P_m}{1+P_m}
\end{aligned}
$$

We can rewrite $\Gamma_{\rm o}$ in terms of normalized message signal and the modulation index a.

$$
\Gamma_{\rm o} = \overline{\Gamma}\ \frac{a^2 P_{m_n}}{1+a^2 P_{m_n}}
$$

As shown in class $\Gamma_{\rm o} \leq \overline{\Gamma}/2$. The upper-bound is achieved when $a = 1$ and $P_{m_n} = 1$. In practice, both of these quantities are less than 1, and $\Gamma_{\rm o}$ obtained is usually at least an order of magnitude smaller than $\overline{\Gamma}$.

The above $\Gamma_{\rm o}$ is for coherent detection. If we use an envelope detector at the receiver (which is the main reason for using conventional AM modulation) the SNR degrades even further. The SNR analysis for an envelope detector is quite complicated and beyond the scope of this class.



$$
y_2(t) = \text{Envelope of } y_1(t) = \sqrt{(\alpha\, A_c[1+m(t)] + n_c(t))^2 + n_s^2(t)}
$$

For small noise (i.e., $\overline{\Gamma} > 10$), we may approximate $y_2(t)$ as:

$$
y_2(t) \approx \alpha\, A_c[1+m(t)] + n_c(t)
$$

and it is easy to show that $\Gamma_{\rm o}$ is the same as that obtained above for coherent demodulation.

In the medium noise case where $1 < \overline{\Gamma} < 10$, $\Gamma_{\rm o}$ is not easily analyzed. Finally, in the small noise case where $\overline{\Gamma} < 1$, it can be shown that $\Gamma_{\rm o} \approx 0$, i.e. the signal is lost in the noise. This is a nonlinear *threshold* phenomenon which we will see again when we study FM in noise.