ECE 359 **Spring 2003**

Handout # 19 April 24, 2003

HOMEWORK ASSIGNMENT 10

Reading: Handouts 18, 20

Due Date: May 6, 2003 (in class)

1. Let $s(t)$ and $r(t)$ be two signals that are both non-zero only on the interval $[0, T]$. Suppose we correlate these two signals and form the output:

$$z = \int_0^T s(t)r(t)dt.$$

Show that z can also be formed by passing $r(t)$ through a LTI system and sampling the output at time T. Find the impulse response of this LTI system in terms of $s(t)$.

2. Suppose the noise in a binary baseband communication system is such that the input Z to the decision device is on the form:

$$\begin{array}{lll} H_0: & \text{bit '0' is sent}: & Z = \tilde{X} \\ H_1: & \text{bit '1' is sent}: & Z = 1 + \tilde{X} \end{array}$$

where $\tilde{X}$ is Laplacian noise (some underwater acoustic channels have this type of noise) with p.d.f.

$$p_{\tilde{X}}(x) = \frac{1}{2}\mathrm{e}^{-|x|}.$$

Find the maximum likelihood decision rule. Simplify your answer as much as possible.

Hint: Begin by drawing a picture of the pdfs

3. In class we argued that the ML decision rule was a likelihood ratio test of the form

$$\frac{p_1(z)}{p_0(z)} \underset{0}{\overset{1}{\gtrless}} 1$$

In this problem you will establish that the above likelihood ratio test also minimizes the average probability of error $\mathrm{P_e} = \mathrm{P}(b \neq \hat{b})$, if the bits are equally likely.

First note that every decision rule δ based on z must divide the real line $\mathbb{R}$ into two disjoint regions Γ_0 and Γ_1, with Γ_i being the region where bit 'i' is chosen.

(a) Assuming that bits '0' and '1' are equally likely to be transmitted, show that the average error probability resulting from the use of decision rule δ can be expressed as

$$\mathrm{P_e} = \frac{1}{2}\int_{\Gamma_1} p_0(z)dz + \frac{1}{2}\int_{\Gamma_0} p_1(z)dz.$$

(b) Use the fact that $\Gamma_0 \cup \Gamma_1 = \mathbb{R}$ and $\Gamma_0 \cap \Gamma_1 = \emptyset$ to show that

$$\mathrm{P_e} = \frac{1}{2} + \frac{1}{2}\int_{\Gamma_1} [p_0(z) - p_1(z)]dz$$

(c) Now argue that $\mathrm{P_e}$ is minimized by picking Γ_1 to be region

$$\Gamma_1^* = \{z \ : \ p_0(z) - p_1(z) < 0\}.$$

4. *Suboptimal receivers:* Suppose binary data is transmitted on an AWGN channel (with PSD $\frac{N_0}{2}$) using antipodal signaling with $s_1(t) = p_T(t)$ and $s_0(t) = -p_T(t)$. We know that the optimum receiver for this signal set correlates the received signal $r(t)$ with the signal $2p_T(t)$ to produce the decision statistic z, and then compares z with 0 to make the bit decision.

   (a) Now suppose we form the decision statistic z in the following suboptimal fashion as

$$z = \int_0^T g(t)r(t)dt$$

   where

$$g(t) = \begin{cases} t & 0 \le t < \frac{T}{2} \\ T - t & \frac{T}{2} \le t < T \\ 0 & \text{otherwise} \end{cases}$$

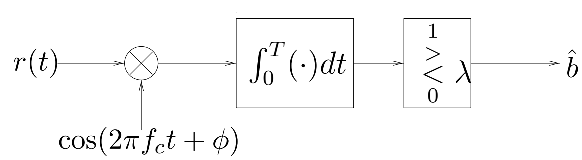
   The statistic z is then compared with 0 to make the bit decision. Find the average error probability $\mathrm{P_e}$ for this suboptimal receiver and compare it with that of the optimal receiver. Is the threshold of 0 optimum for the given suboptimum choice of linear filter?

   (b) Now suppose we form the decision statistic z by simply sampling the received signal at time $\frac{T}{2}$, i.e., $z = r(\frac{T}{2})$. Again suppose z is compared with 0 to make the bit decision. Find the the average error probability $\mathrm{P_e}$ for this suboptimal receiver.
   Hint: This is a trick question!

5. Consider the following signal set.

$$s_1(t) = A_c \sin\left(\frac{\pi t}{T}\right) p_T(t)\, \cos(2\pi f_c t + \phi) \qquad s_0(t) = A_c \sin\left(\frac{2\pi t}{T}\right) p_T(t)\, \cos(2\pi f_c t + \phi)$$

   This signal set is used in a binary communication system with an AWGN channel with PSD $\frac{N_0}{2}$, and the received signal is passed through the receiver shown below:



   Ignore the double frequency components. Answer the following.

   (a) What value of λ minimizes the error probability for the above receiver? What is the corresponding minimum error probability?

   (b) Is the receiver optimum for the given signal set? If not, specify the optimum receiver.

   (c) Find the threshold and error probability for the optimum receiver.

6. *BPSK with Imperfect Phase Reference:* In class we studied the optimum receiver for BPSK and derived its performance. Now assume that a perfect phase reference is not available at the receiver and so the correlation at the receiver is peformed using $\sqrt{2}\cos(2\pi f_c t + \phi')$, where $\phi' \neq \phi$. Show that the error probability for this imperfect receiver is given by:

$$\mathrm{P_e} = Q\left(\sqrt{\frac{2A_c^2 T \cos^2(\phi' - \phi)}{N_0}}\right).$$