1. **Filtering.** Consider a periodic pulse signal $x(t)$ with fundamental frequency of $f_0 = 100$ Hz. We know that $x(t)$ can be expressed by the series:

$$x(t) = \frac{1}{2} + \frac{2}{\pi} \left[ \cos 2\pi f_0 t - \frac{1}{3} \cos 6\pi f_0 t + \frac{1}{5} \cos 10\pi f_0 t - \frac{1}{7} \cos 14\pi f_0 t + \cdots \right]$$

Now suppose $x(t)$ is passed through a (unit gain) Butterworth filter with cut-off frequency $50$ Hz and order $n$. Let the output of the filter be denoted by $y(t)$. Find the smallest value of $n$ such that the power in $y(t)$ at $100$ Hz is less than 1% of the DC power in $y(t)$.

Note: You may need a calculator for this question.

2. **Stereo AM (SAM).** Consider broadcasting stereo AM by using a modulation scheme that forms the signal

$$v_{\text{SAM}}(t) = [A + m_L(t) + m_R(t)] \cos 2\pi f_c t + [m_L(t) - m_R(t)] \sin 2\pi f_c t$$

where $m_L(t)$ and $m_R(t)$ are the left and right audio signals, respectively.

(a) Show that a coherent detector that multiplies $v_{\text{SAM}}(t)$ by $\cos(2\pi f_c t + \phi)$ can be used to recover $m_L(t) - m_R(t)$. (Hint: What value of $\phi$ should you use?)

(b) Assuming that $A \gg |m_L(t)| + |m_R(t)|$ for all $t$, show that envelope detection can be used to produce the sum signal $m_L(t) + m_R(t)$.

(c) How are the desired $m_L(t)$ and $m_R(t)$ finally obtained?

(d) An alternative way to send stereo AM is to use the modulated signal $v'_{\text{SAM}}(t)$ that is given by

$$v'_{\text{SAM}}(t) = [A + m_L(t)] \cos 2\pi f_c t + m_R(t) \sin 2\pi f_c t$$

Give at least one reason why you may choose to use $v_{\text{SAM}}(t)$ over $v'_{\text{SAM}}(t)$.

3. **SSB Demodulation.** Consider the LSB signal

$$v_L(t) = A_L m(t) \cos 2\pi f_c t + A_L \dot{m}(t) \sin 2\pi f_c t$$

We would like to demodulate this signal coherently at the receiver; however for some reason we end up with a phase offset $\phi$. Thus the receiver multiplies $v_L(t)$ with $\cos(2\pi f_c t + \phi)$ and low-pass filters the product to produce the demodulated signal $y(t)$. 

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(a) Find $y(t)$ in terms of $m(t)$ and $\hat{m}(t)$.

(b) Comment on the distortion in the demodulated signal introduced by the phase offset. How is it different from the distortion experienced by a DSB-SC signal due to a phase offset at the receiver?

(c) Show that the output spectrum $Y(f)$ is given by:

$$Y(f) = \begin{cases} \frac{A_c}{2} M(f) e^{j\phi} & \text{if } f \geq 0 \\ \frac{A_c}{2} M(f) e^{-j\phi} & \text{if } f < 0 \end{cases}$$

4. True or False (you must explain your answer to get credit):

(a) Any scheme that can be used to demodulate DSB-SC can also be used to demodulate AM.

(b) If $m_1(t)$ has bandwidth of 10 kHz and $m_2(t)$ has bandwidth of 5 kHz, then $m_1(t)m_2(t)$ has bandwidth 50 kHz.

(c) A transmitter transmits an AM signal with a carrier frequency of 1600 kHz (the IF frequency is 455 kHz). An inexpensive receiver can receive this station at a dial setting of 2510 kHz as well.