ECE 359 **Spring 2003**

February 18, 2003

PRACTICE EXAM 1

The following exam contains questions similar to those you will see on the first exam. The actual exam will be a *closed book* exam, but you will be allowed to use 1 sheet of notes (8.5" $\times$ 11"; both sides). You may need a calculator, so bring one to the exam.

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1. *Filtering.* Consider a periodic pulse signal $x(t)$ with fundamental frequency of $f_0 = 100$ Hz. We know that $x(t)$ can be expressed by the series:

   $$x(t) = \frac{1}{2} + \frac{2}{\pi}\left[\cos 2\pi f_0 t - \frac{1}{3}\cos 6\pi f_0 t + \frac{1}{5}\cos 10\pi f_0 t - \frac{1}{7}\cos 14\pi f_0 t + \cdots\right]$$

   Now suppose $x(t)$ is is passed through a (unit gain) Butterworth filter with cut-off frequency 50 Hz and order n. Let the output of the filter be denoted by $y(t)$.

   Find the smallest value of n such that the power in $y(t)$ at 100 Hz is less than 1% of the DC power in $y(t)$.

   Note: You may need a calculator for this question.

2. *Stereo AM (SAM).* Consider broadcasting stereo AM by using a modulation scheme that forms the signal

   $$v_{\text{SAM}}(t) = [A + m_\ell(t) + m_r(t)]\cos 2\pi f_c t \;+\; [m_\ell(t) - m_r(t)]\sin 2\pi f_c t$$

   where $m_\ell(t)$ and $m_r(t)$ are the left and right audio signals, respectively.

   (a) Show that a coherent detector that multiplies $v_{\text{SAM}}(t)$ by $\cos(2\pi f_c t + \phi)$ can be used to recover $m_\ell(t) - m_r(t)$. (Hint: What value of ϕ should you use?)

   (b) Asumming that $A \gg |m_\ell(t)| + |m_r(t)|$ for all t, show that envelope detection can be used to produce the sum signal $m_\ell(t) + m_r(t)$.

   (c) How are the desired $m_\ell(t)$ and $m_r(t)$ finally obtained?

   (d) An alternative way to send stereo AM is to to use the modulated signal $v'_{\text{SAM}}(t)$ that is given by

   $$v'_{\text{SAM}}(t) = [A + m_\ell(t)]\cos 2\pi f_c t \;+\; m_r(t)\;\sin 2\pi f_c t\,.$$

   Give at least one reason why you may choose to use $v_{\text{SAM}}(t)$ over $v'_{\text{SAM}}(t)$.

3. *SSB Demodulation.* Consider the LSB signal

   $$v_\ell(t) = A_c m(t)\cos 2\pi f_c t + A_c \hat{m}(t)\sin 2\pi f_c t$$

   We would like to demodulate this signal coherently at the receiver; however for some reason we end up with a phase offset ϕ. Thus the receiver multiplies $v_\ell(t)$ with $\cos(2\pi f_c t + \phi)$ and low-pass filters the product to produce the demodulated signal $y(t)$.

(a) Find $y(t)$ in terms of $m(t)$ and $\hat{m}(t)$.

(b) Comment on the distortion in the demodulated signal introduced by the phase offset. How is it different from the distortion experienced by a DSB-SC signal due to a phase offset at the receiver?

(c) Show that the output spectrum $Y(f)$ is given by:

$$Y(f) = \begin{cases} \frac{A_c}{2} M(f) e^{j\phi} & \text{if } f \geq 0 \\ \frac{A_c}{2} M(f) e^{-j\phi} & \text{if } f < 0 \end{cases}$$

4. *True or False* (you must explain your answer to get credit):

(a) Any scheme that can be used to demodulate DSB-SC can also be used to demodulate AM.

(b) If $m_1(t)$ has bandwidth of 10 kHz and $m_2(t)$ has bandwidth of 5 kHz, then $m_1(t)m_2(t)$ has bandwidth 50 kHz.

(c) A transmitter transmits an AM signal with a carrier frequency of 1600 kHz (the IF frequency is 455 kHz). An inexpensive receiver can receive this station at a dial setting of 2510 kHz as well.