ECE 359 **Spring 2003**

April 3, 2003

PRACTICE EXAM 2

The following exam contains questions similar to those you will see on the Exam 2. Exam 2 will be a *closed book* exam, but you will be allowed to use 2 sheets of notes (8.5" × 11"; both sides). It will cover material relevant to HW 1 – HW 8. You may need a calculator, so bring one to the exam.

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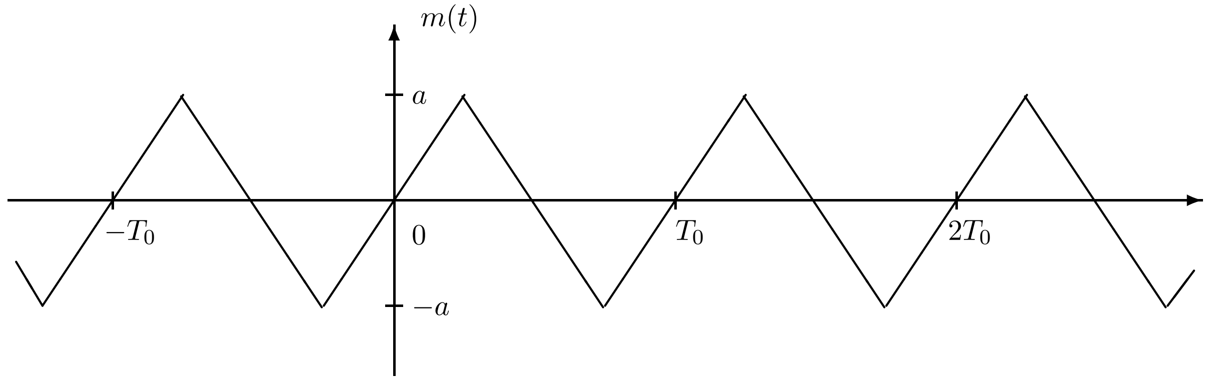
1. *Angle Modulation*:



Consider the *periodic* message signal $m(t)$ shown in the picture, and assume initially that $a = 1$ and $T_0 = 10^{-3}$.

Consider the (essential) bandwidth of the signal W to be the frequency corresponding to the fifth harmonic, i.e. $W = \frac{5}{T_0}$ Hz. Now, suppose $m(t)$ is angle modulated on a 1 MHz carrier.

(a) Find the bandwidth of an FM signal with $k_f = 100$.

(b) Find the bandwidth of a PM signal with $k_p = 2\pi \times 10^2$.

(c) Find the bandwidth of an FM signal with $k_f = 10^4$.

(d) Modify your answers to parts (i)–(iii) for the case where $a = 2$ and $T_0 = 10^{-3}$.

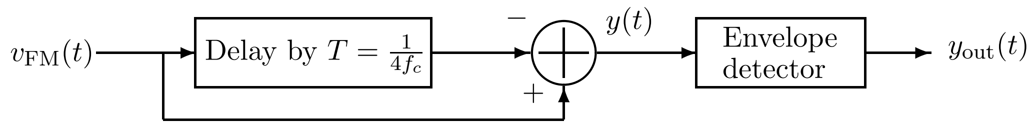
(e) Modify your answers to parts (i)–(iii) for the case where $a = 1$ and $T_0 = 0.5 \times 10^{-3}$.

2. *FM demodulation:* Consider frequency modulation with a sinusoidal message signal (tone modulation) $m(t) = a\cos(2\pi f_m t)$. The modulated signal is:

$$v_{\text{FM}}(t) = A_c \cos[2\pi f_c t + \beta_f \sin(2\pi f_m t)].$$

with $\beta_f = k_f a / f_m$ being the modulation index.

Suppose we pass $v_{\text{FM}}(t)$ through the following system:



That is $y(t)$ in the figure above is given by

$$y(t) = v_{\text{FM}}(t) - v_{\text{FM}}(t - T),$$

with $T = \frac{1}{4f_c}$

Now assume that $f_c \gg f_m$ so that $f_m T \ll 1$.

Show that the output $y_{\text{out}}(t)$ is approximately proportional to $\cos(2\pi f_m t - \pi f_m T)$

Hint: If $\theta \ll 1$, then $\cos(\theta) \approx 1$, and $\sin(\frac{\pi}{4} + \theta) \approx \frac{1}{\sqrt{2}}(1 + \theta)$.

3. *Random Processes*: A zero-mean WGN process with PSD of $\frac{N_0}{2}$ is passed through a unit gain *first* order ($n = 1$) Butterworth filter with cut-off frequency f_0 Hz. Let $Y(t)$ denote the output process.

   (a) Find the autocorrelation function of $Y(t)$.
   Hint: You may find the following Fourier transform pair to be useful

   $$e^{-\alpha|\tau|} \Longleftrightarrow \frac{2\alpha}{\alpha^2 + (2\pi f)^2} \qquad \alpha > 0$$

   (b) Find the average power in $Y(t)$

   (c) Suppose $Y(t)$ is sampled at time instant t_0. Find an expression for $\text{P}(Y(t_0) > \sqrt{2N_0 \pi f_0})$ in terms of the $Q(\cdot)$ function.

4. *True or False* (you must explain your answer to get credit):

   (a) A carrier phase offset of $\pi/4$ in a SSB demodulator still produces a scaled version of the message signal at the output.

   (b) Without prior knowledge of $m(t)$, one can tell by looking at an angle modulated signal if it is an FM signal or a PM signal.

   (c) If $X(t)$ is a WSS process, then $\text{E}[(X(t) - X(t-1))^2]$ is independent of t.

   (d) Consider the transmission of a 5 kHz message signal on a baseband communications channel with a power attenuation of 50 dB, and $N_0 = 10^{-12}$ Watts/Hz. If the baseband SNR $\overline{\Gamma}$ is required to be at at least 20 dB, then $P_{\text{s,t}}$ must be greater than 0.05 Watts.