ECE 371VV **Spring 2001**

Handout # 2 January 23, 2001

HOMEWORK ASSIGNMENT 1

Reading: Paper by V. H. MacDonald "The Cellular Concept" Bell Syst. Tech. J., Jan 1979; Ch. 1 & 2 of Rappaport; Ch. 3 of Bertsekas & Gallager (for queueing theory); paper on priority queueing of handoffs by Hong & Rappaport available on www.ece.sunysb.edu/∼rappap/papers/start_papers.htm.

Include a printout of all required computer programs.

Due Date: Tuesday, January 29, 2001 (in class)

1. *Second-tier approximation for SIR.* Modify the SIR analysis of class to include a second tier of interferers and compare the resulting SIR with the one derived in class for $N = 7$. Consider all 12 second tier interferers, 6 at distance $2D$ and 6 at distance $\sqrt{3}D$. (Use the improved approximations for distances for the first tier only. Assume a path loss exponent of $n = 4$.)

2. *Reverse link SIR approximation.* The improved first-tier SIR analysis done in class was for the link going from the base station to the mobile (forward link). Will the analysis be different for the reverse link? If you answer is yes, compute the reverse link approximation, and compare with the forward link analysis. (Assume $N = 7$ and $n = 4$ for the comparison.)

3. *Erlang-B Formula.* Write a program (in MATLAB or C) to compute the Erlang capacity $C_{\rm E}$ as a function of the GoS and the number of channels using the expression:

$$\text{GoS} = \frac{\rho^S/S!}{\sum_{k=0}^{S} \rho^k/k!}\,, \quad \text{with } \rho = C_{\rm E}\,.$$

   An accuracy of 1 decimal place is good enough.

   For a GoS of 1%, plot $C_{\rm E}$ as a function of S, for S ranging from 1 to 100.

4. *Erlang Capacity of Cellular Systems.* Consider a channelized cellular system with a total of 840 available channels for an operator in a service area. Assume throughout that the GoS provided is 1%, and that each user has an offered traffic of 0.1 Erlangs. Answer the following.

   (a) Find the user capacity per cell for an omni system with a reuse factor of $N = 7$.

   (b) Repeat (a) for a 120 degree sectorized cell with the same reuse factor.

   (c) Explain how can sectorization be exploited to improve capacity, and find the corresponding improved user capacity per cell.

   (d) Explain how zoning can be used to further improve capacity, and find the corresponding user capacity per cell.

5. *Erlang-C system.* Carefully derive the formulas for P{call is queued} and E[W | Queueing] given on page 18 of the notes.

6. *Handoff rates.* Consider a geographical area with 10,000 people per square-kilometer, with 20% of the people subscribing to cell phone service. Assume an average speed of 20 km/hr, an (unsectored) hexagonal cell radius of 2 km, and Erlangs per terminal of 0.05. Answer the following:
   (a) Find the average number of handoffs per hour.
   (b) If each cell is split into 4 cells, keeping the number of subscribers the same, what is the new handoff rate for the smaller cells? By what factor has the handoff rate per square-kilometer increased?
   (c) Redo part (b) for the case where the cell splitting is done in order to accomodate a 4-fold increase in subscribers.

7. *Guard Channels.* Consider a cell in which new calls arrive with an intensity of 20 Erlangs and handoff calls arrive with an intensity of 40 Erlangs. We require that the blocking probability of for handoffs is 0.1%, and that for new calls is 1%.
   (a) Find the average number of handoffs per call.
   (b) Without the use of guard channels, what is the minimum number of channels S needed in order to guarantee the required GoS for both types of calls?
   (c) With the use of appropriately chosen guard channels, what is the minumum value of S? What is the corresponding value of G?

      (You would need to write a computer program to solve this problem. You should be able to use parts of the program you write for problem 3 here.)

Extra Problems (you are not required to turn these in)

1. *Cellular Layout*
   (a) Show that the triangle, square and hexagon are the only regular polygons that tessellate a 2-D surface.
   (b) For a hexagonal cellular structure, prove that any cell can be reached from the cell of interest by moving in a direction perpendicular to one of the edges for a certain number of cells (say i), and then at 60 degrees to this direction for a certain number of cells (say j).
   (c) Use (b) to argue that the clustering algorithm given to you in class works.

2. *Poisson random process.* Prove the statement given in the first line of page 12 of the class slides.

3. *Priority Queueing of handoffs.* Redo the priority queueing analysis for the case where the handoff calls are not only given priority, but they are also queued (in an infinite capacity buffer). New calls are not queued.