ECE 371VV **Spring 2001**

Handout # 3 February 6, 2001

HOMEWORK ASSIGNMENT 2

Reading: Ch. 3 of Rappaport; Ch. 2 of Stuber; B. Gudmundson, "Correlation Model for Shadow Fading in Mobile Radio Systems," Electronic Letters, Nov. 1991; E. Wong and B. Hajek, *Stochastic Processes in Engineering Systems* (for details on isotropic and homogenous random fields.)

Include a printout of all required computer programs.

Due Date: Tuesday, February 13, 2001 (in class)

1. *Mean and median of a lognormal.* Suppose Z is a Gaussian random variable with mean m_Z and variance σ_Z^2, and suppose $S = 10^{Z/10}$.

   (a) Show that the *median* of S is given by:

   $$\mu_S = 10^{m_Z/10}.$$

   Hint: The median μ_X of a (continuous) random variable X is defined by the equation $\mathrm{P}\{X \leq \mu_X\} = 1/2$.

   (b) Now find the mean m_S of S. How are μ_S and m_S related?
   Hint: Use Jensen's inequality

2. *Okumura-Hata model.* Consider the Okumura and Hata model (page 33 of the notes) for a *suburban area* of a *medium city.* Assume a BS antenna height of 50 m, an MS antenna height of 1 m and a carrier frequency of 900 MHz. Also assume antenna gains of 4 and 1.5 at the BS and MS, respectively.

   Use Matlab (or your favorite program) to plot the median link gain $\bar{G}(d)$ [dB] as a function of d, for d ranging from 1 km to 10 km, using a sample spacing of 100 m.

3. *Simulating shadow fading.* Simulate a sample path of a shadow fading process for equispaced samples on a straight line. Assume a correlation distance $D_c = 500$ m and a standard deviation of $\sigma_Z = 7.5$ dB (these values are from Gudmundson's paper). Use a sampling distance of 100 m, and generate enough samples to cover the range from 1 km and 10 km. Add this sample path to $\bar{G}(d)$ [dB] of problem 2 and plot the sum.

4. *Area reliability.* We showed in class that the area reliability of a cell of radius R is given by

   $$F_{\mathrm{u}} = \frac{2}{R^2} \int_0^R Q(a + b \ln \rho) \rho d\rho$$

   where a and b are as defined in class.

   (a) Show that F_{u} has the following closed-form expression:

   $$F_{\mathrm{u}} = Q(a + b \ln R) + \frac{\exp\left(\frac{2}{b^2} - \frac{2a}{b}\right)}{R^2} \left[1 - Q\left(a + b \ln R - \frac{2}{b}\right)\right].$$

   Hint: Use integration by parts

(b) Plot F_u versus R (in meters) for R ranging from 100 m to 10 km, $A_t = 145$ dBm, $B = 35.2$, $\sigma = 8$, and $P_{\text{thresh}} = 23$ dBm.

5. *Area reliability versus Edge Reliability.* The edge reliability of the cell is given by

$$F_{\text{edge}} = 1 - \text{P}_{\text{out}}(R) = Q(a + b \ln R).$$

(a) Show that for a given value of F_{edge}, the area reliability F_{u} is independent of both a and R, and is a function of b only.

(b) Using the fact that $b = B/(\sigma_Z \ln 10)$, show that F_{u} can be written as an explicit function of F_{edge} and $\hat{\sigma}$, where $\hat{\sigma} = 10\sigma_Z/B$ is the ratio of the shadow fading standard deviation and the path loss exponent. Plot F_{u} versus $\hat{\sigma}$ for $\hat{\sigma}$ ranging from 0 to 8, for F_{edge} values ranging from 0.5 to 0.95 in steps of 0.05. (Compare your plot with the one on page 108 of Rappaport's book.) Comment on the nature of your plot.

6. *Moments of lognormals.* Suppose X is a lognormal random variable with mean m_X and second moment δ_X, and suppose $Y = 10 \log X$ has mean m_X and variance σ_Y^2.

(a) Show that

$$m_X = \exp\left(\frac{(\beta\sigma_Y)^2}{2}\right) \exp(\beta m_Y) \quad \text{and}$$

$$\delta_X = \exp\left(2(\beta\sigma_Y)^2\right) \exp(2\beta m_Y),$$

where $\beta = \ln(10)/10$.

(b) From part (a) show that

$$m_Y = 20 \log m_X - 5 \log \delta_X, \quad \text{and that}$$

$$\sigma_Y^2 = \frac{1}{\beta}(10 \log \delta_X - 20 \log m_X).$$

7. *Worst-case SIR with shadow fading.* In class we derived an expression for the statistics of the worst-case SIR with shadow fading using a single-tier crude approximation for omni cells. Use this analysis to compute the probability that the worst-case SIR is less than 18 dB, i.e. $\text{P}\{\epsilon < 18\}$, for $N = 7$, $n = 4$, and $\sigma = 5$. (You would need a function that computes the cdf of a Gaussian distribution for this problem. In Matlab, you can modify the `erf` function appropriately.)

Extra Problems (you are not required to turn these in)

1. *Signal strength prediction.* Consider a mobile that is moving on a straight line path (not necessarily radial) at a constant velocity v. In order to make handoff decisions, the mobile periodically takes pilot power measurements from neighboring BS's. Let us assume that these power measurements are averaged to remove multipath fluctuations, so the the resulting sampled measurements only have a mean component and shadow fading. The k-th sample value of the pilot power (in dBm) from a particular BS is given by:

$$P_{r,k}[\text{dBm}] = \bar{P}_r(d_k) + Z_k = A_t - B \log d_k + Z_k,$$

where d_k is the distance from the BS at the k-th sampling time, and A_t includes the transmitted pilot power. Note that the d_k values are not necessarily equally spaced.

Let us assume isotropic shadow fading with exponential ACF. Since the velocity vector is constant, the random process $\{Z_k, k = 1, 2, \ldots\}$ is a stationary first-order *auto-regressive* (AR) process with

$$\mathrm{E}[Z_k Z_{k+m}] = \sigma_Z^2 a^{|m|},$$

where $a = \exp(-vt_s/D_c)$, t_s is the sampling time, and D_c is the correlation distance.

Handoff decisions are often based on signal strength prediction. Our goal here is to find the MMSE predictor of $P_{r,k+1}$ based on $P_{r,1}, P_{r,2}, \ldots, P_{r,k}$.

(a) Under the assumption that the d_k values are known, show that

$$\hat{P}_{r,k+1}^{\text{MMSE}} = \mathrm{E}[P_{r,k+1}|P_{r,1}, P_{r,2}, \ldots, P_{r,k}] = aP_{r,k} + (1-a)A_t - B\log\left(\frac{d_{k+1}}{d_k^a}\right)$$

and that the corresponding mean-squared error

$$\text{MSE} = \mathrm{Var}[P_{r,k+1}|P_{r,1}, P_{r,2}, \ldots, P_{r,k}] = (1-a^2)\sigma_Z^2.$$

(b) Discuss how you might address the prediction problem if the d_k values were unknown.

2. *Outage with macrodiversity.* Consider a mobile at the midpoint between two base stations in a cellular network. The received signals (in dB-W) from the base stations are given by

$$\begin{aligned} P_{r,1} &= A_t - B\log(D/2) + Z_1, \\ P_{r,2} &= A_t - B\log(D/2) + Z_2, \end{aligned}$$

where Z_1 and Z_2 are $\mathcal{N}(0, \sigma^2)$ random variables.

We define outage with macrodiversity to be the event that both $P_{r,1}$ and $P_{r,2}$ fall below a pre-specified threshold P_{thresh}.

(a) If Z_1 and Z_2 are independent, show that the outage probability is given by

$$\mathrm{P}_{\text{out}} = \left[Q\left(\frac{\Delta}{\sigma}\right)\right]^2,$$

where

$$\Delta \triangleq A_t - B\log(D/2) - P_{\text{thresh}}$$

is the fade margin at the edge of the cell.

(b) Now suppose Z_1 and Z_2 are correlated in the following way.

$$Z_1 = aY_1 + bY \quad \text{and} \quad Z_2 = aY_2 + bY,$$

where Y, Y_1 and Y_2 are *independent* $\mathcal{N}(0, \sigma^2)$ random variables, and a and b are such that $a^2 + b^2 = 1$.

Show that

$$\mathrm{P}_{\text{out}} = \int_{-\infty}^{\infty}\left[Q\left(\frac{\Delta + by\sigma}{a\sigma}\right)\right]^2 \frac{e^{-y^2/2}}{\sqrt{2\pi}}dy.$$

(c) Compare the outage probabilities of (a) and (b) for the special case of $a = b = 1/\sqrt{2}$, $\sigma = 8$ and $\Delta = 5$. (Use a numerical integration routine for P_{out} of (b).)