HOMEWORK ASSIGNMENT 3

Reading: Chap 4 of Rappaport, *CDMA: Principles of Spread Spectrum Communications*, by A. Viterbi.

Due Date: Tuesday, March 6, 2001 (in class)

1. *Small scale fading.* Consider the delay profile with the 13 paths, and the following parameters. The sequence of squared path gains $(\alpha_0^2, \alpha_1^2, \dots, \alpha_{12}^2)$ is given by

$$(0.5, .025, .075, 0.1, .075, .025, .025, .075, .025, .025, .02, .02, .01)$$

and the corresponding sequence of path delays $(\tau_0, \tau_1, \dots, \tau_{12})$ is given by:

$$(0, 1.2, 1.4, 1.6, 1.8, 2.0, 3.4, 3.6, 3.8, 4.0, 5.2, 5.4, 5.6)\ \mu\text{s}$$

   (a) Carefully sketch the delay profile $|c(t;\xi)|$ with the heights of the δ functions drawn approximately to scale to match the path gains.

   (b) Is there an LOS path? If so, what is the Rice factor?

   (c) Argue that a signal with bandwidth $W = 20$ kHz sees the channel as a flat fading channel, and write down an expression for the pdf $p_\alpha(\alpha)$ of the effective channel gain α.

   (d) Argue that a signal with bandwidth $W = 1$ MHz sees the channel as a frequency selective channel, and draw the tapped delay line model for $\frac{1}{W}$-spaced taps.

   (e) In part (d), if we allow the tap spacing to be arbitrary, what is the minimum number of taps needed and what are the corresponding tap delays? What is the (approximate) power gain corresponding to each tap in this model? Is the fading approximately uncorrelated across taps?

   (f) Suppose the carrier frequency $f_c = 1$ GHz, the mobile velocity is 60 km/hr, and the symbol rate is 10^5 symbols/s. Find the coherence time. Is the fading slow?

2. *Simulating Ricean flat fading.* Using the direct approach outlined on page 59 of the notes, simulate a Ricean flat fading process with Rice factor $\kappa = 1$. Assume that the LOS component arrives at angle $\theta_0 = \pi/4$, and that the diffuse components are uniformly distributed in angle and power (isotropic). Let the total number of diffuse components be 12, and assume $f_m = 60$ Hz.

   Plot the channel gain $|V(t)|$ in dB as a function of t, for t ranging from 0 to 250 ms.

   Explain how you could use the direct approach to simulate a frequency selective fading channel, assuming that the taps are uncorrelated.

3. *Binary fading model and diversity.* Consider BPSK signaling in AWGN of the form:

$$r(t) = \pm aA\cos(2\pi f_c t) + n(t), \quad 0 \le t \le T_s,$$

where $n(t)$ is additive white Gaussian noise with PSD $N_0/2$. Let $\mathcal{E} = A^2 T_s / N_0$. Then for a fixed we know (from ECE 359) that

$$\mathrm{P_b} = Q\left(\sqrt{a^2 \frac{\mathcal{E}}{N_0}}\right) .$$

Now suppose the "channel gain" a is random and can take on two values with probability mass function given by

$$\mathrm{P}\{a = 0\} = 0.1 \quad \text{and} \quad \mathrm{P}\{a = 2\} = 0.9.$$

(a) Determine the average probability of error $\overline{\mathrm{P}}_\mathrm{b}$. What value does $\overline{\mathrm{P}}_\mathrm{b}$ approach as $\mathcal{E}/N_0$ approaches infinity?

(b) Suppose the same signal is transmitted on two statistically *independent* channels with gains a_1 and a_2, where

$$\mathrm{P}\{a_1 = 0\} = \mathrm{P}\{a_2 = 0\} = 0.1 \quad \text{and} \quad \mathrm{P}\{a_1 = 2\} = \mathrm{P}\{a_2 = 2\} = 0.9.$$

The noises on the two channels are independent and identically distributed. The receiver performs coherent combining with equal weights. What is $\overline{\mathrm{P}}_\mathrm{b}$ in this case, and what value does it approach as $\mathcal{E}/N_0$ approaches infinity?

(c) Repeat part (b) for maximum ratio combining.

4. *Selection diversity.* Consider the reverse link of a cellular system. Suppose the transmitted signal is received at two basestations with SNR's $\gamma_{\mathrm{b},1}$ and $\gamma_{\mathrm{b},2}$, respectively. It is usually impractical to do coherent combining across basestations, and so instead the basestation with larger SNR is chosen to demodulate the received signal. This is called selection diversity, and it is used for soft handoff in CDMA cellular systems.

   Consider the symmetric situation where the channels to both basestations undergo identically distributed Rayleigh fading, i.e., $\gamma_{\mathrm{b},1}$ and $\gamma_{\mathrm{b},1}$ are i.i.d. exponential random variable with mean $\overline{\gamma}_\mathrm{b}$. Compute an expression for the pdf of the SNR with selection diversity, and find its mean.

5. *CDMA reverse link capacity analysis.* Use the expressions for $\mathrm{E}[X]$ and $\mathrm{Var}(X)$ given on slide 14 of the CDMA notes to solve for c_E, and derive the expression given on slides 15 and 16.

6. *CDMA versus TDMA/FDMA.* Consider the design of a cellular system for a total available bandwidth of $W = 5$ MHz, and an information bit rate of $R = 14.4$ kbps. Assume a GoS of 0.01.

   (a) Compute the Erlang capacity/cell for a TDMA/FDMA system that uses three-sectored cells, with a reuse efficiency of $N = 4$ and bandwidth efficiency of 1 bit/s/Hz. Consider both the cases of zoning and no zoning.

   (b) Compute the reverse link capacity of a CDMA system with admission threshold $\eta = 0.3$, voice activity factor $\rho = 0.45$, and the required γ_b having mean 7 dB and standard deviation of 2.5 dB. Find your answer in Erlangs/cell assuming a three-sectored cell, with a sectorization gain of 2.5.