HOMEWORK ASSIGNMENT 2

Reading: Lecture notes (lectures 4-7), Proakis (Chapter 4), papers referenced in lecture notes.

Due Date: Tuesday, September 26, 2000 (in class)

1. (10 pts) *Simplex Signal Set.* Consider a set of M orthogonal signal waveforms $\{s_m(t)\}_{m=0}^{M-1}$ that have energy $\mathcal{E}$. Define a new set of M waveforms as

$$s'_m(t) = s_m(t) - \frac{1}{M}\sum_{\ell=0}^{M-1} s_\ell(t)\ , \quad m = 0, 1, \ldots, M-1\ .$$

Show that the M signal waveforms have equal energy, given by $\left(1 - \frac{1}{M}\right)\mathcal{E}$, and are equally correlated, with correlation coefficients $\rho_{km} = -\frac{1}{M-1}$ and distances $d_{km} = \sqrt{2\mathcal{E}}$.

2. (20 pts) *Signal Constellation Optimization.* Consider the QAM signal constellation shown in Figure 1.

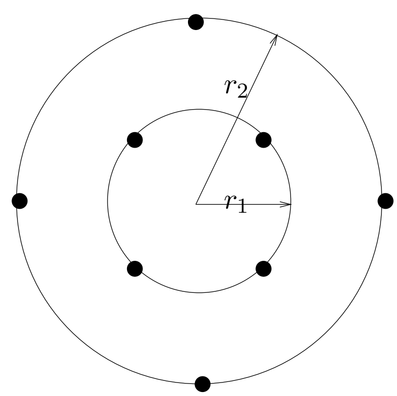

Figure 1: Signal constellation for Problem 2.

(a) (8 pts) Evaluate $\zeta = \frac{d^2_{\min}}{\mathcal{E}_b}$ as a function of $a = \frac{r_2}{r_1}$.

(b) (8 pts) Maximize $\zeta(a)$ over $a \geq 1$ to find the best constellation.

(c) (4 pts) Compare the result in (b) with ζ for 8-PSK.

3. (10 pts) *Phase Trellis for CPM.* Determine the number of states required and draw the state trellis for *binary* CPM with

(a) (4 pts) $h = \frac{1}{3}$, full response, and $q(t) = \frac{t}{T}\,\mathbb{1}_{\{t\in[0,T)\}} + \mathbb{1}_{\{t\in[T,\infty)\}}$.

(b) (6 pts) $h = \frac{1}{2}$, partial response, and $q(t) = \frac{t}{4T}\,\mathbb{1}_{\{t\in[0,2T)\}} + \frac{1}{2}\mathbb{1}_{\{t\in[2T,\infty)\}}$.

4. (15 pts) *Alternative derivation of the PSD of linearly modulated signals.* Let $\{B_n\}$ be a zero-mean discrete-time WSS complex random process with ACF $R_B(k) = \mathrm{E}[B_{n+k}B_n^\star]$ that represents a sequence of digital symbols. Define the PSD of $\{B_n\}$ by $S_B(f) = \sum_{k=-\infty}^{\infty} R_B(k)e^{-j2\pi fk}$ as we did in class. The PSD of the cyclostationary linearly modulated process:

$$s(t) = \sum_{n=-\infty}^{\infty} B_n g(t - nT_s)$$

was derived in class by averaging the periodic ACF $R_s(t+\tau, t)$ over the period T_s, and then evaluating the Fourier transform of the average ACF. An alternative approach is to change the cyclostationary process into a stationary one by adding a random delay Δ (independent of $\{B_n\}$) that is uniformly distributed on $[0, T_s]$ to produce:

$$\bar{s}(t) = \sum_{n=-\infty}^{\infty} B_n g(t - nT_s - \Delta)$$

and defining the PSD of $s(t)$ to be the PSD of the stationary process $\bar{s}(t)$. Show that this method produces the same PSD as the one derived in class, i.e., that

$$S_{\bar{s}}(f) = \frac{S_B(fT_s)|G(f)|^2}{T_s}$$

5. (25 pts) *Bandwidth of Digitally Modulated Signals.* The PSD of $S_s(f)$ of a linearly modulated digital signal $s(t)$ is determined by $|G(f)|^2$, where $G(f)$ is the Fourier transform of the pulse shaping waveform $g(t)$. There are various ways to define the *bandwidth* B of a digitally modulated signal in terms of $S_s(f)$.

   - *Null-to-Null Bandwidth:* The width of the main spectral lobe.
   - *3-dB Bandwidth:* The width of the interval between the two frequencies at which the PSD is 3 dB below its peak value.
   - *x-% Essential Bandwidth:* The width of the (smallest) band of frequencies that contains x-% of the total signal energy.

   To compare the bandwidths of modulation schemes that use different constellation sizes, it is convenient to normalize the bandwidth by the bit rate $1/T_b$. The normalized bandwidth $\beta = BT_b$.

   (a) (18 pts) Compute the normalized bandwidths based on all three definitions for the following digital modulation schemes:

   - QPSK with rectangular signaling pulse:

$$g(t) = \frac{1}{\sqrt{T_s}} p_{T_s}(t)\,.$$

   - BPSK with time-domain raised cosine (TDRC) signaling pulse:

$$g(t) = \sqrt{\frac{2}{3T_s}}\left[1 + \cos\frac{2\pi}{T_s}\left(t - \frac{T_s}{2}\right)\right] p_{T_s}(t)\,.$$

- MSK

Hint: See pg 208 of Proakis for the Fourier transform of the TDRC pulse. Compute the essential bandwidth numerically on Matlab.

(b) (7 pts) Plot the PSD in dB (normalized so that the peak is 0 dB) versus normalized frequency fT_b for all three signaling schemes.

6. (20 pts) *Spectral shaping via precoding.* Let $\{B_n\}$ be a sequence of i.i.d. zero mean complex random variables with $\mathrm{E}[|B_n|^2] = 1$, and suppose we form the sequence $\{D_n\}$ via precoding as

$$D_n = B_n + \alpha B_{n-1}$$

and modulate the sequence $\{D_n\}$ to form the signal:

$$Y(t) = \sum_{n=-\infty}^{\infty} D_n g(t - nT_s - \Delta)$$

where Δ is uniform on $[0, T_s]$ and independent of $\{D_n\}$.

(a) (10 pts) Find α so as to put a spectral null at $f = 1/T$ (i.e., to make $S_Y(1/T) = 0$) regardless of the choice of $g(t)$.

(b) (10 pts) Find a precoding operation that will put spectral nulls at *all* integer multiples of $1/4T$.