Wideband Multiantenna Wireless Channels: Statistical Modeling, Analysis and Simulation

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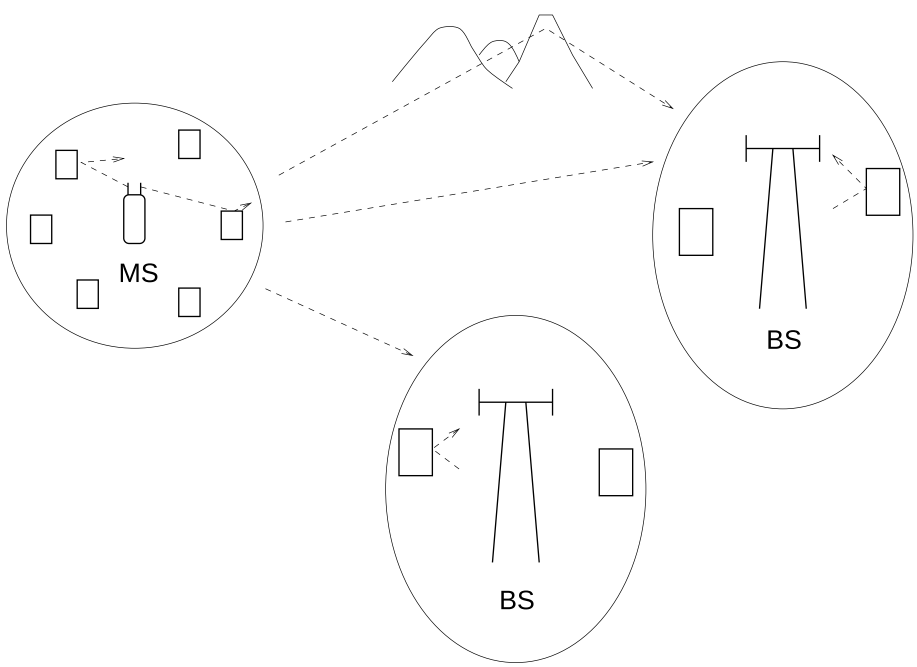
IEEE VTC 2001 Tutorial, October 7, 2001

Overview

- Channel modeling for mobile wireless communications has been studied for over 50 years
- Emphasis of much of this work (see, e.g., Jakes[1])
  - narrowband channels
  - first order statistical modeling
- *Goal of this tutorial:* to provide updated review of channel modeling covering
  - spatial/time correlation
  - role of angular spread
  - wideband channels
  - multiantenna systems

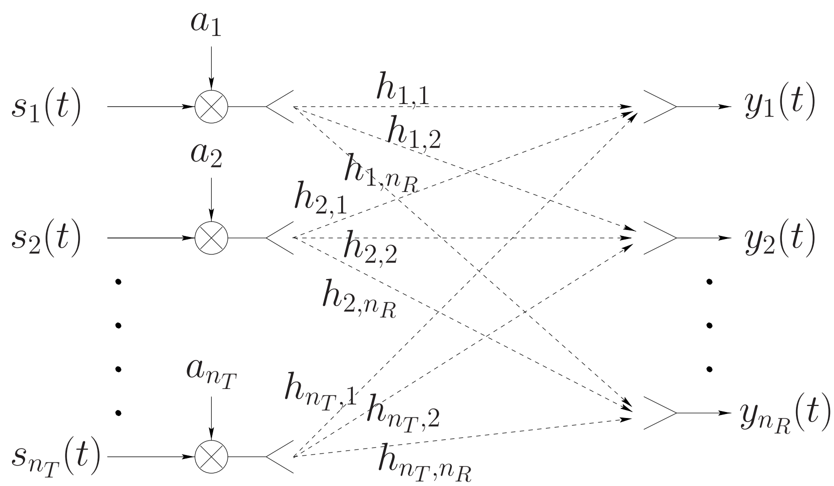
Part I: Introduction and Background

Wireless Propagation Channel



- Channel between mobile and BS network is multipath, multiantenna channel that is described by linear channel matrix

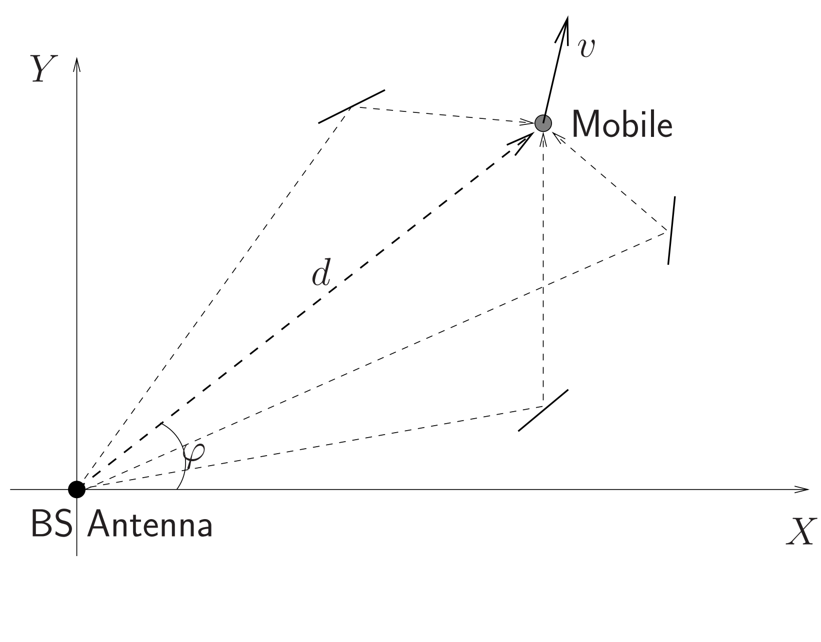
Linear Matrix Channel Between Tx and Rx



$$y_j(t) = \sum_{i=1}^{n_T} a_i \int_0^{\tau_{\mathsf{DS}}} s_i(t-\xi) h_{i,j}(\mathbf{L};\xi) d\xi$$

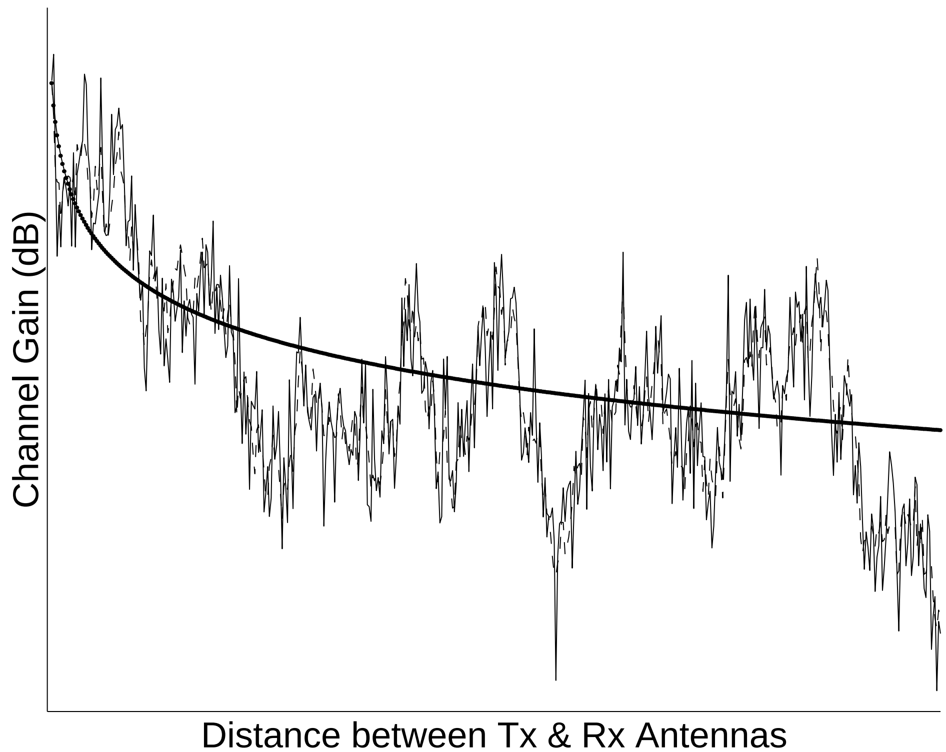
- Mobile moves $\Longrightarrow$ location $\mathbf{L}$ changes with t
- Key to modeling: understanding variations of $h_{i,j}(\mathbf{L},\xi)$ and relationship between various $\{h_{i,j}\}$

Multipath Channel between pair of Tx & Rx Antennas

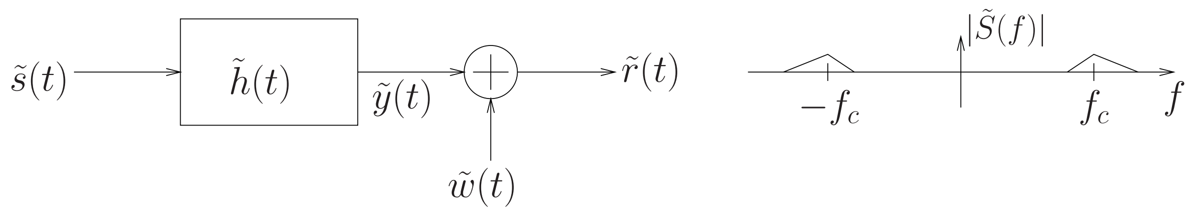


Multipath channel seen at location (d,φ)

Fading on link between pair of Tx & Rx Antennas

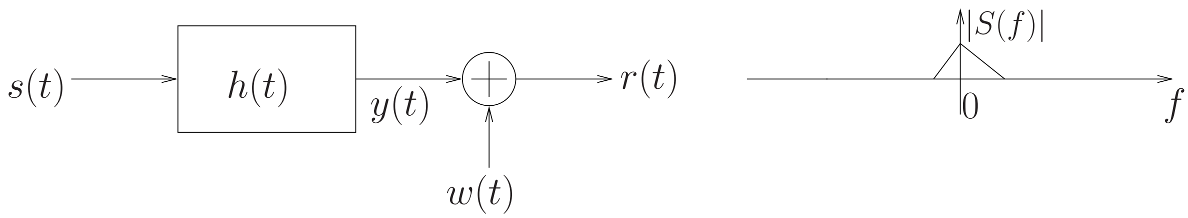


Channel Model for Point-to-Point Communications



Real bandpass channel model

$$\equiv$$



Complex baseband channel model

Passband-Baseband relationships (Proakis[2])

Passband to Baseband	Baseband to Passband
$S(f) = \tilde{S}_+(f+f_c) = \sqrt{2}u(f+f_c)\tilde{S}(f+f_c)$	$\tilde{S}(f) = \frac{S(f-f_c)+S^\star(-f-f_c)}{\sqrt{2}}$
$s(t) = \tilde{s}_+(t)e^{-j2\pi f_c t} = \frac{1}{\sqrt{2}}[\tilde{s}(t) + j\hat{\tilde{s}}(t)]e^{-j2\pi f_c t}$	$\tilde{s}(t) = \mathrm{Re}[\sqrt{2}s(t)e^{j2\pi f_c t}]$
$h(t) = \frac{1}{\sqrt{2}}\tilde{h}_+(t)e^{-j2\pi f_c t}$	$\tilde{h}(t) = 2\mathrm{Re}[h(t)e^{j2\pi f_c t}]$
$w(t) = \tilde{w}_+(t)e^{-j2\pi f_c t} = \frac{1}{\sqrt{2}}[\tilde{w}(t) + j\hat{\tilde{w}}(t)]e^{-j2\pi f_c t}$	$\tilde{w}(t) = \mathrm{Re}[\sqrt{2}w(t)e^{j2\pi f_c t}]$

Properties of Complex Baseband Additive Noise $w(t)$

① If $\tilde{w}(t)$ is zero mean, Gaussian, then $w(t)$ is zero mean, complex Gaussian

② Let $w(t) = w_I(t) + jw_Q(t)$. If $\tilde{w}(t)$ is wide sense stationary (WSS), then $w_I(t)$ and $w_Q(t)$ are jointly WSS, and

$$R_{w_I}(\tau) = R_{w_Q}(\tau)\ , \quad \text{and} \quad R_{w_I w_Q}(\tau) = -R_{w_Q w_I}(\tau)$$

A complex process with this property is called *proper complex*

Define ACF of $w(t)$ by

$$R_w(\tau) = \mathsf{E}[w(t+\tau)w^\star(t)]$$

Then from proper complex property

$$R_w(\tau) = 2R_{w_I}(\tau) + j2R_{w_Q w_I}(\tau)$$

Proper Complex Processes (Neeser & Massey [3])

- Let $\boldsymbol{Y} = \boldsymbol{Y}_I + j\boldsymbol{Y}_Q$ be a complex random vector with

$$\Sigma_{\boldsymbol{Y}_I} = \mathsf{E}[(\boldsymbol{Y}_I - \boldsymbol{m}_{Y_I})(\boldsymbol{Y}_I - \boldsymbol{m}_{Y_I})^\top], \quad \Sigma_{\boldsymbol{Y}_Q} = \mathsf{E}[(\boldsymbol{Y}_Q - \boldsymbol{m}_{Y_Q})(\boldsymbol{Y}_Q - \boldsymbol{m}_{Y_Q})^\top$$

$$\Sigma_{\boldsymbol{Y}_I\boldsymbol{Y}_Q} = \mathsf{E}[(\boldsymbol{Y}_I - \boldsymbol{m}_{Y_I})(\boldsymbol{Y}_Q - \boldsymbol{m}_{Y_Q})^\top], \quad \Sigma_{\boldsymbol{Y}_Q\boldsymbol{Y}_I} = \mathsf{E}[(\boldsymbol{Y}_Q - \boldsymbol{m}_{Y_Q})(\boldsymbol{Y}_I - \boldsymbol{m}_{Y_I}$$

- Complex covariance

$$\Sigma_{\boldsymbol{Y}} = \mathsf{E}\left[(\boldsymbol{Y} - \boldsymbol{m}_Y)(\boldsymbol{Y} - \boldsymbol{m}_Y)^\dagger\right] = \left(\Sigma_{\boldsymbol{Y}_I} + \Sigma_{\boldsymbol{Y}_Q}\right) + j\left(\Sigma_{\boldsymbol{Y}_Q\boldsymbol{Y}_I} - \Sigma_{\boldsymbol{Y}_I\boldsymbol{Y}_Q}\right)$$

- Complex pseudo-covariance

$$\tilde{\Sigma}_{\boldsymbol{Y}} = \mathsf{E}\left[(\boldsymbol{Y} - \boldsymbol{m}_Y)(\boldsymbol{Y} - \boldsymbol{m}_Y)^\top\right] = \left(\Sigma_{\boldsymbol{Y}_I} - \Sigma_{\boldsymbol{Y}_Q}\right) + j\left(\Sigma_{\boldsymbol{Y}_Q\boldsymbol{Y}_I} + \Sigma_{\boldsymbol{Y}_I\boldsymbol{Y}_Q}\right)$$

- **Definition 1.** $\boldsymbol{Y}$ is a *proper* complex vector if $\tilde{\Sigma}_Y = 0$, i.e.

$$\boxed{\Sigma_{\boldsymbol{Y}_I} = \Sigma_{\boldsymbol{Y}_Q} \quad \text{and} \quad \Sigma_{\boldsymbol{Y}_Q\boldsymbol{Y}_I} = -\Sigma_{\boldsymbol{Y}_I\boldsymbol{Y}_Q}}$$

  For proper complex $\boldsymbol{Y}$,

$$\boxed{\Sigma_{\boldsymbol{Y}} = 2\Sigma_{\boldsymbol{Y}_I} + j2\Sigma_{\boldsymbol{Y}_Q\boldsymbol{Y}_I}}$$

- *The scalar case:* If Y is proper complex scalar, then Y_I and Y_Q are uncorrelated and

$$\sigma_Y^2 = \mathsf{E}[|Y - m_Y|^2] = 2\sigma_{Y_I}^2 = 2\sigma_{Y_Q}^2$$

- If $\boldsymbol{Y}$ is proper and Gaussian it is said to be proper complex Gaussian (PCG) or *circularly* complex Gaussian

Useful results on PCG random vectors

- **Result 1.** If $\boldsymbol{Y}$ is a PCG vector, the pdf of Y is given by

$$p_{\boldsymbol{Y}}(\boldsymbol{y}) := p_{\boldsymbol{Y}_I\boldsymbol{Y}_Q}(\boldsymbol{y}_I, \boldsymbol{y}_Q) = \frac{1}{\pi^n|\Sigma_Y|}\exp\left\{-(\boldsymbol{y}-\boldsymbol{m}_Y)^\dagger \Sigma_Y^{-1}(\boldsymbol{y}-\boldsymbol{m}_Y)\right\}$$

  The pdf of $\boldsymbol{Y}$ has circular symmetry.

  Notation: $\boldsymbol{Y} \sim \mathcal{CN}(\boldsymbol{m}_Y, \Sigma)$

- **Result 2.** If $\boldsymbol{Y}$ is PCG, then $\boldsymbol{Z} = \boldsymbol{A}\boldsymbol{Y} + \boldsymbol{b}$ is also PCG.
  The circular property is preserved under linear transformations.

- **Result 3. (Central Limit Theorem).** If $\{\boldsymbol{Y}_k\}$ is a sequence of independent proper complex random vectors (not necessarily Gaussian), then the sum $\sum_k \boldsymbol{Y}_k$ (after appropriate normalization) converges to a PCG vector.

Proper Complex and PCG processes

- Covariance and pseudo-covariance functions of $Y(t) = Y_I(t) + jY_Q(t)$:

$$C_Y(t+\tau, t) = \mathsf{E}\left[(Y(t+\tau) - m_Y(t+\tau))(Y(t) - m_Y(t))^\star\right]$$

$$\tilde{C}_Y(t+\tau, t) = \mathsf{E}\left[(Y(t+\tau) - m_Y(t+\tau))(Y(t) - m_Y(t))\right]$$

- **Definition 2.** $\{Y(t)\}$ is *proper complex* if $\tilde{C}_{YY}(t+\tau,t) = 0$, i.e.

$$\boxed{C_{Y_I}(t+\tau,t) = C_{Y_Q}(t+\tau,t) \quad \text{and} \quad C_{Y_QY_I}(t+\tau,t) = -C_{Y_IY_Q}(t+\tau,t)}$$

  For proper complex $\{Y(t)\}$,

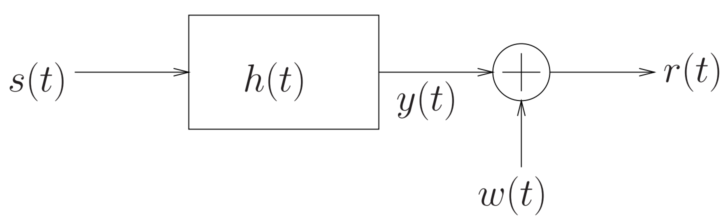
$$\boxed{C_Y(t+\tau,t) = 2C_{Y_I}(t+\tau,t) + j2C_{Y_QY_I}(t+\tau,t)}$$

- **Definition 3.** A proper complex process $\{Y(t)\}$ is PCG if, for all n, and all $t_1, t_2, \dots, t_n$, the samples $Y(t_1), Y(t_2), \dots, Y(t_n)$ are jointly PCG .

- **Result 4.** If a PCG process $\{Y(t)\}$ is passed through a linear system, the output is PCG as well.

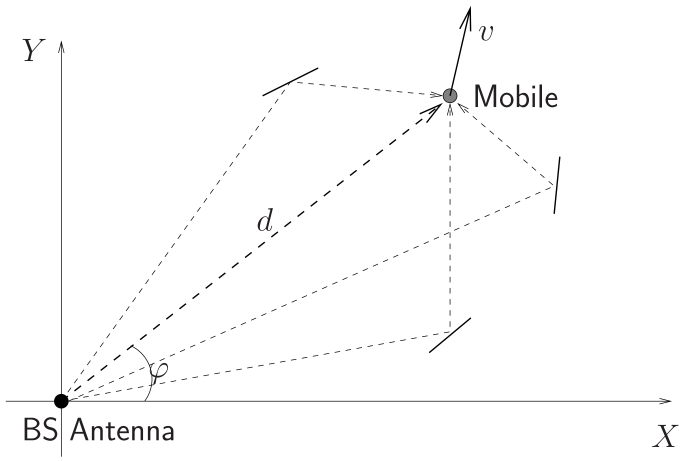
Part II: From Point-to-Point Communications Model to Mobile Communications Channel Model

Complex Baseband Model for Point-to-Point Communications

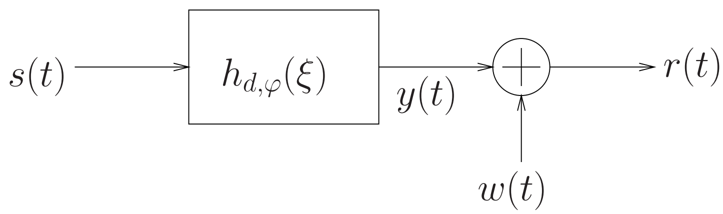


- If bandpass noise $\{\tilde{w}(t)\}$ is AWGN with PSD $N_0/2$, then baseband noise $\{w(t)\}$ is a PCG white process with
  - $R_w(\tau) = \mathsf{E}[w(t+\tau)w^\star(t)] = N_0\delta(\tau)$
  - $R_{w_I w_Q}(\tau) = R_{w_Q w_I}(\tau) = 0 \implies \{w_I(t)\}$; $\{w_Q(t)\}$ independent
  - $R_{w_I}(\tau) = R_{w_Q}(\tau) = \frac{1}{2}R_w(\tau) = \frac{N_0}{2}\delta(\tau)$
  - $S_{w_I}(f) = S_{w_Q}(f) = \frac{1}{2}S_w(f) = \frac{N_0}{2}$ for all f

Mobile Comm. Channel Model for link between Tx/Rx Pair



Causal LTI system corresponding to multipath profile at (d, φ)



Multipath Profile and Channel Impulse Response

- At location (d, φ), n-th path connecting Tx and Rx antennas has
  - amplitude gain of $\beta_n(d, \varphi)$
  - delay of $\tau_n(d, \varphi)$
  - carrier phase shift of $\phi_n(d, \varphi) = -2\pi f_c \tau_n(d, \varphi) +$ constant.

- Thus

$$y(t) = \sum_n \beta_n(d, \varphi)\, e^{j\phi_n(d,\varphi)}\, s(t - \tau_n(d, \varphi))$$

$$\Longrightarrow \quad \boxed{h_{d,\varphi}(\xi) = \sum_n \beta_n(d, \varphi)\, e^{j\phi_n(d,\varphi)}\, \delta(\xi - \tau_n(d, \varphi))}$$

- As MS moves, (d, φ) varies with time $\Longrightarrow$ Time varying channel

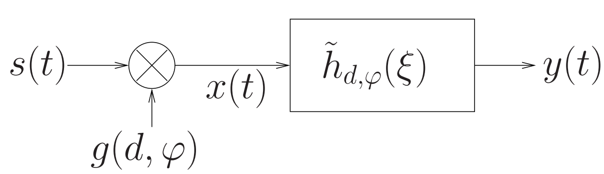
Two scales of variation

- *Small scale variations*
  - ◦ movements of the order of few carrier wavelengths
  - ◦ multipath profile roughly constant – # paths, strengths, delays
  - ◦ channel variations due to phase differences in paths
  - ◦ average power gain in vicinity of (d, φ) is $G(d, \varphi) = \sum_n \beta_n^2$
  - ◦ typical values: $f_c = 1$ G Hz $\implies$ $\lambda_c = 0.3$ m.

- *Large scale variations*
  - ◦ variations in $G(d, \varphi)$ that result from changing multipath profile
  - ◦ scale of distance between objects in environment
  - ◦ typical values: 10's of meters (outdoor)

Separation of scales

- *Small and large scale variations separated*



  - ◦ $\tilde{h}_{d,\varphi}(\xi)$ is $h_{d,\varphi}(\xi)$ normalized to have average power gain of 1, i.e.

$$\tilde{h}_{d,\varphi}(\xi) = \sum_n \tilde{\beta}_n(d, \varphi)\, e^{j\phi_n(d,\varphi)}\, \delta(\xi - \tau_n(d, \varphi))$$

with

$$\sum_n \tilde{\beta}_n^2(d, \varphi) = 1$$

  - ◦ amplitude gain (real) $g(d, \varphi)$

$$G(d, \varphi) = g^2(d, \varphi)$$

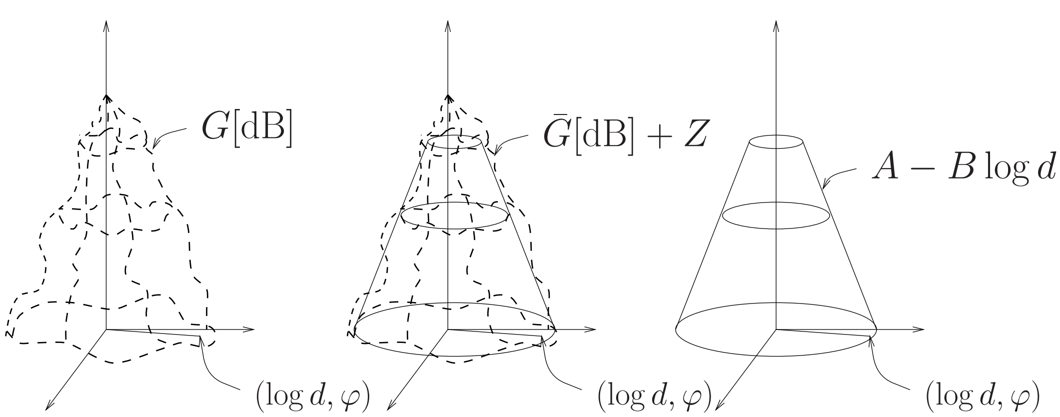
Part III: Large Scale Variations

Large Scale Variations in Gain

- Measurements of $G(d, \varphi)$ obtained from drive tests or simulations
- Physics $\implies$ variation of G should be roughly of the form ad^{-b}.

$$G(d, \varphi)[\mathrm{dB}] = \bar{G}(d)[\mathrm{dB}] + Z(d, \varphi) = A - B \log d + Z(d, \varphi)$$

- "Best" A and B obtained by curve fitting $G(d, \varphi)$ measurements. For generic estimates for various scenario, see e.g. Okumura&Hata [4]



- Piece-wise linear model for $\bar{G}(d)[\mathrm{dB}]$ will produce smaller error Z

Shadow Fading

- $Z(d, \varphi)$ is the variation in gain around monotonic $\bar{G}(d)\mathrm{dB}]$
- Variations due to loss or gain of paths $\implies$ Z is called *shadow fading*
- Model for $\{Z(d, \varphi)\}$ – *deterministic* vs. *stochastic*
- Stochastic Model
  - $\{Z(d, \varphi)\}$ is *random field* in spatial location (d, φ)
  - For a least-squares curve fit E$[Z(d, \varphi)] \approx 0$
  - Central Limit Theorem $\implies$ $\{Z(d, \varphi)\} \sim$ Gaussian field
  - First order statistics: $Z \sim \mathcal{N}(0, \sigma_Z^2)$

Lognormal statistics

- Shadow fading as multiplicative factor

$$G(d, \varphi)[\mathrm{dB}] = Z(d, \varphi) + \bar{G}(d)[\mathrm{dB}] \quad \implies \quad G(d, \varphi) = S(d, \varphi)\bar{G}(d)$$

where

$$S(d, \varphi) = 10^{\frac{Z(d,\varphi)}{10}} \text{ and } Z(d, \varphi) = 10 \log S(d, \varphi)$$

- Z is Gaussian $\implies$ S is *lognormal*

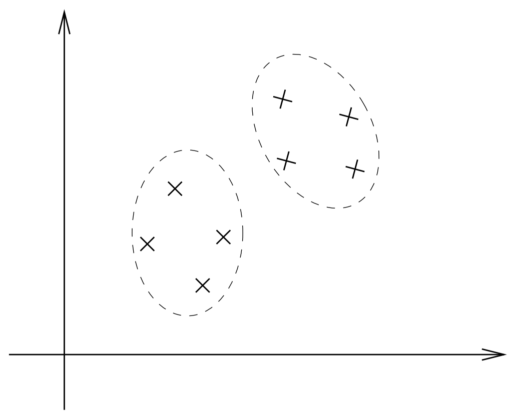
$$p_S(s) = \frac{10}{s \ln(10)} \frac{1}{\sigma_Z \sqrt{2\pi}} \exp\left(-\frac{(10 \log s)^2}{2\sigma_Z^2}\right) u(s)$$

- Why $\bar{G}(d)$ is referred to as the *median* link gain:

mean of $G(d, \varphi) \neq \bar{G}(d)$, but median of $G(d, \varphi) = \bar{G}(d)$

Random Field Model

- **Definition 4.** [5] A random field $V(x,y)$ is said to be *homogeneous* and *isotropic* if the joint distribution of $V(x_1,y_1),\dots,V(x_n,y_n)$ is unchanged by *rigid body movements* of $(x_1,y_1),\dots,(x_n,y_n)$



- For a homogeneous and isotropic random field

  $$\mathsf{E}[V(x_1,y_1)\,V(x_2,y_2)] = R_V(\Delta), \quad \text{where } \Delta = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}.$$

Correlation Model for Shadow Fading

- Stochastic Model for Z over cell (sector):

$\{Z(x,y)\}$ is zero mean, Gaussian, homogeneous, isotropic, random field

- AR-1 Model for $R_Z(\Delta)$ (Gudmundson [6]):

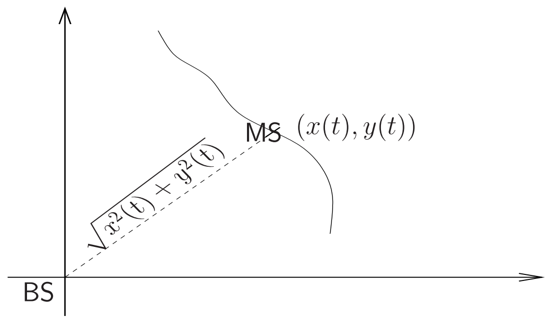
  $$R_Z(\Delta) = \sigma_Z^2 \exp\left(-\frac{|\Delta|}{D_c}\right)$$

  where D_c is the *correlation distance*

- Value of D_c can be determined via field trials or simulations

- Higher order AR models

Shadow Fading from Perspective of User Moving in Cell



- As (x, y) change with t, the user sees time variations in power gain

$$G_t = \bar{G}\left(\sqrt{x^2(t)+y^2(t)}\right) + \tilde{Z}(t)$$

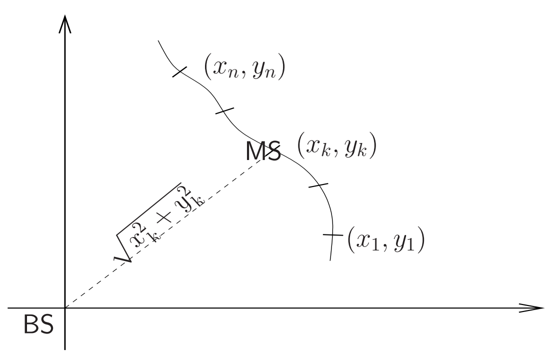
where $\{\tilde{Z}(t)\}$ is zero mean Gaussian random process with

$$\mathsf{E}[\tilde{Z}(t)\tilde{Z}(t+\tau)] = R_Z\left(\sqrt{(x(t+\tau)-x(t))^2 + (y(t+\tau)-y(t))^2}\right)$$

- For user with constant velocity, $\{\tilde{Z}(t)\}$ is stationary with

$$R_{\tilde{Z}}(\tau) = R_Z(v\tau) = \sigma_Z^2 \exp\left(-\frac{v|\tau|}{D_c}\right) \text{ (for AR-1 model).}$$

Discrete Time (Distance) Model



- Link gain at k-th sample

$$G_k = \bar{G}\left(\sqrt{x_k^2+y_k^2}\right) + Z_k$$

- For samples equally spaced with distance d_s on a straight line trajectory, the discrete process $\{Z_k\}$ is *stationary*, zero mean, Gaussian with

$$R_Z[m] = \mathsf{E}[Z_k Z_{k+m}] = R_Z(md_s)$$
$$= \sigma_Z^2\, e^{-\frac{|m|d_s}{D_c}} = \sigma_Z^2\, a^{|m|} \text{ (for AR-1 model)}$$

where $a = e^{-\frac{d_s}{D_c}}$ is the *correlation coefficient*

Simulating Shadow Fading

- To generate n samples of $\{Z(d,\phi)\}$ (or $\{Z(x,y)\}$) over a given region
  ① Create covariance matrix Σ of samples using $R_Z(\Delta)$
  ② Set $[Z_1 \dots Z_n]^\top = \mathbf{Z} = \Sigma^{1/2}\mathbf{W}$, with $\mathbf{W} \sim \mathcal{N}(0, \mathbf{I})$
- To generate stationary discrete time process Z_k for equidistant samples on a straight line
  ① Compute the Z-transform $S_Z(z)$ of $R_Z[m]$
  ② Find $H(z)$ s.t. $H(z)H(z^{-1}) = S_Z(z)$ using spectral factorization
  ③ Filter sequence $\{W_k\}$ of i.i.d. $\mathcal{N}(0,1)$'s (WGN):

$$\{W_k\} \longrightarrow \boxed{H(z)} \longrightarrow \{Z_k\} \qquad H(z) = \frac{\sigma_Z\sqrt{1-a^2}}{1-az^{-1}}$$

for AR-1 model

Also, for AR-1 model

$$Z_0 = \sigma_Z W_0$$
$$Z_{k+1} = aZ_k + \sigma_Z\sqrt{1-a^2}W_k, \quad k = 0, 1, \dots$$

Applications of Large Scale Variation Models

- Large scale gain variations in a cell (sector) can be parameterized using
  ① median gain parameters A and B
  ② shadow fading variance σ_Z^2
  ③ correlation parameters (e.g. D_c)
- Applications include:
  ① Cellular area reliability and edge reliability
  ② SIR in channelized systems with shadow fading
  ③ Capacity and coverage analysis of CDMA systems with or without soft handoff
  ④ Channel gain prediction for radio resource management using pilot measurements – exploit correlation structure for prediction
  ⑤ Performance analysis of radio resource management schemes

Part IV: Small Scale Variations in Gain – Basics

Small Scale Variations

- Small scale variations are captured in $h_{d,\varphi}$

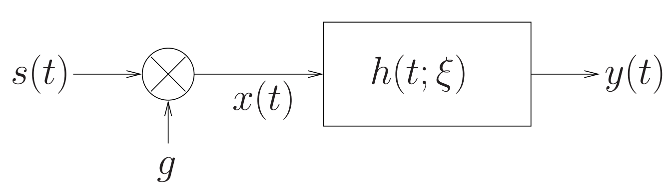
$$h_{d,\varphi}(\xi) = \sum_n \beta_n(d,\varphi)\, e^{j\phi_n(d,\varphi)}\, \delta(\xi - \tau_n(d,\varphi))$$

  where $\{\beta_n(d,\varphi)\}$ are normalized so that $\sum_n \beta_n^2(d,\varphi) = 1$

- As (d,φ) changes with t, channel becomes time varying:

$$h(t;\xi) := h_{d(t),\varphi(t)}(\xi) = \sum_n \beta_n(t)\, e^{j\phi_n(t)}\, \delta(\xi - \tau_n(t))$$

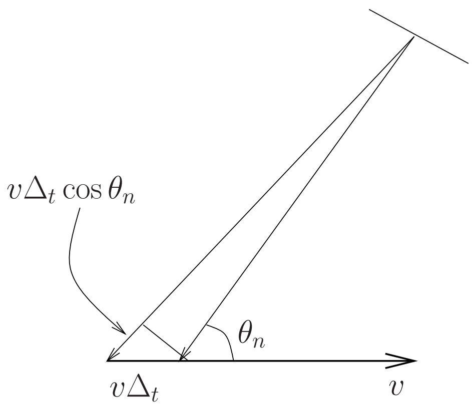
- Assume $g(d,\varphi)$ is constant over small scale



$$\boxed{y(t) = \int_0^\infty h(t;\xi)x(t-\xi)d\xi}$$

Doppler shifts in phase

- For movements of the order a few wavelengths
  - $\{\beta_n(t)\}$ and $\{\tau_n(t)\}$ are roughly constant
  - **BUT** $\phi_n(t) = -2\pi f_c \tau_n(t) + \text{const.}$ changes significantly
- Doppler shift is function of angle θ_n of path w.r.t. velocity vector



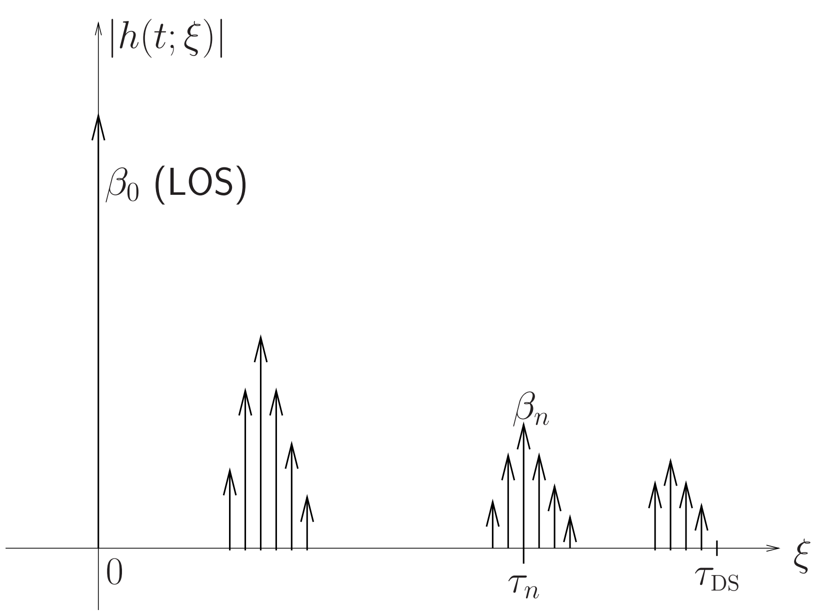
$$\phi_n(t+\Delta_t) - \phi_n(t) \approx \frac{2\pi f_c v \Delta_t \cos\theta_n}{c} = \frac{2\pi v \Delta_t \cos\theta_n}{\lambda_c} = 2\pi f_{\max} \Delta_t \cos\theta_n$$

- $f_{\max} = v/\lambda_c$ is the *maximum Doppler frequency*

Delay Profile of Channel

- β_n and τ_n are roughly independent of t, i.e. time variations are mainly due to changes in ϕ_n

$$\Longrightarrow \quad h(t;\xi) = \sum_n \beta_n \, e^{j\phi_n(t)} \, \delta(\xi - \tau_n)$$



Part V: Flat Fading

Frequency-Nonselective (Flat) Fading

- **Definition 5.** The quantity $\tau_{\text{DS}} = \max \tau_n - \min \tau_n$ is called the *delay spread* of the channel.

  w.l.o.g. assume $\min \tau_n = 0$. Then $\max \tau_n = \tau_{\text{DS}}$ and

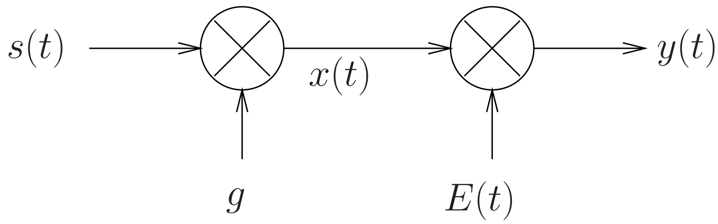
$$\boxed{y(t) = \int_0^{\tau_{\text{DS}}} h(t;\xi)x(t-\xi)d\xi}$$

- If passband bandwidth of $s(t)$, $W \ll \frac{1}{\tau_{\text{DS}}}$, then $x(t)$ is roughly constant over time intervals of order of τ_{DS}

$$\Longrightarrow \; y(t) \approx x(t)\int_0^{\tau_{\text{DS}}} h(t;\xi)d\xi = x(t)\sum_n \beta_n e^{j\phi_n(t)} = x(t)E(t)$$

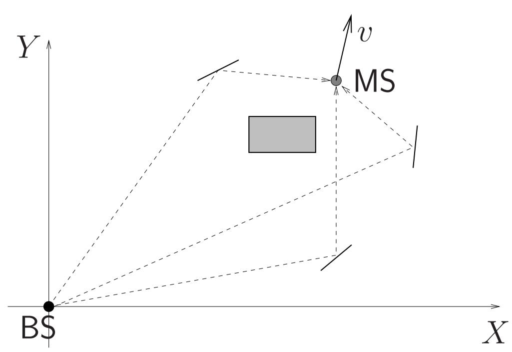
$$\Longrightarrow \; \boxed{h(t;\xi) \approx E(t)\delta(\xi)}$$

Channel Model for Flat Fading



$$\boxed{E(t) = \int_0^{\tau_{\mathrm{DS}}} h(t;\xi)d\xi = \sum_n \beta_n e^{j\phi_n(t)}}$$

Purely Diffuse (no LOS) Scattering – Rayleigh fading



- No LOS path or no single path dominates all other paths
- If we model $\{\phi_n\}$ as independent random $\mathrm{Unif}[0, 2\pi]$, then $\{E(t)\}$ is zero mean process
- The process $\{\beta_n e^{j\phi_n(t)}\}$ is *proper* complex
- If number of paths is large, by CLT (Result 3),

$$\boxed{\{E(t)\} \text{ is a Proper Complex Gaussian (PCG) random process}}$$

First Order Statistics for Purely Diffuse Scattering

- For fixed t, $E(t) = E_I(t) + jE_Q(t)$ is PCG r.v. with

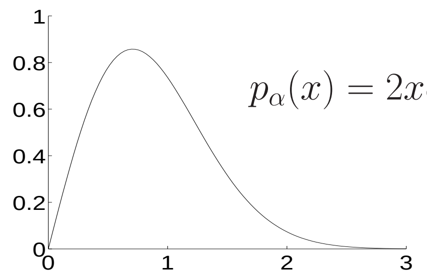
$$\mathsf{E}\left[|E(t)|^2\right] = \sum_n \beta_n^2 = 1$$

$\Longrightarrow$ $E_I(t)$ and $E_Q(t)$ are independent $\mathcal{N}(0, 1/2)$ r.v.'s

- Envelope and phase

$$\alpha(t) = |E(t)| = \sqrt{E_I^2(t) + E_Q^2(t)}\,, \quad \text{and } \phi(t) = \tan^{-1}\left(\frac{E_Q(t)}{E_I(t)}\right)$$

- For fixed t, $\alpha(t)$ and $\phi(t)$ are independent, $\alpha(t)$ has a *Rayleigh* pdf and $\phi(t)$ is $\mathrm{Unif}[0, 2\pi]$.



Autocorrelation function of $\{E(t)\}$

$$\begin{aligned}
R_E(t, t+\tau) &= \mathsf{E}\left[E^\star(t)E(t+\tau)\right] \\
&= \mathsf{E}\left[\sum_n \beta_n e^{-j\phi_n(t)} \sum_i \beta_i e^{j\phi_i(t+\tau)}\right] \\
&= \sum_n \beta_n^2\, \mathsf{E}\left[e^{j[\phi_n(t+\tau) - \phi_n(t)]}\right] \\
&\approx \sum_n \beta_n^2\, e^{j2\pi f_{\max}\tau\cos\theta_n} = R_E(\tau)
\end{aligned}$$

- $\{E(t)\}$ is approximately stationary

- In-phase and Quadrature components have correlation functions

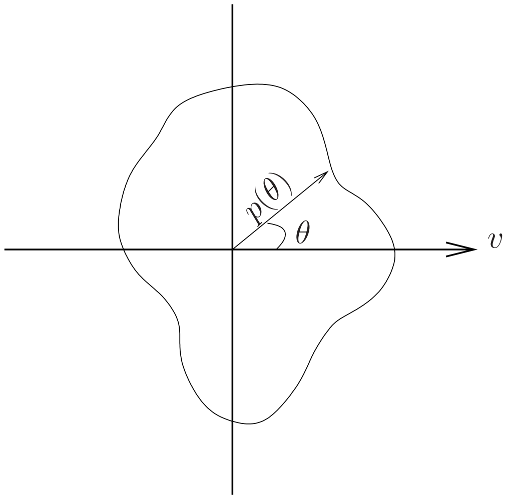
$$R_{E_I}(\tau) = R_{E_Q}(\tau) = \frac{1}{2}\mathrm{Re}\left\{R_E(\tau)\right\} = \frac{1}{2}\sum_n \beta_n^2 \cos(2\pi f_{\max}\tau\cos\theta_n)$$

$$R_{E_Q E_I}(\tau) = -R_{E_I E_Q}(\tau) = \frac{1}{2}\mathrm{Im}\left\{R_E(\tau)\right\} = \frac{1}{2}\sum_n \beta_n^2 \sin(2\pi f_{\max}\tau\cos\theta_n)$$

Angular Gain Density and Continuum-of-Paths Model

$$\boxed{R_E(\tau) = \sum_n \beta_n^2 \, e^{j2\pi f_{\max}\tau\cos\theta_n}}$$

β_n^2 is fraction of power corresponding to path n



Define *angular gain density* $p(\theta)$ by

$$p(\theta) = \sum_n \beta_n^2 \delta(\theta - \theta_n)$$

Then

$$R_E(\tau) = \int_{-\pi}^{\pi} p(\theta) e^{j2\pi f_{\max}\tau\cos\theta} d\theta$$

Isotropic Scattering Environment

"Isotropic" scattering may be approximated by continuum of paths with

$$p(\theta) = \frac{1}{2\pi} \quad \text{(uniform)}$$

to get

$$R_E(\tau) = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j2\pi f_{\max}\tau\cos\theta} d\theta = J_0(2\pi f_{\max}\tau)$$

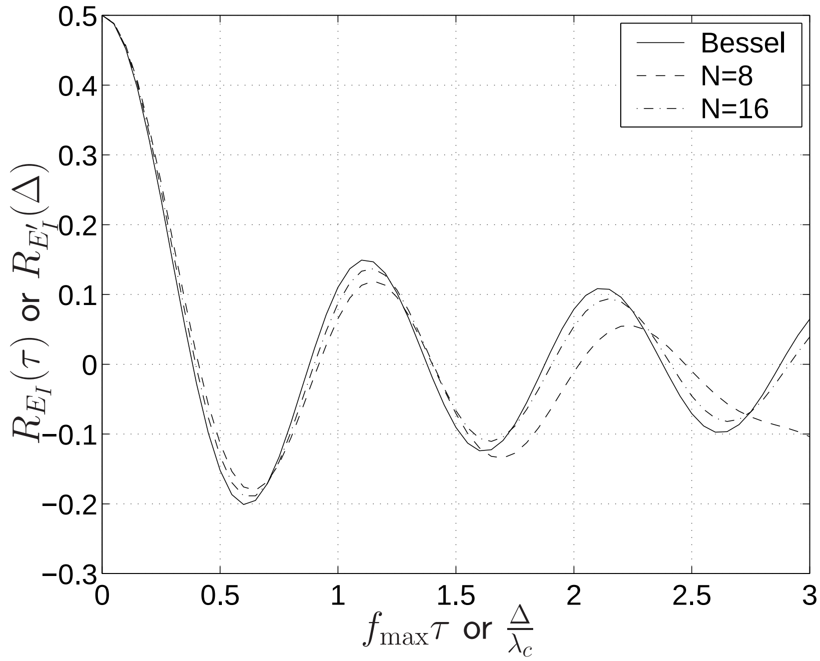
where $J_0(\cdot)$ is the *zeroth order Bessel function of the first kind* [7]

$$J_0(x) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(x\cos\theta) d\theta$$

Also

$$R_{E_I}(\tau) = R_{E_Q}(\tau) = \frac{1}{2} R_E(\tau)$$
$$R_{E_Q E_I}(\tau) = R_{E_I E_Q}(\tau) = 0$$

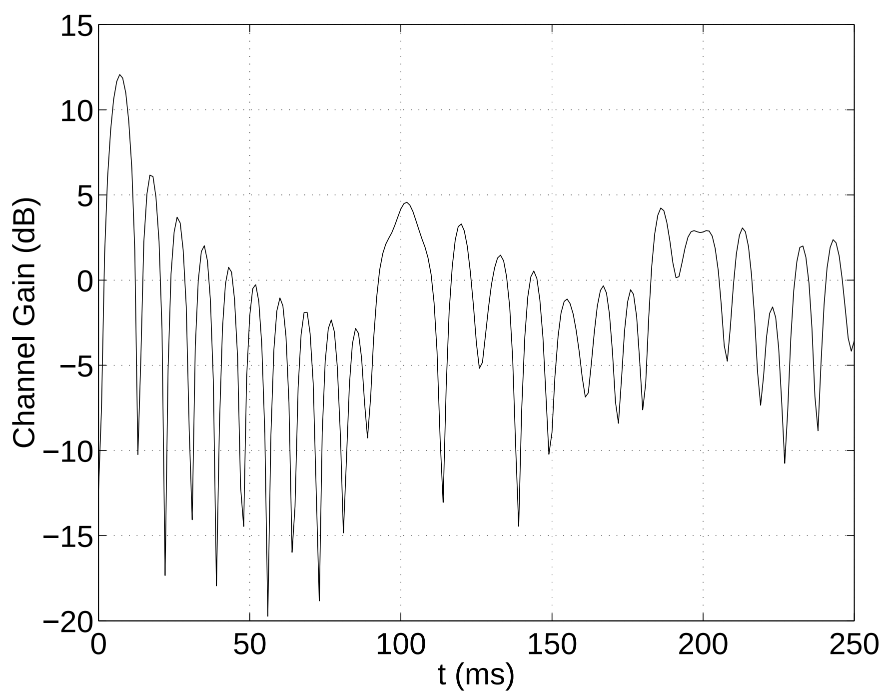
Accuracy of Bessel Approximation



Even for a few uniformly distributed discrete paths ($N = 8, 16$) we get an ACF that is well approximated by a Bessel function

Rayleigh Fading Example

$$v = 72 \text{ km/hr} = 20 \text{ m/s};\ f_c = 900 \text{ MHz} \Rightarrow \lambda_c = 1/3 \text{ m}$$
$$\Rightarrow \boxed{f_{\max} = 60 \text{ Hz}}$$



ACF of Envelope and Squared-Envelope for Rayleigh Fading

$$y(t) = E(t)x(t) = \alpha(t)\, e^{j\phi(t)} x(t)$$

- Envelope fluctuations directly influence receiver SNR
- Envelope covariance function has complicated form but may be approximated as

$$C_\alpha(\tau) \approx \frac{\pi}{16} |R_E(\tau)|^2$$

- Squared-envelope $\alpha^2(t)$ has *exponential* first-order pdf

$$p_{\alpha^2}(x) = \frac{p_\alpha(\sqrt{x})}{2\sqrt{x}} = e^{-x} u(x)$$

- Squared-envelope covariance is computed exactly as

$$C_{\alpha^2}(\tau) = |R_E(\tau)|^2$$

Scattering with LOS Component

- If LOS (specular) path with parameters θ_0, β_0 and $\phi_0(t)$, then

$$E(t) = \beta_0\, e^{j\phi_0(t)} + \sqrt{1 - \beta_0^2}\, \check{E}(t)$$

where $\{\check{E}(t)\}$ is zero mean PCG, Rayleigh fading process

- *Note:* $\{E(t)\}$ is zero-mean process, but not Gaussian since LOS component dominates diffuse components in power
- *Rice Factor:*

$$\kappa = \frac{\text{power in the specular component}}{\text{total power in diffuse components}} = \frac{\beta_0^2}{1 - \beta_0^2}$$

From the definition of κ it follows that

$$\beta_0 = \sqrt{\frac{\kappa}{\kappa + 1}}, \quad \text{and} \quad 1 - \beta_0^2 = \frac{1}{(\kappa + 1)}$$

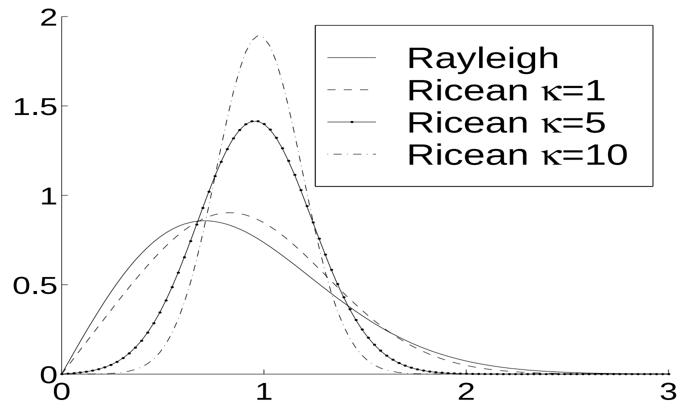
Ricean Fading

For fixed t, the envelope has Ricean pdf (Rice [8])

$$\boxed{p_\alpha(x) = 2x(\kappa+1)\ I_0\left(2x\sqrt{\kappa(\kappa+1)}\right)\ \exp\left[-x^2(\kappa+1)-\kappa\right]\ u(x)}$$

where $I_0(\cdot)$ is *zeroth order modified Bessel function of 1st kind* [7]

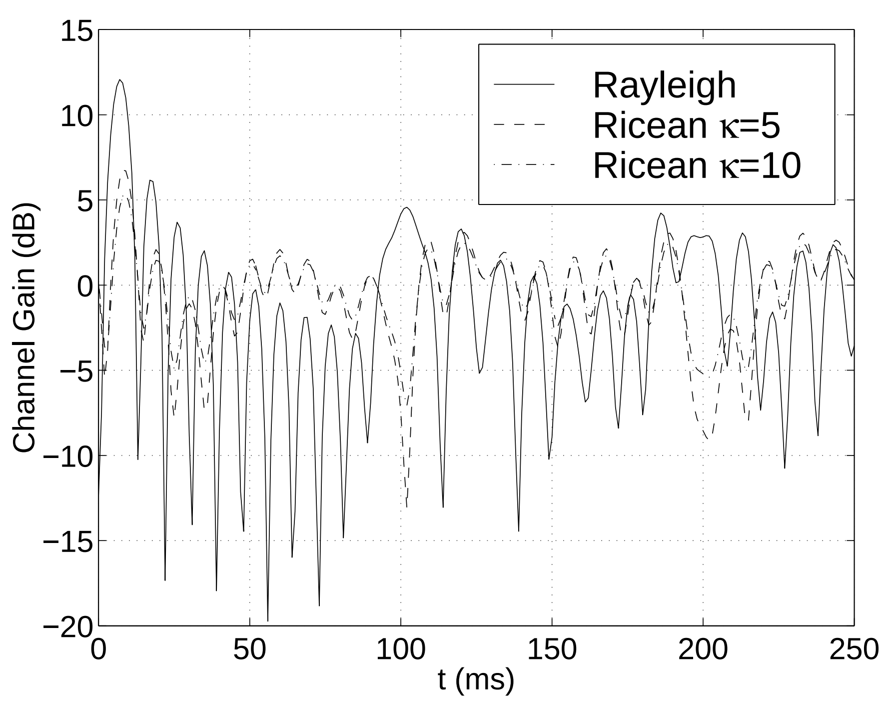
$$I_0(y) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \exp(y\cos\phi)d\phi\ .$$



Ricean Fading Example

v = 72 km/hr = 20 m/s; $f_c = 900$ MHz $\Rightarrow \lambda_c$ = 1/3 m

$\Rightarrow \boxed{f_{\max} = 60 \text{ Hz}}$



ACF for Ricean Fading

- Autocorrelation function of $\{E(t)\}$

$$R_E(\tau) = \frac{\kappa}{\kappa+1}\, e^{j2\pi f_{\max}\tau\cos\theta_0} + \frac{1}{\kappa+1}\, R_{\check{E}}(\tau)$$

where

$$R_{\check{E}}(\tau) = \int_{-\pi}^{\pi} p(\theta) e^{j2\pi f_{\max}\tau\cos\theta} d\theta$$

with $p(\theta)$ being angular gain density of diffuse components.

- Squared-envelope autocovariance

$$C_{\alpha^2}(\tau) = \left(\frac{1}{\kappa+1}\right)^2 \left[|R_{\check{E}}(\tau)|^2 + 2\kappa \mathsf{Re}\left[R_{\check{E}}(\tau)e^{-j2\pi f_{\max}\tau\cos\theta_0}\right]\right]$$

Note that $C_{\alpha^2}(\tau) \to 0$ as $\kappa \to \infty$

Fading Process in Distance Variable

- Fading process is fundamentally a function of location
- To get fading as function of time we assumed MS is traveling along constant velocity vector $\vec{v}$.
- Distance ζ relative to location at time 0 equals vt
- Fading process in ζ is given by

$$E'(\zeta) = E(\zeta/v)$$

- Fading ACF over distance variable is given by

$$\begin{aligned} R_{E'}(\Delta) &= \mathsf{E}\left[E'(\zeta+\Delta)E'^{\star}(\zeta)\right] \\ &= R_E\left(\tfrac{\Delta}{v}\right) = R_E\left(\tfrac{\Delta}{f_{\max}\lambda_c}\right) \\ &= \int_{-\pi}^{\pi} p(\theta)\, e^{\frac{j2\pi\Delta\cos\theta}{\lambda_c}} d\theta \\ &= J_0\left(2\pi\tfrac{\Delta}{\lambda_c}\right) \quad \text{(isotropic Rayleigh fading)} \end{aligned}$$

Fading ACF and Coherence Distance/Time

- *Coherence distance* Δ_c is measure of distance separation over which E' remains roughly unchanged
- Δ_c can be defined more precisely in terms of the ACF as (say):

$$\Delta_c = \text{largest } \Delta \text{ such that } |R_{E'}(\Delta)| > 0.9 R_{E'}(0) = 0.9$$

- *Coherence time*

$$T_c = \frac{\Delta_c}{v}$$

- For isotropic Rayleigh fading

$$\Delta_c \approx 0.1\lambda_c \ , \ \text{ and } \ T_c \approx \frac{0.1}{f_{\max}} \ .$$

- Fading is said to be *slow* if $T_c \gg T_s$, where T_s is the symbol period

Angular Spread of Scattering and Coherence Distance/Time

- The ACF of $\{E(t)\}$ is a strong function of the angular spread (and angular location) of the propagation
- If support of $p(\theta)$ restricted to $\theta \in [\theta_1, \theta_2]$, then

$$R_{\check{E}}(\tau) = \int_{\theta_1}^{\theta_2} p(\theta) e^{j2\pi f_{\max}\tau \cos\theta} d\theta \ .$$

- Extreme case: single (LOS) path at angle θ with angular spread of 0

$$E(t) = \beta \, e^{-j(2\pi f_{\max} t \cos\theta + \phi)}$$

$\Longrightarrow$ no envelope fluctuation; only Doppler shift $\Longrightarrow$ no fading

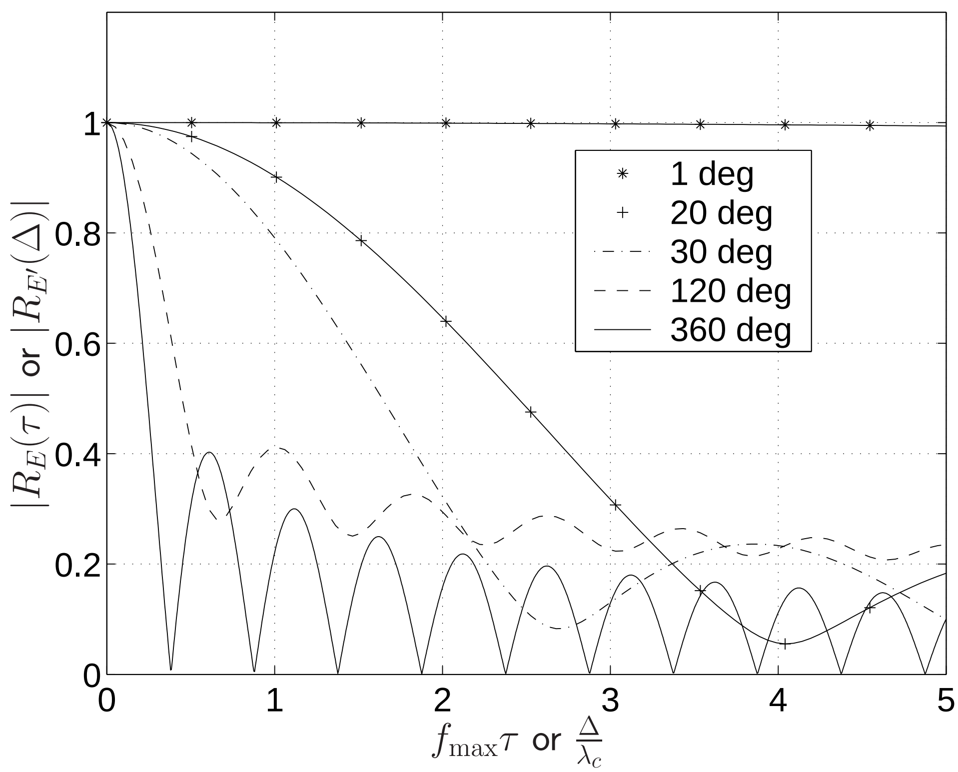
From Paulraj & Papadias [9]

Environment	Spread
Flat Rural	1 deg
Urban	20 deg
Hilly	30 deg
Mall	120 deg
Indoors	360 deg

Smaller angular spreads usually associated with LOS component

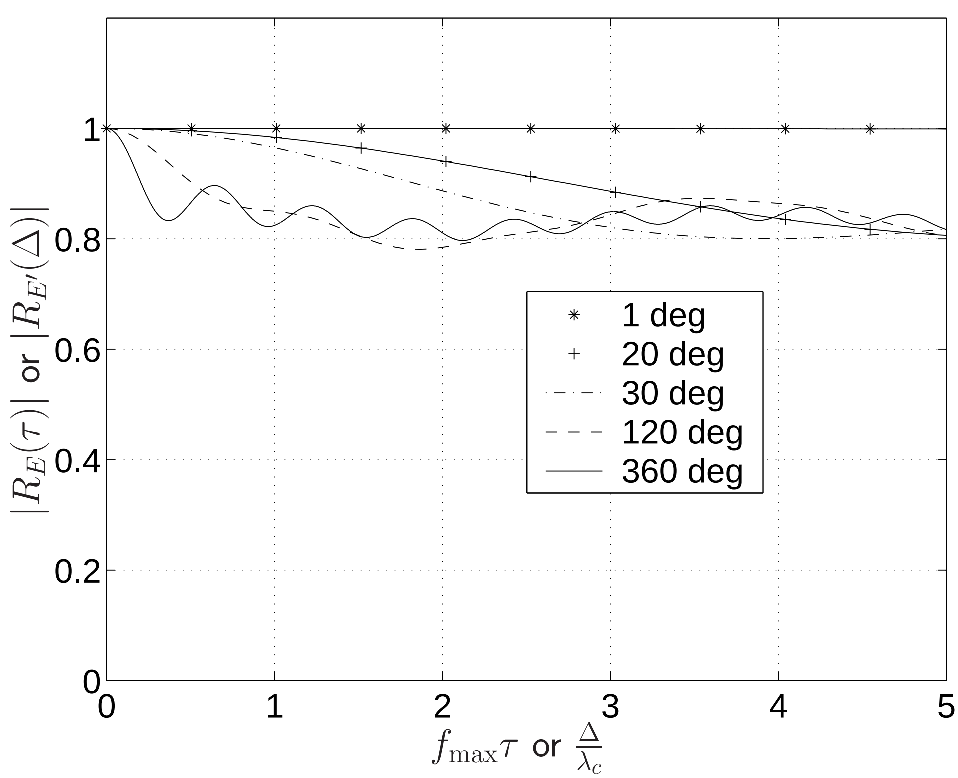
Correlation for Various Angular Spreads – Rayleigh Fading

Angular spread centered around $\theta = 45$ deg



Correlation for Various Angular Spreads – Ricean Fading

Angular spread centered around $\theta = 45$ deg; LOS at $\theta = 45$ deg; $\kappa = 5$



Power Spectrum of $\{E(t)\}$

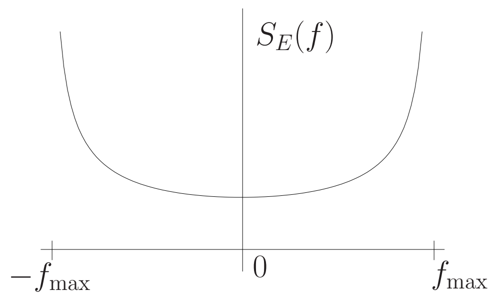
- For arbitrary angular gain density $p(\theta)$:

$$S_{\check{E}}(f) = \begin{cases} \dfrac{p(\cos^{-1}(f/f_{\max})) + p(-\cos^{-1}(f/f_{\max}))}{\sqrt{f_{\max}^2 - f^2}} & \text{for } |f| < f_{\max} \\ 0 & \text{otherwise} \end{cases}$$

$$S_E(f) = \frac{\kappa}{\kappa+1}\,\delta(f - f_{\max}\cos\theta_0) + \frac{1}{\kappa+1}\,S_{\check{E}}(f)$$

- For isotropic fading

$$S_{\check{E}}(f) = \begin{cases} \frac{1}{\pi\sqrt{f_{\max}^2 - f^2}} & \text{if } |f| < f_{\max} \\ 0 & \text{otherwise} \end{cases}$$



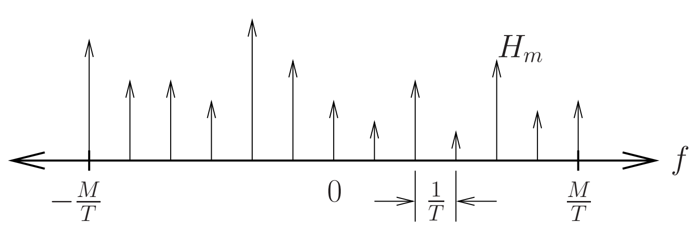
Discrete-Tone Approximation for (Rayleigh) Flat Fading

- $\{E(t)\}$ is bandlimited to $f_{\max}$
- *Doppler bandwidth* $B_d = 2f_{\max}$
- For finite observation window $[0, T]$

$$E(t) \approx \sum_{m=-M}^{M} H_m\, e^{\frac{j2\pi mt}{T}}$$

where $2M + 1 = \lceil B_d T \rceil$ and

$$\mathsf{E}[H_m H_i^\star] = (-1)^{m-i} \int_{-\pi}^{\pi} p(\theta)\,\mathrm{sinc}\left[T\left(\frac{m}{T} - f_{\max}\cos\theta\right)\right] \mathrm{sinc}\left[T\left(\frac{i}{T} - f_{\max}\cos\theta\right)\right] d\theta$$



Discrete-Time Discrete-Tone Model for Flat Fading

- Assume fading is slow, i.e. $T_s \ll \frac{1}{f_{\max}}$
- $E(t) \approx$ constant over symbol interval

$$E_n \triangleq E(nT_s) = \sum_{m=-M}^{M} H_m \, e^{\frac{j2\pi mnT_s}{T}}$$

- For observation covering N symbols, i.e. $T = NT_s$,

$$E_n = \sum_{m=-M}^{M} H_m \, e^{\frac{j2\pi mn}{N}}$$

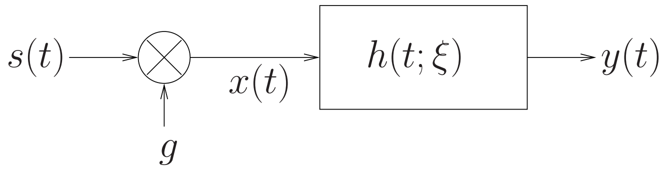
- Degrees of freedom in time:

$$\frac{2M+1}{N} = \frac{\lceil B_d T \rceil}{T/T_s} = \lceil B_d T_s \rceil \ll 1$$

$\Longrightarrow$ long range prediction!

Part VI: Frequency-Selective Fading

Frequency-Selective Fading



From Paulraj et al [9]

Environment	Spread
Flat Rural	0.5 μs
Urban	5 μs
Hilly	20 μs
Mall	0.3 μs
Indoors	0.1 μs

- $s(t)$ has passband bandwidth of W
- If $W \ll \frac{1}{\tau_{\mathrm{DS}}}$, then fading is flat
- If $W > \frac{1}{\tau_{\mathrm{DS}}}$, then fading is frequency selective
- $s(t)$ has baseband bandwidth of $W/2 \implies x(t)$ has bandwidth $W/2$
- By Sampling Theorem (sinc interpolation formula)

$$x(t-\xi) = \sum_{\ell=-\infty}^{\infty} x\left(t - \ell/W\right) \operatorname{sinc}\left[W\left(\xi - \ell/W\right)\right]$$

$$\begin{aligned} y(t) &= \int_0^{\tau_{\mathrm{DS}}} h(t;\xi)\, x(t-\xi) d\xi \\ &= \sum_{\ell=-\infty}^{\infty} x\left(t-\ell/W\right) \int_0^{\tau_{\mathrm{DS}}} h(t;\xi) \operatorname{sinc}\left[W\left(\xi-\ell/W\right)\right] d\xi \end{aligned}$$

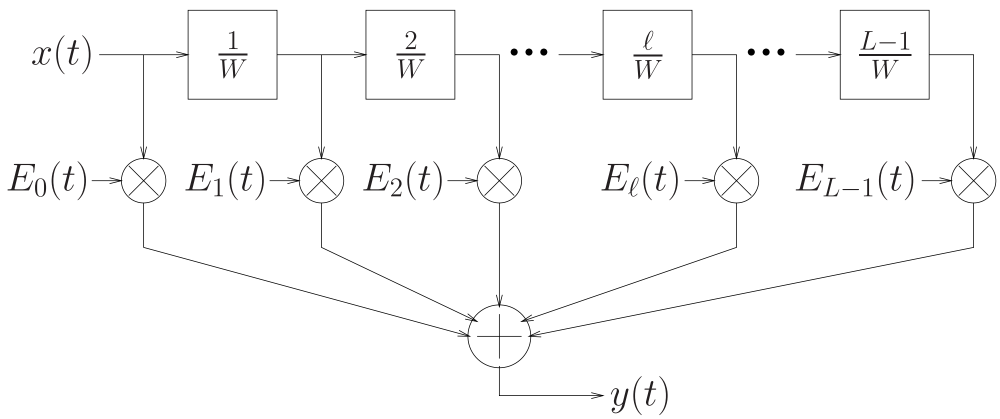
$$\boxed{E_\ell(t) = \int_0^{\tau_{\mathrm{DS}}} h(t;\xi) \operatorname{sinc}\left[W\left(\xi-\ell/W\right)\right] d\xi}$$

- $E_\ell(t) \approx 0$ for $\ell < 0$ and for $\ell/W > \tau_{\mathrm{DS}}$. If $L = \lceil \tau_{\mathrm{DS}} W \rceil$ then

$$y(t) \approx \sum_{\ell=0}^{L-1} x\left(t-\ell/W\right) E_\ell(t)$$

$$\implies h(t;\xi) \approx \sum_{\ell=0}^{L-1} E_\ell(t) \delta\left(\xi - \ell/W\right)$$

Tapped Delay Line Model

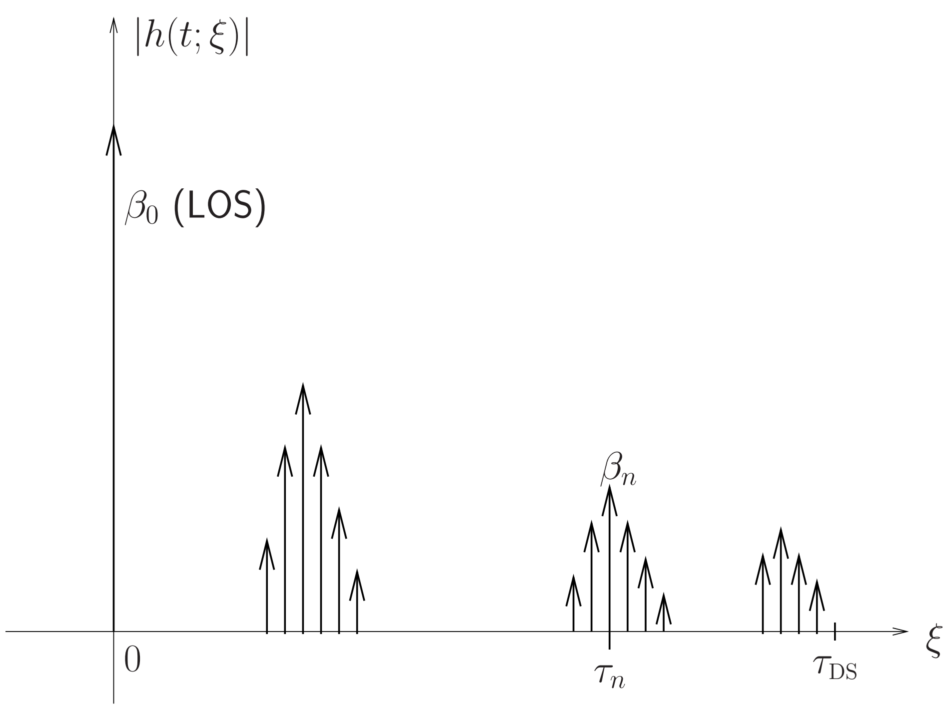


- Recall that $h(t;\xi) = \sum_n \beta_n \, e^{j\phi_n(t)} \, \delta(\xi - \tau_n)$

$$\Longrightarrow \quad E_\ell(t) = \int_0^{\tau_{\mathrm{DS}}} h(t;\xi)\, \mathrm{sinc}\left[W\left(\xi - \ell/W\right)\right] d\xi$$

$$= \sum_n \beta_n \, e^{j\phi_n(t)} \, \mathrm{sinc}\left[W\left(\tau_n - \ell/W\right)\right]$$

Sinc Mask



Statistical Model for $\{E_\ell(t)\}$

- If $\{E_\ell(t)\}$ includes a dominant LOS component, then it has Ricean envelope; else it has Rayleigh envelope
- Autocorrelation function of $\{E_\ell(t)\}$

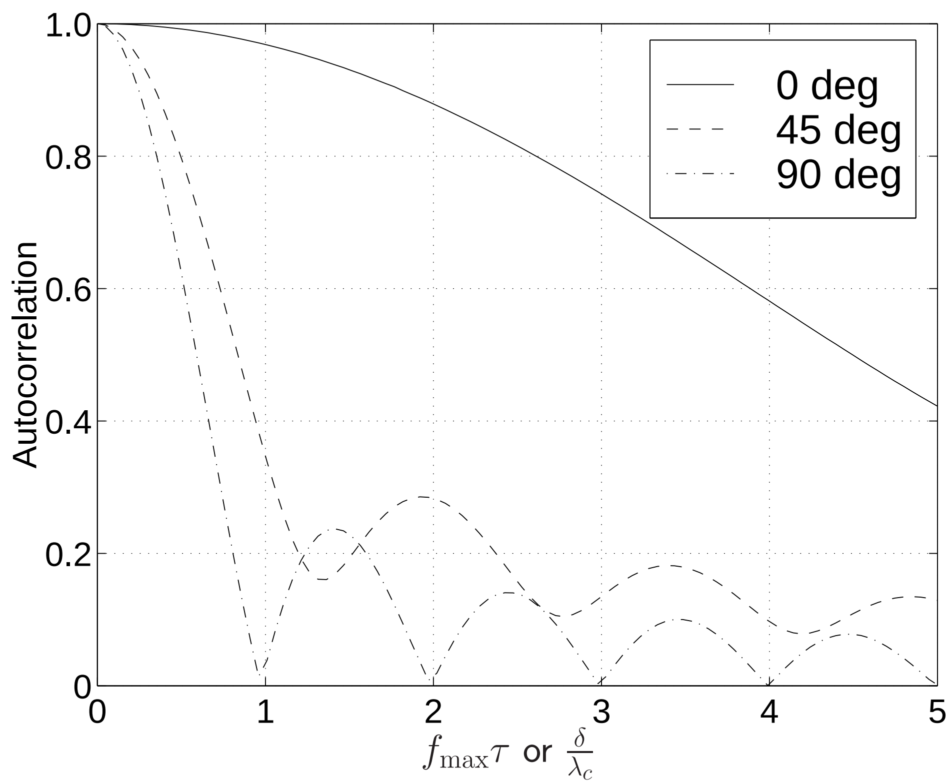
$$R_{E_\ell}(\tau) = \mathsf{E}\left[(t)E_\ell(t+\tau)E_\ell^\star\right]$$
$$\approx \sum_n \beta_n^2 \, e^{j2\pi f_{\max}\tau\cos\theta_n} \;\; \mathrm{sinc}^2\left[W\left(\tau_n - \ell/W\right)\right]$$

- *Check:* If the fading is flat, i.e. $\tau_{\mathrm{DS}} \ll \frac{1}{W}$, $E_\ell(t) \approx 0$ for $\ell \neq 0$, and

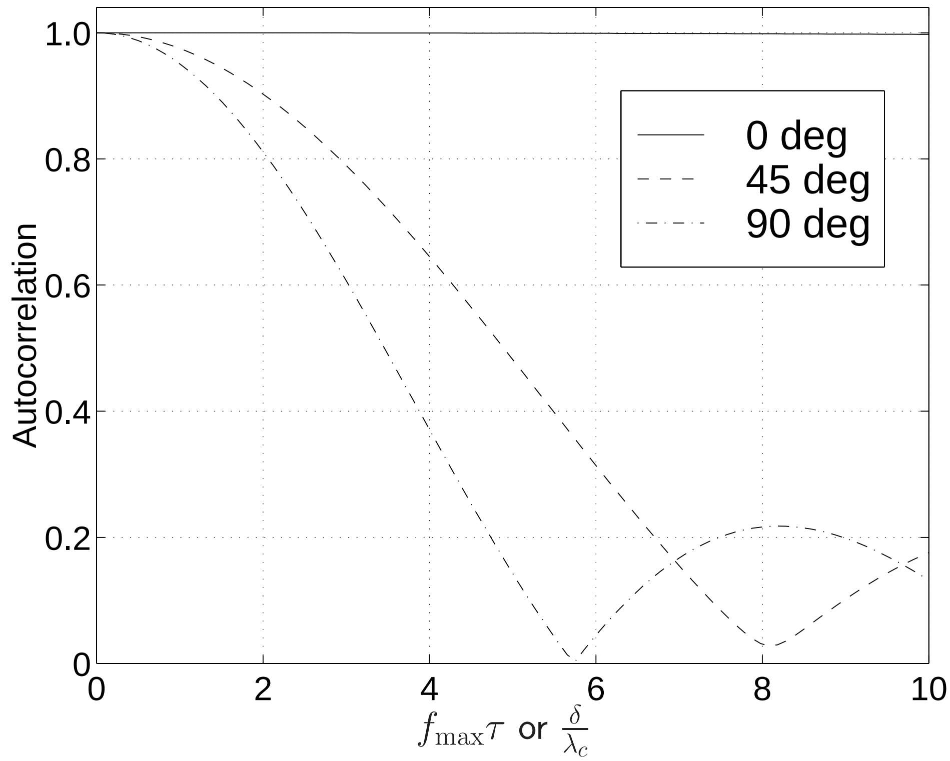
$$R_{E_0}(\tau) \approx \sum_n \beta_n^2 \, e^{j2\pi f_{\max}\tau\cos\theta_n} \, \mathrm{sinc}^2\left[W\tau_n\right]$$
$$\approx \sum_n \beta_n^2 \, e^{j2\pi f_{\max}\tau\cos\theta_n} \approx R_E(\tau)$$

- Nature of $R_{E_\ell}(\tau)$ depends on angular location and spread of paths contributing to tap ℓ

$|R_{E_\ell}(\tau)|$ for various angular locations for spread of 60 deg



$|R_{E_\ell}(\tau)|$ for various angular locations for spread of 10 deg



Cross-correlation Between Taps

$$R_{E_k E_\ell}(\tau) = \mathsf{E}\left[E_k(t+\tau)E_\ell^\star(t)\right]$$
$$\approx \sum_n \beta_n^2\, e^{j2\pi f_{\max}\tau\cos\theta_n}\ \operatorname{sinc}\left[W\left(\tau_n - \ell/W\right)\right]\ \operatorname{sinc}\left[W\left(\tau_n - k/W\right)\right]$$

- Frequency diversity depends on cross-correlation between taps
- Fading in neighboring taps can be highly correlated
- If tap delays are chosen to match cluster centers in delay profile, then taps will be less correlated
- Cluster model for channel

$$h(t;\xi) \approx \sum_{\ell=0}^{L_c-1} E_\ell(t)\delta\left(\xi - \tau_\ell\right)$$

where L_c is number of clusters; τ_ℓ delay of cluster ℓ

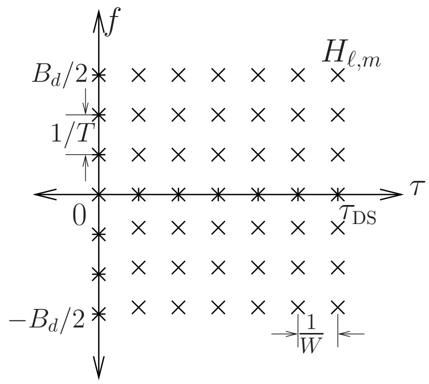
Canonical Model for Frequency-Selective (Rayleigh) Fading

- For finite observation window $[0, T]$

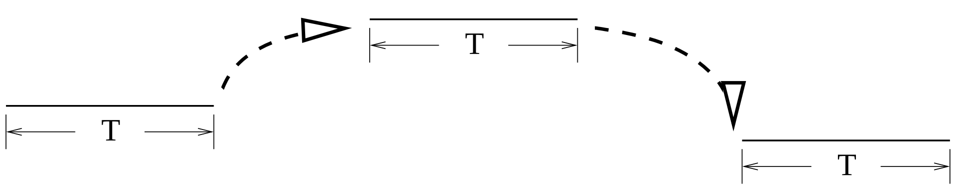
$$E_\ell(t) \approx \sum_{m=-M}^{M} H_{\ell,m}\, e^{\frac{j2\pi mt}{T}}$$

$$\Longrightarrow \quad h(t;\xi) \approx \sum_{\ell=0}^{L-1} \sum_{m=-M}^{M} H_{\ell,m}\, e^{\frac{j2\pi mt}{T}} \delta\left(\xi - \ell/W\right)$$

- $\{H_{\ell,m}\}$ zero mean PCG correlated (in general)



Block Fading Channel



- TDMA or frequency hopping system

- Blocklength T is such that $TB_d < 1$
  $\Longrightarrow$ one coefficient per tap sufficient ($M = 0$)

- Within each block

$$h(t;\xi) \approx \sum_{\ell=0}^{L-1} H_\ell\, \delta\left(\xi - \ell/W\right) \stackrel{\Delta}{=} h(\xi)$$

  Channel is LTI within each block!

- With sufficient time/frequency separation between blocks, channel is independent from block to block

Comment on GWSSUS Channel Model

- Gaussian Wide Sense Stationary Uncorrelated Scattering (Bello[10])
- For no LOS, model $h(t;\xi)$ as zero-mean PCG field in $(t;\tau)$
- Autocorrelation function:

$$\mathsf{E}\left[h(t_1,\tau_1)h^\star(t_2,\tau_2)\right] = \Phi_h(t_1-t_2;\tau_1)\,\delta(\tau_1-\tau_2)$$

  - ○ with each delay τ_1 is associated a WSS fading process with ACF $\Phi_h(t_1-t_2;\tau_1)$
  - ○ fading processes are uncorrelated across delays
- For LOS add $\beta_0 e^{j\phi_0(t)}\delta(\tau)$
- Can derive flat and frequency-selective channel models from GWSSUS model, BUT physical interpretation and angular dependence hidden

Simulation of Small Scale Variations

- *Method 1: Direct Approach*

  ① Pick path gains and angles of arrival for discrete set of paths corresponding to desired angular spread and angular location

  ② Approximate $\phi_n(t)$ by

$$\phi_n(t) = \hat{\phi}_n + 2\pi f_{\max} t \cos\theta_n,$$

  where $\hat{\phi}_n$ are chosen to be i.i.d. Unif$[0,2\pi]$.

  ③ Sum up paths with appropriate "sinc mask" to generate $\{E_\ell(t)\}$

- *Method 2: Filtered White Gaussian Noise (for independent taps)*

  ① Find discrete-time approximation for R_{E_ℓ}

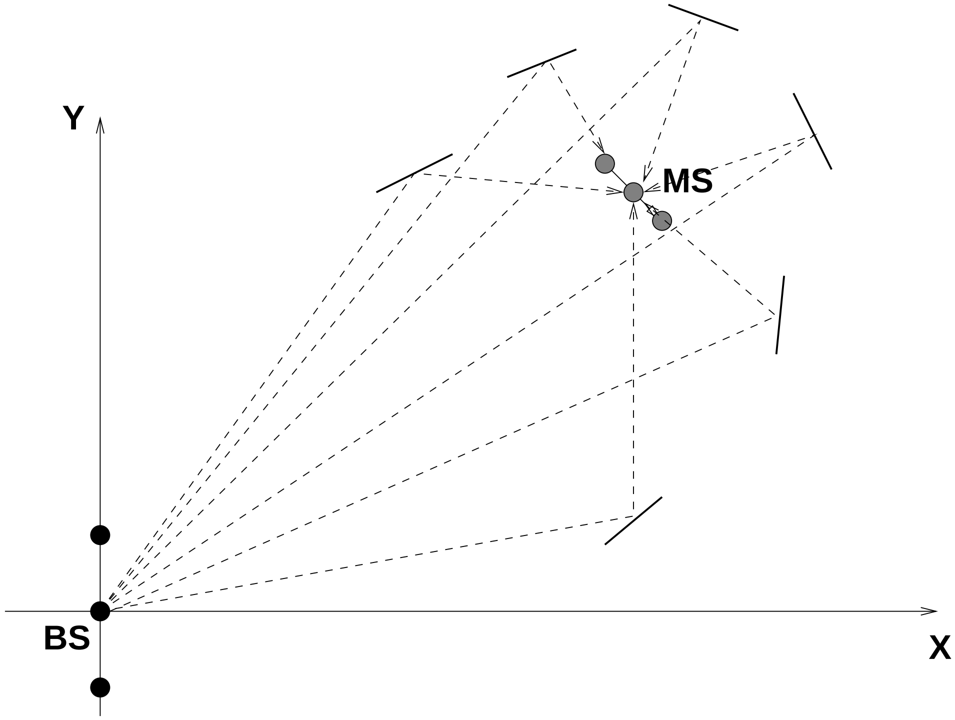
  ② Find "square-root" filter using spectral factorization

  ③ Pass complex WGN sequence through filter to produce samples of $E_\ell(t)$

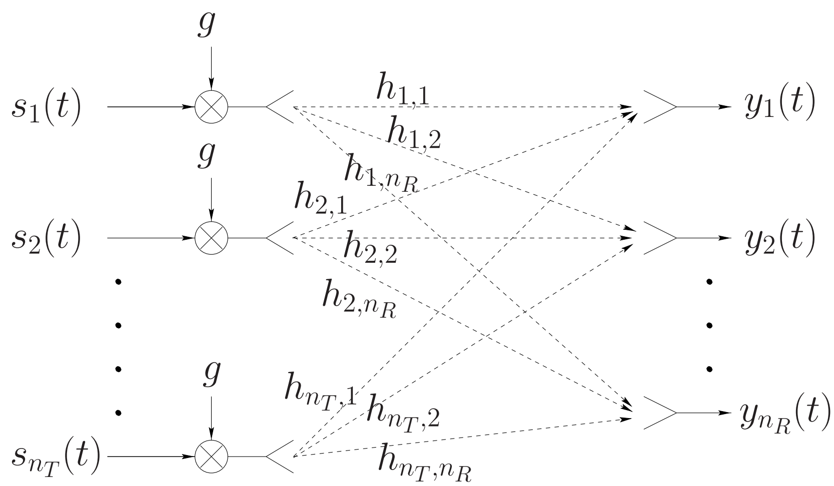
- *Method 3: Use Canonical Model*

Part VII: Multiantenna Systems

Multiple Antennas at BS and MS



Linear Matrix Channel



$$y_j(t) = g \sum_{i=1}^{n_T} \int_0^{\tau_{\mathsf{DS}}} s_i(t-\xi) h_{i,j}(t;\xi) d\xi$$

- Large scale variations identical at each antenna (for microdiversity)
- Small scale variations captured in $\{h_{i,j}(t;\xi)\}$

Correlation Models

- For input signals with bandwidth W

$$h_{i,j}(t;\xi) \approx \sum_{\ell=0}^{L-1} E_{\ell,i,j} \delta\left(\xi - \ell/W\right)$$

- Autocorrelation function

$$R_{E_{\ell,i,j}}(\tau) = R_{E_\ell}(\tau) \text{ for all } i,j$$

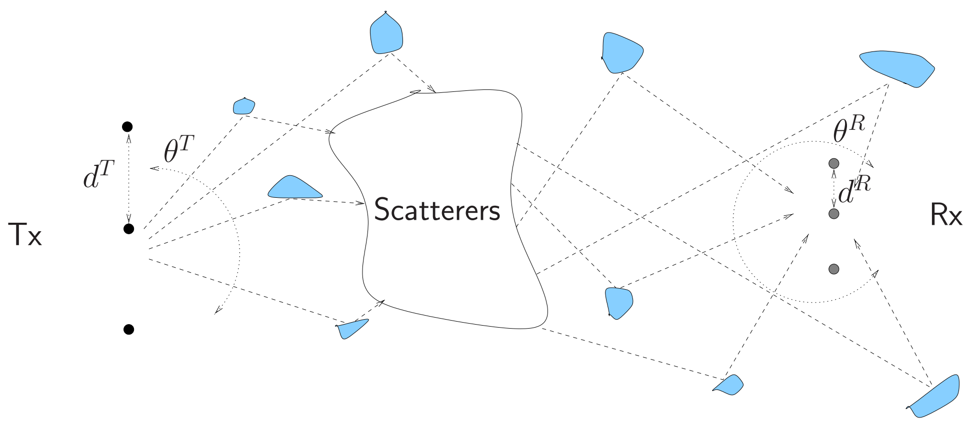
- Crosscorrelation
  - ◦ complicated – depends on spacing between antennas and "two-dimensional" angular gain density
  - ◦ angular spread may be different at BS and MS
  - ◦ if antenna separation is sufficient then uncorrelated $\{E_{\ell,i,j}\}$
  - ◦ $\lambda_c/2$ spacing is rarely sufficient at BS

Special Case: Flat, Block Fading Channel

- Within each block, fading is flat and constant:

$$h_{i,j}(t;\xi) \approx H_{i,j}\delta(\xi)$$

- How are $\{H_{i,j}\}$ correlated?



General Equation for Correlation

- In general

$$\mathsf{E}\left[H_{i,j}H_{p,q}^{\star}\right] = \sum_n \beta_n^2 \exp\left[\frac{\jmath 2\pi d^T|p-i|\cos\theta_n^T}{\lambda_c}\right] \exp\left[\frac{\jmath 2\pi d^R|q-j|\cos\theta_n^R}{\lambda_c}\right]$$

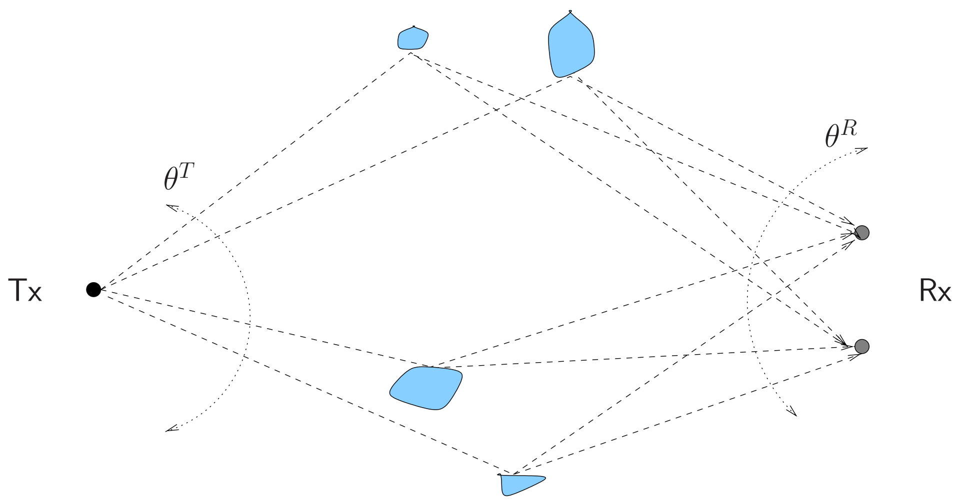
$$= \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} p(\theta^T,\theta^R) \exp\left[\frac{\jmath 2\pi d^T|p-i|\cos\theta^T}{\lambda_c}\right] \cdot \exp\left[\frac{\jmath 2\pi d^R|q-j|\cos\theta^R}{\lambda_c}\right] d\theta^T\, d\theta^R$$

where $p(\theta^T,\theta^R)$ is a two-dimensional angular gain density

- Marginals

$$p(\theta^T) = \int_{-\pi}^{\pi} p(\theta^T,\theta^R)\, d\theta^R \qquad p(\theta^R) = \int_{-\pi}^{\pi} p(\theta^T,\theta^R)\, d\theta^T$$

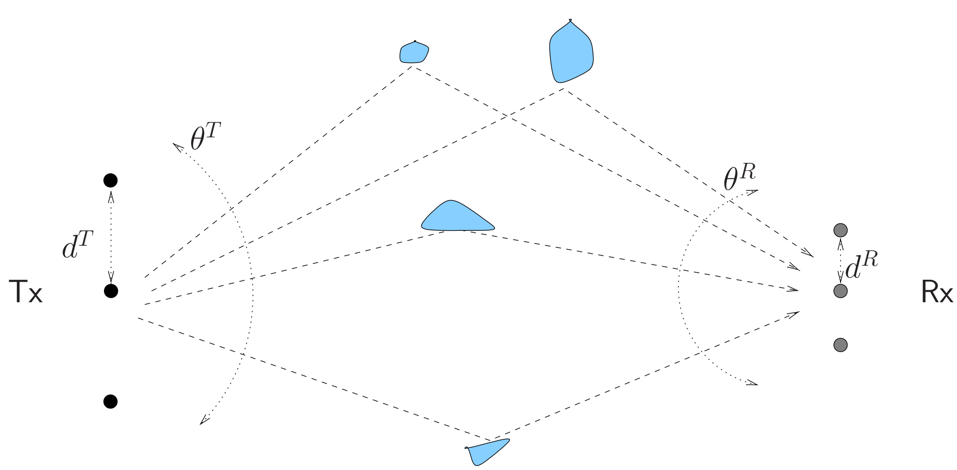
Correlation at Rx Antennas for Links to Any One Tx Antenna



$$\rho^R_{j,q} = \mathsf{E}\left[H_{i,j} H^\star_{i,q}\right] = \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} p(\theta^T,\theta^R)\ \exp\left[\frac{\jmath 2\pi d^R |q-j| \cos\theta^R}{\lambda_c}\right]\ d\theta^T\ d\theta^R$$
$$= \int_{-\pi}^{\pi} p(\theta^R) \exp\left[\frac{\jmath 2\pi d^R |q-j| \cos\theta^R}{\lambda_c}\right]\ d\theta^R \quad \text{for all } i$$

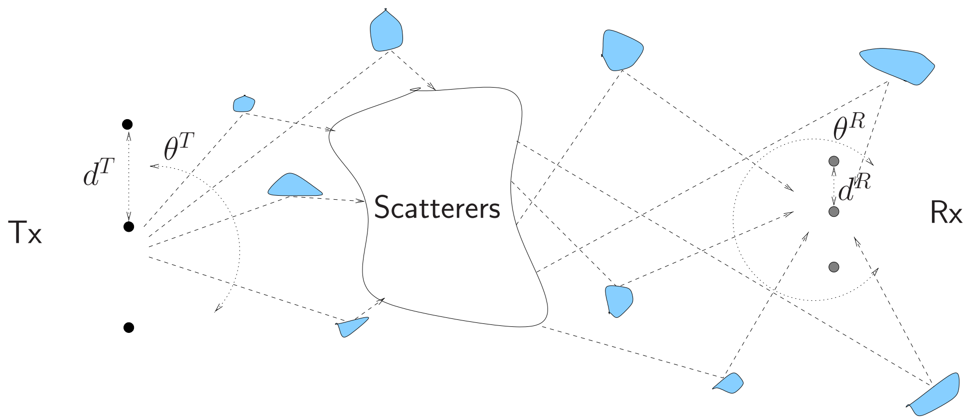
Similar form for $\rho^T_{i,p}$ for link from Tx to any one Rx antenna

General Correlation Function



$$p(\theta^T,\theta^R) \neq p(\theta^T)\,p(\theta^R) \quad \Longrightarrow \quad \mathsf{E}\left[H_{i,j} H^\star_{p,q}\right] \neq \rho^T_{i,p}\rho^R_{j,q}$$

Product Form Correlation Function



$$p(\theta^T, \theta^R) = p(\theta^T)\, p(\theta^R) \quad \Longrightarrow \quad \mathsf{E}\left[H_{i,j} H^{\star}_{p,q}\right] = \rho^T_{i,p} \rho^R_{j,q}$$

Applications

- *First Order Statistics*

  ① Performance of digital communications in slow fading

  ② Performance gains obtained from diversity

- *Correlation Models*

  ① Channel estimation schemes

  ② Finite State Markov Channel (FSMC) approximations

  ③ Spatial correlation and multiantenna design

  ④ Space-Time channel capacity and code design

  ⑤ Space-Time receiver design and decoding

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