Communication on Flat Fading Channels

Signaling Through Slow Flat Fading Channels

◇ We assume that the long-term variations in the channel are absorbed into $\mathcal{E}_m$. Then $\mathcal{E}_m$ represents the average received symbol energy (for symbol m) over the time frame for which the multipath profile may be assumed to be constant. Then the received signal is given by:

$$y(t) = E(t)s(t) + w(t) = \beta(t)e^{j\phi(t)}s(t) + w(t) \tag{1}$$

where $\mathsf{E}[\beta^2(t)] = 1$.

◇ For slow fading, $\beta(t)$ and $\phi(t)$ may be assumed to be constant over each symbol period. Thus, for memorlyless modulation and symbol-by-symbol demodulation, $y(t)$ for demodulation over symbol period $[0, T_s]$ may be written as

$$y(t) = \beta e^{j\phi}s_m(t) + w(t) \text{ (conditioned on symbol } m \text{ being transmitted)} \tag{2}$$

Average probability of error for slow, flat fading

◇ The error probability is a function of the received signal-to-noise ratio (SNR), i.e., the received symbol energy divided by the noise power spectral density. We denote the symbol SNR by γ_s, and the corresponding bit SNR by γ_b, where $\gamma_\mathrm{b} = \gamma/\nu$ and $\nu = \log_2 M$.

◇ For slow, flat fading, the received SNR is

$$\gamma_\mathrm{s} = \frac{\beta^2\mathcal{E}_\mathrm{s}}{N_0} . \tag{3}$$

The average SNR (averaging over β^2) is given by

$$\overline{\gamma}_\mathrm{s} = \mathsf{E}[\beta^2]\frac{\mathcal{E}_\mathrm{s}}{N_0} = \frac{\mathcal{E}_\mathrm{s}}{N_0} . \tag{4}$$

The corresponding bit SNR's are given by

$$\gamma_\mathrm{b} = \frac{\gamma}{\nu}, \text{ and } \overline{\gamma}_\mathrm{b} = \frac{\mathcal{E}_\mathrm{s}}{N_0\nu} = \frac{\mathcal{E}_\mathrm{b}}{N_0} . \tag{5}$$

◇ Suppose the symbol error probability with SNR γ_s is denoted by $\mathrm{P_e}(\gamma_\mathrm{s})$. Then the average error probability (averaged over the fading) is

$$\overline{\mathrm{P}}_\mathrm{e} = \int_0^\infty \mathrm{P_e}(x)p_{\gamma_\mathrm{s}}(x)dx \tag{6}$$

where $p_{\gamma_\mathrm{s}}(x)$ is the pdf of γ_s.

◇ For Rayleigh fading, β^2 is exponential with mean 1; hence γ_s is exponential with mean $\overline{\gamma}_\mathrm{s}$, i.e.,

$$p_{\gamma_\mathrm{s}}(x) = \frac{1}{\overline{\gamma}_\mathrm{s}} \exp\left[-\frac{x}{\overline{\gamma}_\mathrm{s}}\right] \mathbb{1}_{\{x\geq 0\}} . \tag{7}$$

⋄ For Ricean fading, β^2 has a "Ricean-squared" pdf, and hence $\gamma_{\rm s}$ has pdf

$$p_{\gamma_{\rm s}}(x) = \frac{\kappa+1}{\overline{\gamma}_{\rm s}}\, I_0\left(2\sqrt{\frac{x\kappa(\kappa+1)}{\overline{\gamma}_{\rm s}}}\right)\, \exp\left[-\frac{x(\kappa+1)}{\overline{\gamma}_{\rm s}} - \kappa\right]\, 1\!\!1_{\{x\geq 0\}}\, . \tag{8}$$

⋄ $\overline{P}_{\rm e}$ *for Rayleigh Fading*

∘ Useful result (see problem 4 of HW#2):

$$\int_0^\infty Q(\sqrt{x}\,)\,\frac{e^{-x/\gamma}}{\gamma}dx = \frac{1}{2}\left[1-\sqrt{\frac{\gamma}{2+\gamma}}\,\right]\, . \tag{9}$$

∘ BPSK

$$\mathrm{P_b}(\gamma_{\rm b}) = Q(\sqrt{2\gamma_{\rm b}}\,)\, . \tag{10}$$

Using (7) and (9), we get

$$\overline{\mathrm{P}}_{\rm b} = \int_0^\infty Q(\sqrt{2x}\,)p_{\gamma_{\rm b}}(x)dx = \frac{1}{2}\left[1-\sqrt{\frac{\overline{\gamma}_{\rm b}}{1+\overline{\gamma}_{\rm b}}}\,\right] \approx \frac{1}{4\overline{\gamma}_{\rm b}} \text{ (for large}\overline{\gamma}_{\rm b}). \tag{11}$$

∘ Binary coherent orthogonal modulation (e.g. FSK)

$$\mathrm{P_b}(\gamma_{\rm b}) = Q(\sqrt{\gamma_{\rm b}}\,)\, . \tag{12}$$

Here $\overline{\mathrm{P}}_{\rm b}$ is the same as that for BPSK with $\overline{\gamma}_{\rm b}$ replaced by $\overline{\gamma}_{\rm b}/2$, i.e.,

$$\overline{\mathrm{P}}_{\rm b} = \frac{1}{2}\left[1-\sqrt{\frac{\overline{\gamma}_{\rm b}}{2+\overline{\gamma}_{\rm b}}}\,\right] \approx \frac{1}{2\overline{\gamma}_{\rm b}} \text{ (for large } \overline{\gamma}_{\rm b}). \tag{13}$$

∘ Binary DPSK

$$\mathrm{P_b}(\gamma_{\rm b}) = \frac{1}{2}e^{-\gamma_{\rm b}}\, . \tag{14}$$

In this we case we may integrate directly to get

$$\overline{\mathrm{P}}_{\rm b} = \int_0^\infty \frac{1}{2}e^{-x}\,\frac{e^{-x/\overline{\gamma}_{\rm b}}}{\overline{\gamma}_{\rm b}}dx = \frac{1}{2(1+\overline{\gamma}_{\rm b})} \approx \frac{1}{2\overline{\gamma}_{\rm b}} \text{ (for large } \overline{\gamma}_{\rm b}). \tag{15}$$

∘ Binary noncoherent orthogonal modulation (FSK)

$$\mathrm{P_b}(\gamma_{\rm b}) = \frac{1}{2}e^{-\gamma_{\rm b}/2}\, . \tag{16}$$

Here $\overline{\mathrm{P}}_{\rm b}$ is the same as that for DPSK with $\overline{\gamma}_{\rm b}$ replaced by $\overline{\gamma}_{\rm b}/2$, i.e.,

$$\overline{\mathrm{P}}_{\rm b} = \frac{1}{2+\overline{\gamma}_{\rm b}} \approx \frac{1}{\overline{\gamma}_{\rm b}} \text{ (for large } \overline{\gamma}_{\rm b}). \tag{17}$$

∘ Similar expressions may be derived for other M-ary modulation schemes. Note that without fading the error probabilities decrease exponentially with SNR, whereas with fading the error probabilities decrease much more slowly with SNR (inverse linear in case of Rayleigh fading).

$\diamond$ *$\overline{\mathrm{P}}_{\mathrm{e}}$ for Ricean Fading*

- *Direct approach:* Compute $\overline{\mathrm{P}}_{\mathrm{e}}$ using (6) and (8). This is cumbersome except in some special cases.
- *Nakagami-m approach:* Approximate $p_{\gamma_{\mathrm{s}}}(x)$ by a Nakagami-m distribution for which integration of P_{e} to produce $\overline{\mathrm{P}}_{\mathrm{e}}$ is relatively easy. (See Problem 7 of HW#2).
- *Complex Gaussian approach:* We begin by rewriting γ_{s} of (3) as

$$\gamma_{\mathrm{s}} = \beta^2 \overline{\gamma}_{\mathrm{s}} = (E_I^2 + E_Q^2)\overline{\gamma}_{\mathrm{s}} = Z_I^2 + Z_Q^2 = |Z|^2 \,, \tag{18}$$

where $Z_I = \sqrt{\overline{\gamma}_{\mathrm{s}}}\, E_I$, $Z_Q = \sqrt{\overline{\gamma}_{\mathrm{s}}}\, E_Q$, and $Z = Z_I + jZ_Q$ is PCG, with mean m_Z and variance σ_Z^2, conditioned on the LOS phase ϕ_0, given by (see channel modeling notes)

$$m_Z = \sqrt{\overline{\gamma}_{\mathrm{s}}}\, \beta_0 e^{j\phi_0} = \sqrt{\frac{\overline{\gamma}_{\mathrm{s}}\kappa}{\kappa+1}}\, e^{j\phi_0}, \ \text{ and } \sigma_Z^2 = \mathsf{E}[|Z|^2] = \frac{\overline{\gamma}_{\mathrm{s}}}{\kappa+1} \,. \tag{19}$$

- Without loss of generality, we may assume that $\phi_0 = 0$, since the pdf of γ_{s} is independent of ϕ_0.
- *General expression for $\overline{P}_e$*

$$\begin{aligned} \overline{\mathrm{P}}_{\mathrm{e}} &= \int_0^\infty \mathrm{P}_{\mathrm{e}}(x) p_{\gamma_{\mathrm{s}}}(x) dx = \int_{z\in\mathbb{C}} \mathrm{P}_{\mathrm{e}}\left(|z|^2\right) p_Z(z)\, dz \\ &= \frac{1}{\pi\sigma_Z^2} \int_{z\in\mathbb{C}} \mathrm{P}_{\mathrm{e}}\left(|z|^2\right)\, \exp\left(-\frac{|z-m_Z|^2}{\sigma_Z^2}\right)\, dz \end{aligned} \tag{20}$$

- *Useful result 1*

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\theta}\right) d\theta \ \text{ (problem 3 of HW\#4)} \,. \tag{21}$$

This alternative representation was introduced recently by Simon and Divsalar [1] as a way to compute general expressions for the error rates for digital modulation on fading channels. For more recent results, see the book by Simon and Alouini [2].

- *Useful result 2* The following result is also very useful in computing closed-form expressions for the error probability in some special cases.

$$I_n(c) = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^n d\theta = [A(c)]^n \sum_{i=0}^{n-1} \binom{n-1+i}{i} [1-A(c)]^i \tag{22}$$

with $A(c) = \frac{1}{2}\left[1 - \sqrt{c/(1+c)}\right]$. This result is derived in [3]. Note that $I_n(c)$ also has the following alternative expression whose form is similar to that obtained in Problem 7 of HW#2.

$$I_n(c) = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^n d\theta = \frac{1}{2} - \left[\frac{1}{2} - A(c)\right] \sum_{i=0}^{n-1} \binom{2i}{i} [A(c)]^i\, [1-A(c)]^i \,. \tag{23}$$

◦ $\overline{P}_b$ *for Binary Signaling with Ricean Fading*

- BPSK

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi\sigma_Z^2}\int_{z\in\mathbb{C}} Q\left(\sqrt{2|z|^2}\right)\exp\left(-\frac{1}{\sigma_Z^2}|z-m_Z|^2\right)\,dz$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\left[\frac{1}{\pi\sigma_Z^2}\int_{z\in\mathbb{C}}\exp\left(-\frac{|z|^2}{\sin^2\theta}\right)\,\exp\left(-\frac{|z-m_Z|^2}{\sigma_Z^2}\right)\,dz\right]\,d\theta \tag{24}$$

$$= \frac{1}{\pi}\int_0^{\pi/2}\frac{(\kappa+1)\sin^2\theta}{\overline{\gamma}_{\mathrm{b}}+(\kappa+1)\sin^2\theta}\,\exp\left(-\frac{\overline{\gamma}_{\mathrm{b}}\kappa}{(\kappa+1)\sin^2\theta+\overline{\gamma}_{\mathrm{b}}}\right)\,d\theta\;.$$

where the last line follows after completion of squares inside the exponential to compute the complex Gaussian integral.
Note that for $\kappa = 0$ (i.e. Rayleigh fading), we have

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2}\frac{\sin^2\theta}{\overline{\gamma}_{\mathrm{b}}+\sin^2\theta}\,d\theta\;. \tag{25}$$

Using (22) with $n = 1$, we can immediately see that the above expression is the same as the one obtained in (11). Also, for $\kappa \to \infty$, we see that we get back AWGN performance.

- Binary coherent FSK. Same as BPSK with $\overline{\gamma}_{\mathrm{b}}$ replaced by $\overline{\gamma}_{\mathrm{b}}/2$.
- Binary DPSK.

$$\begin{aligned}\overline{\mathrm{P}}_{\mathrm{b}} &= \frac{1}{\pi\sigma_Z^2}\int_{z\in\mathbb{C}}\frac{1}{2}\exp\left(-|z|^2\right)\,\exp\left(-\frac{1}{\sigma_Z^2}|z-m_Z|^2\right)\,dz \\ &= \frac{\kappa+1}{2(\kappa+1+\overline{\gamma}_{\mathrm{b}})}\,\exp\left[-\frac{\kappa\overline{\gamma}_{\mathrm{b}}}{\kappa+1+\overline{\gamma}_{\mathrm{b}}}\right]\;.\end{aligned} \tag{26}$$

where the second line follows easily by completion of squares as done in class. Again, it is easy to check that we get the Rayleigh result when $\kappa = 0$ and the AWGN result as $\kappa \to \infty$.

- Binary noncoherent FSK. Same as DPSK with $\overline{\gamma}_{\mathrm{b}}$ replaced by $\overline{\gamma}_{\mathrm{b}}/2$.

Diversity Techniques for Flat Fading Channels

- Performance with fading is considerably worse than without fading, especially when the fading is Rayleigh.
- Performance may be improved by sending the same information on many (independently) fading channels
- For signaling on L channels, the received signal on the ℓ-th channel is:

$$y_\ell(t) = \beta_\ell e^{j\phi_\ell} s_{m,\ell}(t) + w_\ell(t), \quad \ell = 1, 2, \ldots, L, \;\; m = 0, 1, \ldots, M-1\,. \tag{27}$$

  where the noise $w_\ell(t)$ is assumed to be independent across channels.

- If $\{\beta_\ell e^{j\phi_\ell}\}_{\ell=1}^{L}$ are independent, we get maximum diversity against fading.
- How do we guarantee independence of channels? By separating them either in time, frequency or space.
  - ◦ frequency separation must be $\gg \frac{1}{\tau_{\text{ds}}}$, where τ_{ds} is the delay spread
  - ◦ time separation must be $\gg \frac{1}{f_m}$, where f_m is the maximum Doppler frequency
  - ◦ spatial separation must be $\gg \frac{\lambda_c}{2}$, where λ_c is the carrier wavelength.

Memoryless linear modulation with diversity

- When symbol m is sent on the channels

$$y_\ell(t) = \beta_\ell e^{j\phi_\ell} \sqrt{\mathcal{E}_{\text{s},\ell}}\, a_m\, e^{j\theta_m}\, g_\ell(t) + w_\ell(t), \;\; \ell = 1, 2, \ldots, L, \; m = 0, 1, \ldots, M-1\,, \tag{28}$$

  where $g_\ell(t)$ is a (possibly complex) unit energy shaping function on channel ℓ, $\mathcal{E}_{\text{s},\ell}$ is the average symbol energy on channel ℓ, and the a_m's are normalized so that $\sum_m a_m^2 = 1$. We assume that the fading and noise are independent across channels. Note that $\{w_\ell(t)\}$ are independent PCG processes with PSD N_0.

- *Optimum receiver:* If we assume that the phases $\{\phi_\ell\}$ and the amplitudes $\{\beta_\ell\}$ are estimated perfectly at the receiver, the optimum test statistic is formed by Maximal Ratio Combining (MRC) as

$$Y = \sum_{\ell=1}^{L} \beta_\ell \sqrt{\mathcal{E}_\ell}\, e^{-j\phi_\ell} \int y_\ell(t) g_\ell(t) dt\,. \tag{29}$$

  We proved that his was optimum in class; also see [4, 5] and Problem 1 of HW#3.

- The sufficient statistic y may be rewritten as

$$Y = \sum_{\ell=1}^{L} \beta_\ell^2 \sqrt{\mathcal{E}_{\text{s},\ell}}\, a_m\, e^{j\theta_m} + \sum_{\ell=1}^{L} \beta_\ell \sqrt{\mathcal{E}_{\text{s},\ell}}\, W_\ell\,, \tag{30}$$

  where $\{W_\ell\}$ are independent $\mathcal{CN}(0, N_0)$.

- The MPE (ML) decision rule is the same as without diversity except that the constellation is scaled in amplitude based on the fading on the channels.

Special Case: BPSK with diversity

- The sufficient statistic in this case takes the form

$$Y = \pm \sum_{\ell=1}^{L} \beta_\ell^2 \mathcal{E}_{b,\ell} + W\,, \tag{31}$$

where $W = \sum_{\ell=1}^{L} \beta_\ell \sqrt{\mathcal{E}_{b,\ell}}\, W_\ell$ is PCG with

$$\mathsf{E}[|W|^2] = N_0 \sum_{\ell=1}^{L} \mathcal{E}_{b,\ell}\, \beta_\ell^2\,. \tag{32}$$

- The MPE decision rule for equal priors (or the ML decision rule)is to decide $+1$ (bit '1') if $Y_I > 0$, and -1 (bit '0') if $Y_I < 0$.

- For fixed $\{\beta_\ell\}$,

$$\begin{aligned} \mathrm{P_b} = \mathsf{P}(\{Y_I > 0\} \mid \{\text{bit '0' sent}\}) &= \mathsf{P}\left\{ W_I > \sum_{\ell=1}^{L} \beta_\ell^2 \mathcal{E}_{b,\ell} \right\} \\ = Q\left(\sqrt{2\sum_{\ell=1}^{L} \frac{\beta_\ell^2 \mathcal{E}_{b,\ell}}{N_0}} \right) = Q\left(\sqrt{2\sum_{\ell=1}^{L} \gamma_{b,\ell}} \right) &= Q\left(\sqrt{2\gamma_b}\right) \end{aligned} \tag{33}$$

where $\gamma_{b,\ell}$ is the received bit SNR on the ℓ-th channel, and $\gamma_b = \sum_{\ell=1}^{L} \gamma_{b,\ell}$ is the total received bit SNR.

- The average BER is given by

$$\overline{\mathrm{P}}_\mathrm{b} = \int_0^\infty Q\left(\sqrt{2x}\right) p_{\gamma_b}(x)dx\,. \tag{34}$$

Thus, we may evaluate $\overline{\mathrm{P}}_\mathrm{b}$ by first finding the pdf $p_{\gamma_b}(x)$. This works well for Rayleigh fading. However, as shown below, $\overline{\mathrm{P}}_\mathrm{b}$ is more easily evaluated in the general case of Ricean fading using the complex Gaussian approach of (20), and we get the Rayleigh fading result as a special case.

- *General Ricean analysis using the complex Gaussian approach:*

Assume that β_ℓ is Ricean with Rice factor κ_ℓ. Write $\gamma_{b,\ell} = |Z_\ell|^2$ where $\{Z_\ell\}$ are PCG with means and variances:

$$m_\ell = \sqrt{\overline{\gamma}_{b,\ell}}\, \beta_{0,\ell}\, e^{j\phi_{0,\ell}} = \sqrt{\frac{\overline{\gamma}_{b,\ell}\, \kappa_\ell}{\kappa_\ell + 1}}\, e^{j\phi_{0,\ell}}, \ \text{ and } \sigma_\ell^2 = \mathsf{E}[|Z_\ell|^2] = \frac{\overline{\gamma}_{b,\ell}}{\kappa_\ell + 1}\,. \tag{35}$$

Then

$$\begin{aligned}
\overline{\mathrm{P}}_{\mathrm{b}} &= \int_0^\infty Q\left(\sqrt{2x}\,\right) p_{\gamma_b}(x)dx \\
&= \int_{\boldsymbol{z}} Q\left(\sqrt{2\sum_{k=1}^{L}|z_\ell|^2}\right)\ \prod_{\ell=1}^{L} \frac{1}{\pi\sigma_\ell^2}\ \exp\left(-\frac{|z_\ell - m_\ell|^2}{\sigma_\ell^2}\right)\ dz_1 \ldots dz_L \\
&= \frac{1}{\pi}\int_0^{\pi/2} \prod_{\ell=1}^{L} \left[\int_{z_\ell} \exp\left(-\frac{|z_\ell|^2}{\sin^2\theta}\right)\ \exp\left(-\frac{|z_\ell - m_\ell|^2}{\sigma_\ell^2}\right)\ dz_\ell\right]\ d\theta \\
&= \frac{1}{\pi}\int_0^{\pi/2} \prod_{\ell=1}^{L} \frac{(\kappa_\ell + 1)\sin^2\theta}{\overline{\gamma}_{b,\ell} + (\kappa_\ell+1)\sin^2\theta}\ \exp\left(-\frac{\overline{\gamma}_{b,\ell}\,\kappa_\ell}{(\kappa_\ell+1)\sin^2\theta + \overline{\gamma}_{b,\ell}}\right)\ d\theta\ .
\end{aligned} \tag{36}$$

This is best we can do for general Ricean fading. Further simplification is possible for Rayleigh fading.

- *Special case: Rayleigh fading.* If the fading is Rayleigh on all channels, i.e., $\kappa_\ell = 0$, for $\ell = 1, 2, \ldots, L$, then

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2} \prod_{\ell=1}^{L} \frac{\sin^2\theta}{\overline{\gamma}_{b,\ell} + \sin^2\theta}\ \ d\theta\ . \tag{37}$$

  ◦ Case 1: $\overline{\gamma}_{b,\ell}$'s are distinct for $\ell = 1, 2, \ldots, L$. Here

$$\prod_{\ell=1}^{L} \frac{\sin^2\theta}{\overline{\gamma}_{b,\ell} + \sin^2\theta} = \sum_{\ell=1}^{L} C_\ell \frac{\sin^2\theta}{\overline{\gamma}_{b,\ell} + \sin^2\theta}\ , \tag{38}$$

  where

$$C_\ell = \prod_{i \neq \ell} \frac{\overline{\gamma}_{b,\ell}}{\overline{\gamma}_{b,\ell} - \overline{\gamma}_{b,i}}\ . \tag{39}$$

  Thus

$$\overline{\mathrm{P}}_{\mathrm{b}} = \sum_{\ell=1}^{L} C_\ell \frac{1}{\pi}\int_0^{\pi/2} \frac{\sin^2\theta}{\overline{\gamma}_{b,\ell} + \sin^2\theta} d\theta = \sum_{\ell=1}^{L} \frac{C_\ell}{2}\left[1 - \sqrt{\frac{\overline{\gamma}_{b,\ell}}{1 + \overline{\gamma}_{b,\ell}}}\ \right] \tag{40}$$

  where the second equality follows from (22)

  ◦ Case 2: $\overline{\gamma}_{b,\ell}$'s are identical, i.e. $\overline{\gamma}_{b,\ell} = \overline{\gamma}_{\mathrm{b}}/L$ for all ℓ. Here

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2} \left(\frac{\sin^2\theta}{\frac{\overline{\gamma}_{\mathrm{b}}}{L} + \sin^2\theta}\right)^L d\theta = \left[A\left(\frac{\overline{\gamma}_{\mathrm{b}}}{L}\right)\right]^L\ \sum_{\ell=0}^{L-1} \binom{L-1+\ell}{\ell} \left[1 - A\left(\frac{\overline{\gamma}_{\mathrm{b}}}{L}\right)\right]^\ell \tag{41}$$

  with

$$A\left(\frac{\overline{\gamma}_{\mathrm{b}}}{L}\right) = \frac{1}{2}\left[1 - \sqrt{\frac{\overline{\gamma}_{\mathrm{b}}}{L + \overline{\gamma}_{\mathrm{b}}}}\ \right]\ . \tag{42}$$

  Note that the equation for $\overline{\mathrm{P}}_{\mathrm{b}}$ given in (41) is identical to that for BPSK in Nakagami-m fading with $m = L$ (see Problem 7 of HW#2).

  For large $\overline{\gamma}_{\mathrm{b}}$,

$$A\left(\frac{\overline{\gamma}_{\mathrm{b}}}{L}\right) \approx \frac{L}{4\overline{\gamma}_{\mathrm{b}}}\ \ \text{and}\ \ 1 - A\left(\frac{\overline{\gamma}_{\mathrm{b}}}{L}\right) \approx 1\ . \tag{43}$$

Thus

$$\overline{\mathrm{P}}_{\mathrm{b}} \approx \left(\frac{L}{4\overline{\gamma}_{\mathrm{b}}}\right)^{L} \sum_{\ell=1}^{L} \binom{L-1+\ell}{\ell} = \left(\frac{L}{4\overline{\gamma}_{\mathrm{b}}}\right)^{L} \binom{2L-1}{L}. \tag{44}$$

Note that with diversity $\overline{\mathrm{P}}_{\mathrm{b}}$ decreases at $(\overline{\gamma}_{\mathrm{b}})^{-L}$ which is a significant improvement over the inverse linear performance obtained without diversity. (See Figure 1.)

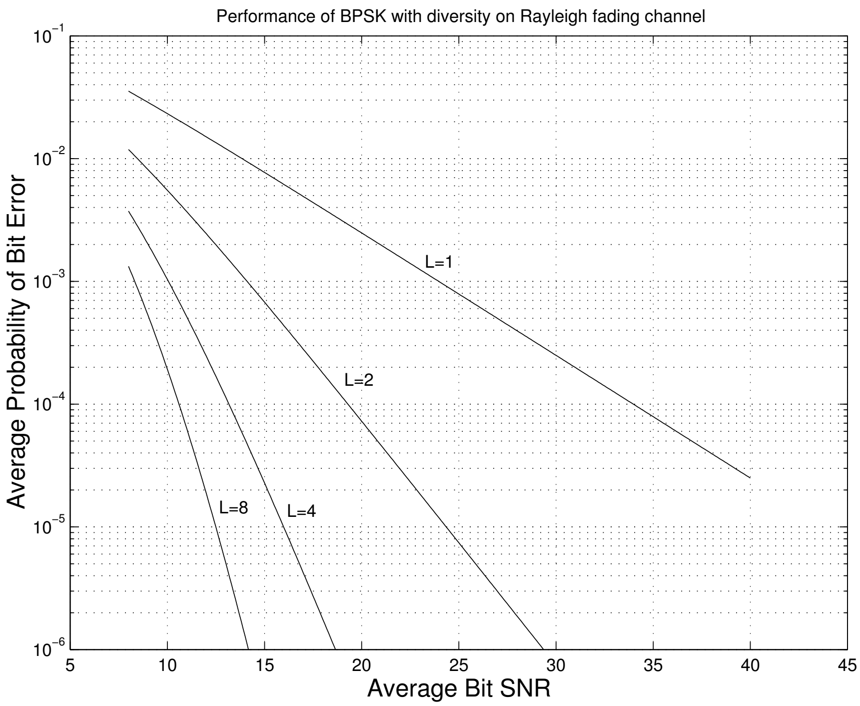

Figure 1: BPSK with diversity on Rayleigh fading channels.

References

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