Wireless Communications
ECE 559-VV

Low Complexity Multiuser Detection

Final Report

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Introduction

Multiuser detection (MUD) refers to the joint decoding of users' signals in wireless systems and was first proposed by Verdu in [1]. Instead of viewing the users as interference noise, MUD tries to cancel the effect that each user has on the other to achieve significant capacity increase and near/far resistance. MUD provides significant benefits in Code Division Multiple Access (CDMA) system. First, the increase in performance with MUD results in the increase the system capacity as more interference can be tolerated. Second, since MUD allows weaker users to be detected in presence of strong interferes, less stringent power control is needed and a better near-far resistance is achieved [2,3]. Inspite of these improvements different limitations hinder the application of MUD in wireless systems. Intercell interference limits the capacity improvement with MUD. Unless the users from neighboring cell are considered resulting in a larger increase in complexity, the capacity increase in the system is limited. Another factor to consider in applying MUD is the spreading coding trade-off [4] where coding can narrow the gap in performance between low complexity receivers and more advanced MUD receivers.

In this report we present an overview of different receivers in MUD as outlined in Fig. 1. All these type of receivers present a trade-off between complexity and performance. Optimum maximum-likelihood (ML) multiuser detector provides close performance to the single user channel but at an exponential increase in complexity [1]. Suboptimal approaches have been proposed to reduce the detector complexity and can be classified into three main categories: linear, interference cancellation, and near-ML detectors.

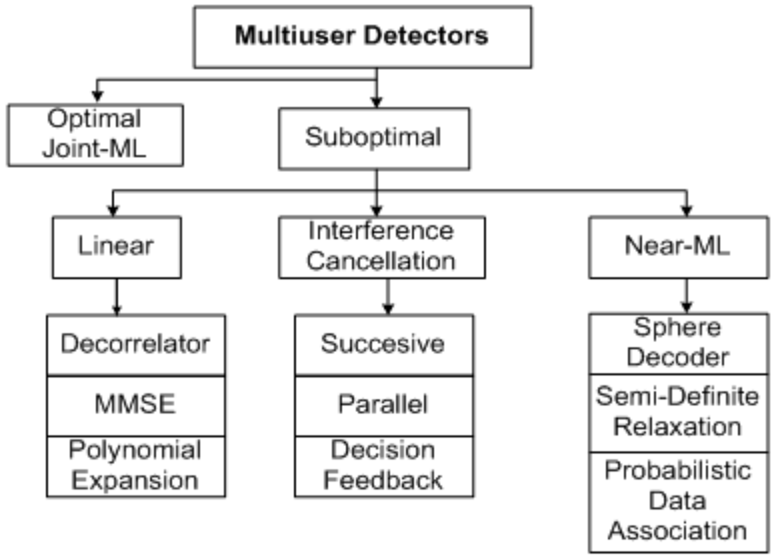

Figure 1: Different types of MUD receivers

Problem Setup and Optimum Detection

DS-CDMA allows multiple users to transmit at the same time using the whole available spectrum. Users are distinguished by their spreading code. In the following, we assume that there are K users in the system and all received signal are synchronous. Each user is transmitting a symbol z from a constellation (Ω) with amplitude *a* and a signature code waveform *g(t).* The signal at the receiver is the sum of all user signals corrupted by complex Gaussian noise n(t) with mean zero and variance No and is given by:

$$r(t) = \sum_{l=1}^{K} a_l z_l g_l(t) + n(t) \quad (1)$$

Conventional receivers treat the interfering users as noise. The codes are designed so that their autocorrelations are greater than the correlation between different code waveforms. Conventional receivers perform matched filtering (MF) on r(t) as shown in Fig. 2.

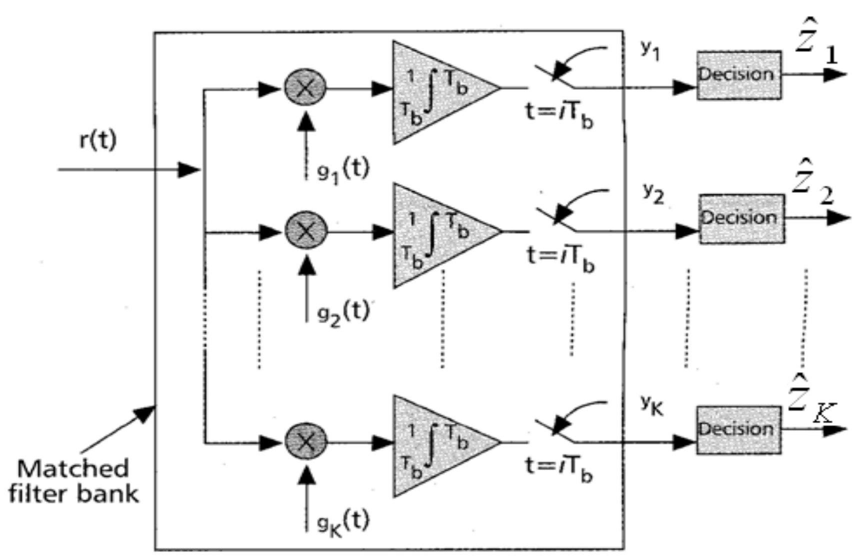

Fig.2: Conventional Receiver in DS-CDMA [3].

The output of the *kth* correlator is given by:

$$\begin{aligned} y_k &= \langle r(t), g_k(t) \rangle \\ &= a_k z_k + \sum_{l \neq k} a_l z_l \rho_{lk} + w_k \qquad where \quad \rho_{lk} = \langle g_l(t), g_k(t) \rangle \end{aligned} \quad (2)$$

The decision on the symbol of a user k depends on the correlation of the kth user only and the rest of the correlations (Multiple Access Interference (MAI)) treated as noise. Therefore, large degradation in performance can be observed as the number of interfering users increases. This loss in performance is more severe with the near-far problem where users communicate with different amplitudes due to their different geographical locations from the base station. Stronger users will start to overwhelm the weaker users and the need for MUD arises.

Multiuser detection was proposed by [1] in 1986. Optimum multiuser detection involves the joint detection of the different users in presence of noise. From (1) since n(t) is Gaussian it can be easily seen that maximum likelihood decoding is given by:

$$
\begin{aligned}
\mathbf{z}_{ML} &= \arg\max_{\mathbf{z}\in\Omega^K} \int_0^T \left[y(t) - \sum_{l=1}^{K} A_l z_l s_l(t) \right]^2 dt \\
&= \arg\max_{\mathbf{z}\in\Omega^K} \; 2\mathbf{z}^* A\mathbf{y} - \mathbf{z}^* ARA\mathbf{z} \qquad \textit{where} \quad R = \left[\rho_{lk}\right]_{\substack{l=1:K \\ k=1:K}} \\
&= \arg\max_{\mathbf{z}\in\Omega^K} \; 2\mathbf{z}^* A\mathbf{y} - \mathbf{z}^* H\mathbf{z} \qquad\qquad A = diag(a_1, ..., a_K)
\end{aligned}
$$

The search space in ML decoding is exponential in the number of users so that an exhaustive search is hard to implement. In asynchronous CDMA, the Viterbi algorithm can be applied for the ML decoding with a complexity still exponential in K [5]. Several suboptimal MUD schemes have been proposed to decrease the receiver complexity. Three main categories can be found for suboptimal receivers: Linear detectors, Subtractive-interference cancellation, and Near-ML detectors. In the following, we present an overview of the detectors in each category.

Linear Detectors

These detectors apply a linear transformation (T) to the soft output of the conventional MF inorder to minimize a certain criteria such as MAI and mean square error as shown in Fig. 3. This gives rise to two popular detectors decorrelating detector and minimum mean square error detector which will be presented next in addition to the polynomial expansion detector. The overview presented here is mainly based on information presented in [2, 3, and 5].

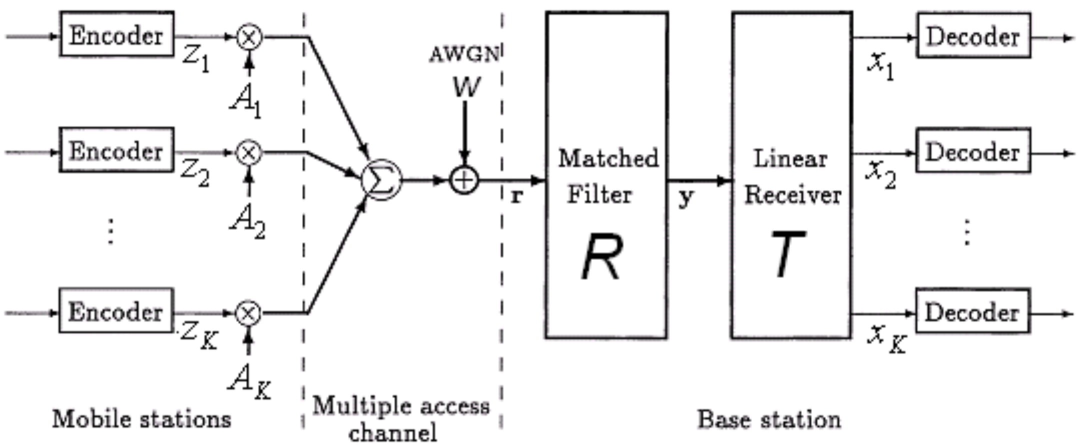

Fig 3.: Linear Receiver for multiuser detection

Decorrelating Detector

The linear transformation for the decorrelating is the inverse of the correlation matrix R. The decision under the decorrelating detector is

$$\mathbf{x} = R^{-1}\mathbf{y} = A\mathbf{z} + R^{-1}\mathbf{w}$$
$$\Rightarrow x_k = A_k z_k + \tilde{w}_k$$

This detector completely eliminates MAI but at the expense of the increase in the noise power due to the multiplication by the inverse of R. This detector is characterized by its low complexity and best near-far resistance and also it can be shown that it achieves ML detection when A is not known at the receiver [3].

Minimum Mean-Squared Error Detector

The MMSE is designed to minimize the mean-squared error (MSE) between its output and the transmitted symbol, specifically

$$T_{MMSE} = \arg\min_{T} E\left[\|T\mathbf{y} - \mathbf{z}\|^2\right]$$

It can be shown that (for example see [5]) the MMSE detector is given by

$$\mathbf{x} = T_{MMSE}{}^{-1}\mathbf{y} \;\; where \;\; T_{MMSE} = \left(R + N_0 A^{-2}\right)^{-1}$$

That is, the MMSE detector takes into account the MAI and the background noise resulting in a better performance than the decorrelating detector. For example, if the noise standard deviarion (No) is small then the MMSE detector will approximate the decorrelating detector. The MMSE detector suffers some loss in near-far resistance over the decorrelating detector as it depends on the signal amplitudes (A).

Polynomial Expansion Detector

In the polynomial expansion (PE) scheme [6], the detector is a linear combination of powers of the correlation matrix R and is expressed as:

$$\mathbf{x} = L_{PE}\mathbf{y} \;\; where \;\; L_{PE} = \sum_{i=0}^{N} w_i R^i$$

The weighs (w) can be fixed or updated adaptively to minimize a performance criterion such as MSE or approximate other linear detectors. Using Cayley-Hamilton theorem, for example, the weights can be chosen for the PE detector to approximate a decorrelating detector. The attractive feature of the PE detector is that it avoids matrix inversions and lends itself to a regular implementation (see Fig. 4).

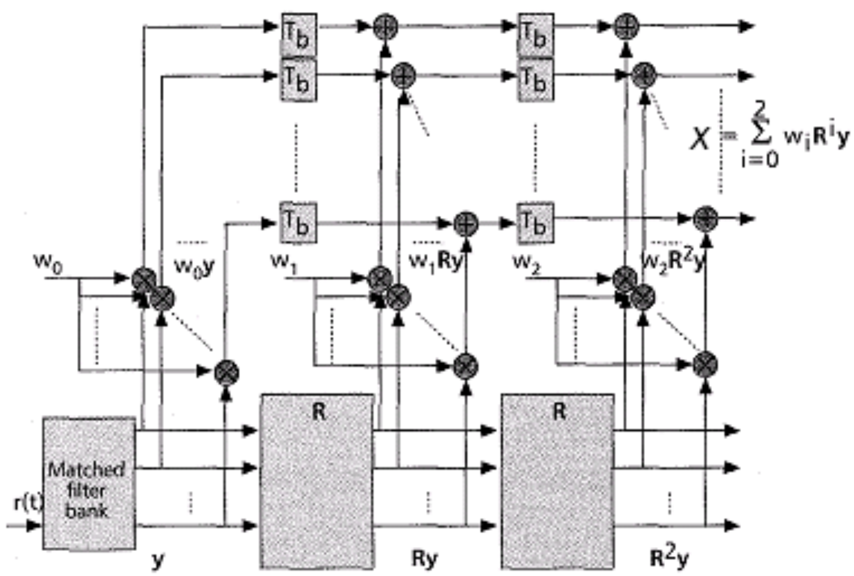

Figure 4: Polynomial expansion detector for MUD [3].

Subtractive-Interference Cancellation

In this class of MUD, the decisions of different users are utilized to cancel MAI. They are often implemented using multiple stages with the decisions improving at each stage. Three major types of detectors are found in this class and will be discussed next. A combination of these different can also be utilized to obtain different schemes of interference cancellation. The overview presented here is mainly based on information presented in [2] and [3].

Successive Interference Cancellation

Successive interference cancellation (SIC) cancels MAI serially. At each stage, decision on a single user is made after using the decision of the already detected users to cancel their contribution to MAI. Therefore, remaining users will experience less MAI. Before starting SIC, users need to be arranged in descending order of their power so that decisions at the starting stages are more reliable. SIC detectors require only a minimal additional amount of hardware over the conventional receiver (see Fig. 5). As the number of stages increases to improve performance, the latency of the system increases. A trade-off exists between the latency and the number of users to cancel. Another implementation difficulty of SIC is the need to reorder the users when the power profile of the system changes. One of the limitation of the SIC is the effect of wrong initial decisions on the performance as they are propagated across successive users.

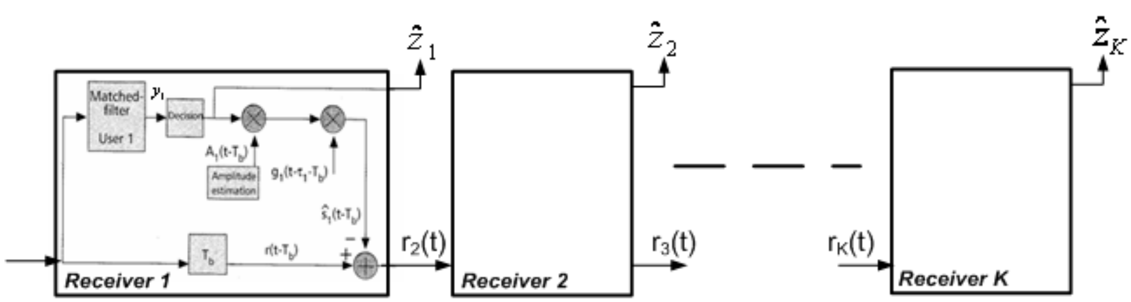

Figure 5: Successive interference cancellation detector

Parallel Interference Cancellation

Parallel interference cancellation (PIC) subtracts the MAI contributions of all users in parallel. An initial stage based on a MF, decorrelator, or a MMSE is used to generate initial decisions on the interfering users. PIC detection is based on multiple stages with decisions improving at every stage. At each stage, the decisions of the previous stage are used to estimate the MAI for each user which are then subtracted from the received signal to generate updated decisions for all users in parallel as shown in Fig. 6 and the decision-update equation ate stage *i* is given by: $\hat{\mathbf{z}}(i+1) = \mathbf{y} - GA.\hat{\mathbf{z}}(i)$

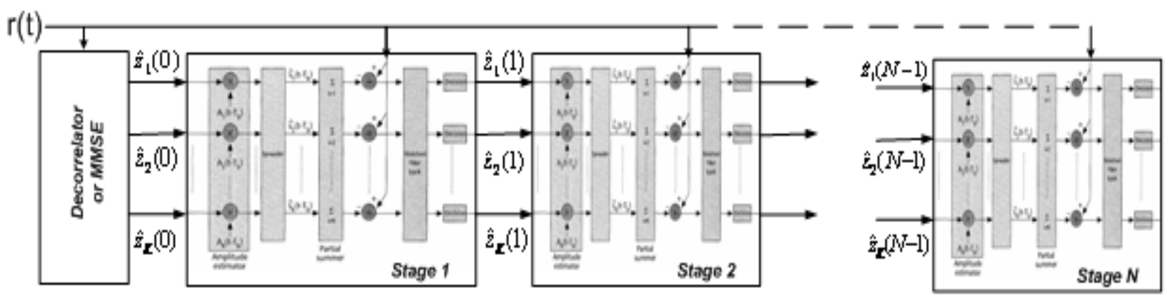

Figure 6: Parallel interference cancellation with N stages.

Variations of the PIC detector include the multistage decision feedback detector where instead of using the decisions at the previous stage, the decisions of the already detected users in the current stage are used [7].

Another alternative includes partial PIC which takes into account the fact that the decisions at the starting stages are unreliable so that less MAI cancellation is used at these stages with this cancellation increasing in consecutive stages [8]. The decision update in this case can be derived based on $\mathbf{p}\left(\hat{\mathbf{z}}(i+1) \middle| \mathbf{y}, GA.\hat{\mathbf{z}}(i), \hat{\mathbf{z}}(i)\right)$ and is expressed as

$$\hat{\mathbf{z}}(i+1) = p_i\left(\mathbf{y} - GA.\hat{\mathbf{z}}(i)\right) + (1 - p_i)\hat{\mathbf{z}}(i)$$

where p_i determines the contribution of the MAI cancellation and is a function of the standard deviations and correlations between decisions at consecutive stages and can be chosen based on simulations. Figure 7 shows a large improvement in system performance by using partial PIC where degradation refers to the increase in the SNR needed to accommodate additional user in the system while maintaining an average BER of 10^{-3}.

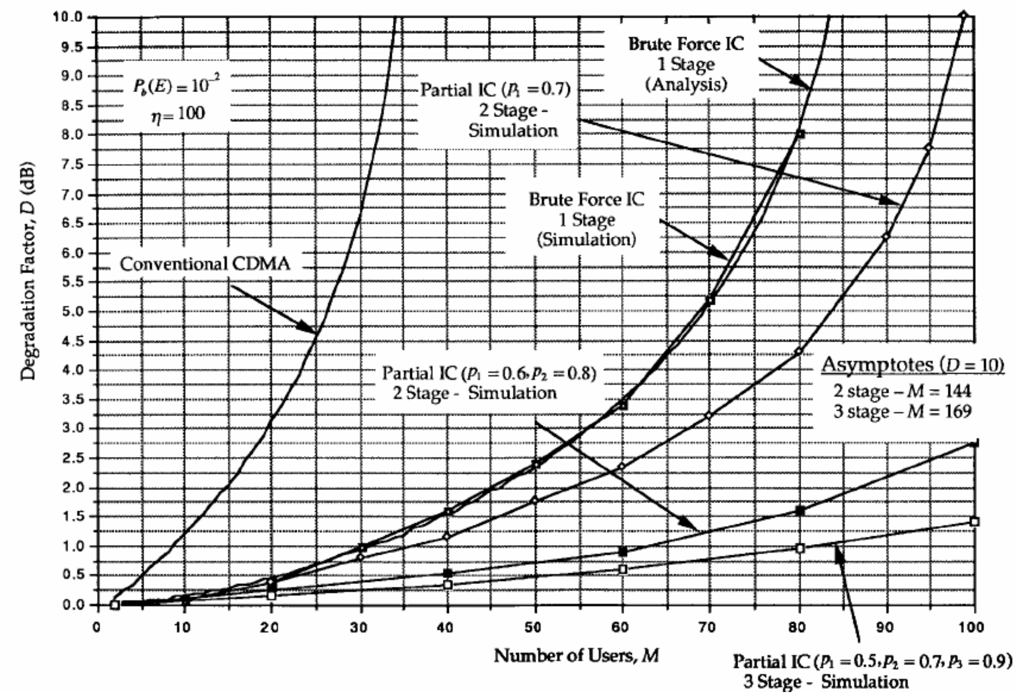

Figure 7: SNR needed to accommodate the increase in the number of users at a 10^{-3} BER [8].

Decision Feed-back Detectors

The decision feedback for MUD was proposed in [9] and consists of two stages (see Fig. 8):

1. A linear transformation is applied to decorrelate the noise across the different users without noise enhancement. This accomplished by using a Cholesky decomposition of R=F*F where F is lower triangular. The received vector can then be expressed as

$$\left(F^*\right)^{-1}\mathbf{y} = \tilde{\mathbf{y}} = F\mathbf{Az} + \mathbf{n}$$ where **n** is white Gaussian noise.

2. A SIC operation is applied to cancel the MAI for each user based on the lower triangular form of the matrix F: $\hat{z}_k = \tilde{y}_k - \sum_{i=1}^{k-1} f_{i,k} A_i \hat{z}_i$

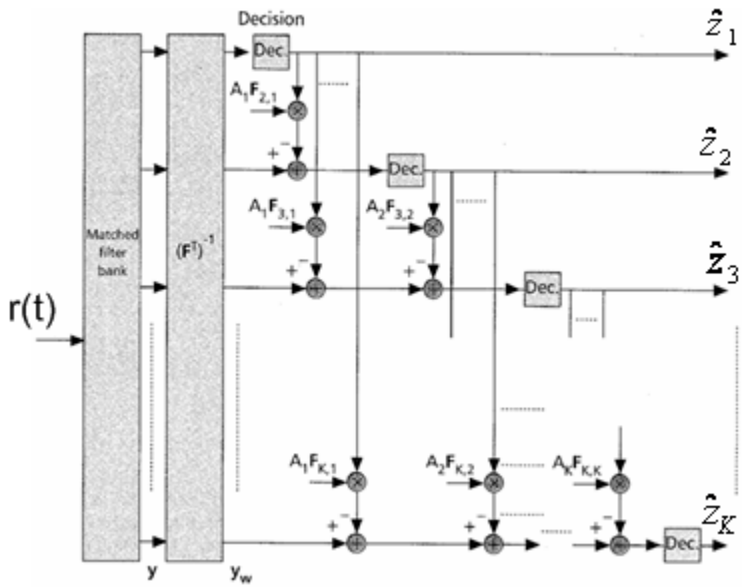

Figure 8: Decision feedback detector [3]

The decision feed-back can also be extended to include more than one stage with decision improving at each stage. Figure 9 shows performance of decision feedback in a 2 user system. Another extension is to apply a feedforward and feedback filter (transformation) on the received vector where the coefficients are chosen to minimize the mean square error.

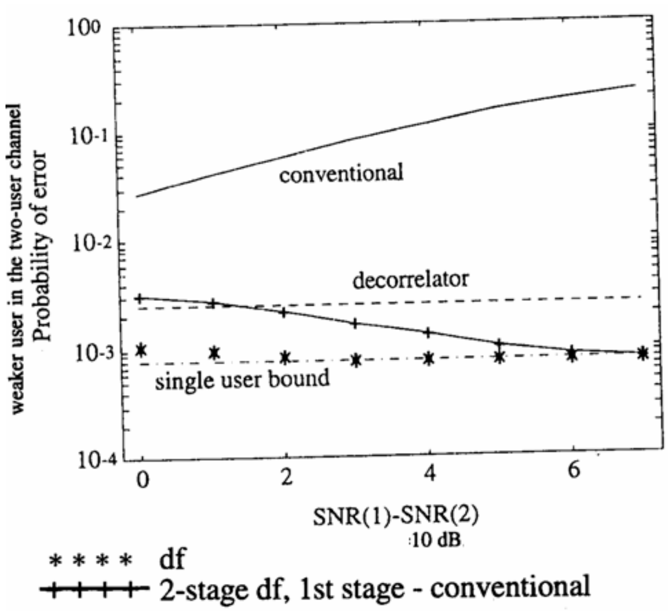

Figure 9: Decision feed-back detector in 2 user system. BER for user 2 whose SNR is fixed [9].

Near-ML Multiuser Detection

This class of detectors provides better performance than the detectors discussed so far but at an increased complexity. In this class we will discuss sphere decoder that tries to reduce the search space for ML decoding, semi-definite relaxation that relaxes the original ML problem so that the solution is easier to find, and probabilistic data association that treats interfering users as random variables.

Sphere Decoders

Sphere decoder (SD) or lattice decoders are considered a promising alternative for low complexity and high throughput near ML- decoding [10]. The basic idea is to restrict the search space to a given sphere around the received vector. The general detection problem for SD can be derived for the Gaussian channel. For MUD detection, a preprocessing step is needed to transform the channel to Gaussian as will be discussed later. With Gaussian noise the received vector is given by

$$\mathbf{y} = H\mathbf{z} + \mathbf{n}$$

Where H is the channel matrix and **n** is additive white Gaussian noise. ML Detection will search the whole space to minimize ML probability so that the ML decision is:

$$\mathbf{z}_{ML} = \arg\min_{\mathbf{z}\in\Lambda} \left\| \mathbf{y} - H\mathbf{z} \right\|^2$$

If we assume that the transmitted symbol is **z~** then

$$\begin{aligned} \mathbf{z}_{ML} &= \arg\min_{\mathbf{z}\in\Lambda} \left\| H\tilde{\mathbf{z}} - H\mathbf{z} \right\|^2 \\ &= \arg\min_{\mathbf{z}\in\Lambda} (\mathbf{z} - \tilde{\mathbf{z}})^* H^* H (\mathbf{z} - \tilde{\mathbf{z}}) \end{aligned}$$

SD will limit the value of the metric to be minimized to R^2 so that only symbols **z** satisfying the following equation will be considered:

$$(\mathbf{z} - \tilde{\mathbf{z}})^* H^* H (\mathbf{z} - \tilde{\mathbf{z}}) \leq R^2$$

That is the search is restricted to a sphere of radius R and center z~ (see Fig. 10).

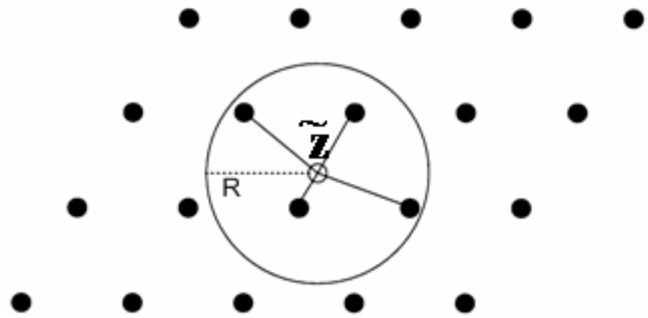

Figure 10: SD restriction of search space.

A preliminary step before SD is to triangularize the channel to enable layered decoding, i.e. channel H must be changed to a lower or upper triangular matrix. In Gaussian noise, QR decomposition is used to transform H into a unitary matrix Q and upper triangular matrix R. Therefore, multiplying by inverse of Q will result in the desired channel form with white Gaussian noise as follows

$$\mathbf{y} = H\mathbf{z} + \mathbf{n} = QR\mathbf{z} + \mathbf{n}$$

$$\underbrace{Q^{-1}\mathbf{y}}_{\textbf{New received vector}} = R\mathbf{z} + \underbrace{Q^{-1}\mathbf{n}}_{\textbf{Still AWGN with equal variance}}$$

In MUD, a Cholesky decomposition to transform H to a lower triangular matrix (L) and upper triangular (L*) is sufficient to triangularize the channel with white Gaussian noise [11] and the received vector will be

$$\tilde{\mathbf{y}} = \left(\mathbf{L}^*\right)^{-1}\mathbf{y} = \mathbf{Lz} + \mathbf{n}$$

Having expressed the channel in a lower (or upper) triangular noise with white Gaussian noise, the metric to minimize in SD can be expressed as a sum of partial Euclidian distance (C) in a layered manner and is given by

$$
\begin{aligned}
d(\mathbf{z}) &= \|\mathbf{y} - L\mathbf{z}\|^2 \\
&= C_{L_1}(z_1) + C_{L_2}(z_1, z_2) + C_{L_3}(z_1, z_2, z_3) \\
&\quad + ... + C_{L_K}(z_1, z_2, ..., z_K)
\end{aligned}
$$

Where, $C_{L_i}(z_1,...,z_i) = \left| y_i - l_{i,1}z_1 - l_{i,2}z_2 - .... - l_{i,i}z_i \right|^2$

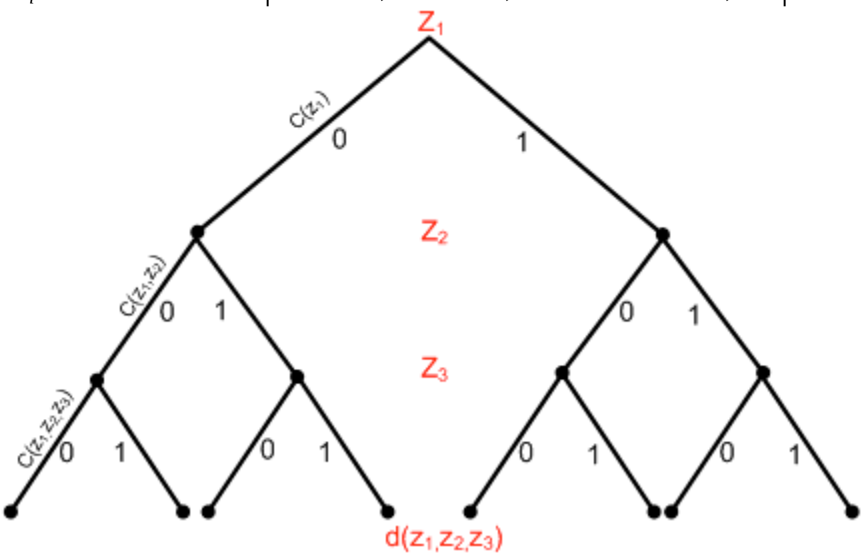

Figure 11: Tree based decoding

Therefore, the ML search can now be expressed in a tree form in Fig. 11 [12]. At each level the decision for one user is considered given the decision of the users in the upper levels. Each branch corresponds to the partial distance of the decision. Each pass through the tree from the root to one of its leaves will correspond to a hypothesis on the decisions and summing the partial Euclidean distances on each branch in the path will result in its ML metric. A full search ML will go through all paths from the root to all the leaves in the tree and then choose the path with the minimum metric. In SD, restricting the search to a radius R will discard the paths with metrics greater than the given value. SD proceeds through the tree in a downward manner through the tree and accumulates the partial Euclidean distance. The path is discarded if it has a value less than R. The radius R can also be updated after each pass through the tree to the minimum value found so far. This type of search is referred to as depth-first search [12]. Another type of restriction in SD is the number of path to keep at each level. This is referred to as breadth-first search or K-best search [12]. At each level, the paths with the best K metrics found so far are kept and the rest are discarded. A tradeoff between complexity and performance exists by choosing R or K. Increasing their values will improve performance as more candidates are considered but this comes at the cost of increase in complexity. Figure 12 and 13 shows that SD performs better than the decoders discussed so far and table 1 shows a 1000 times decrease in complexity over the full search ML decoder by using the SD.

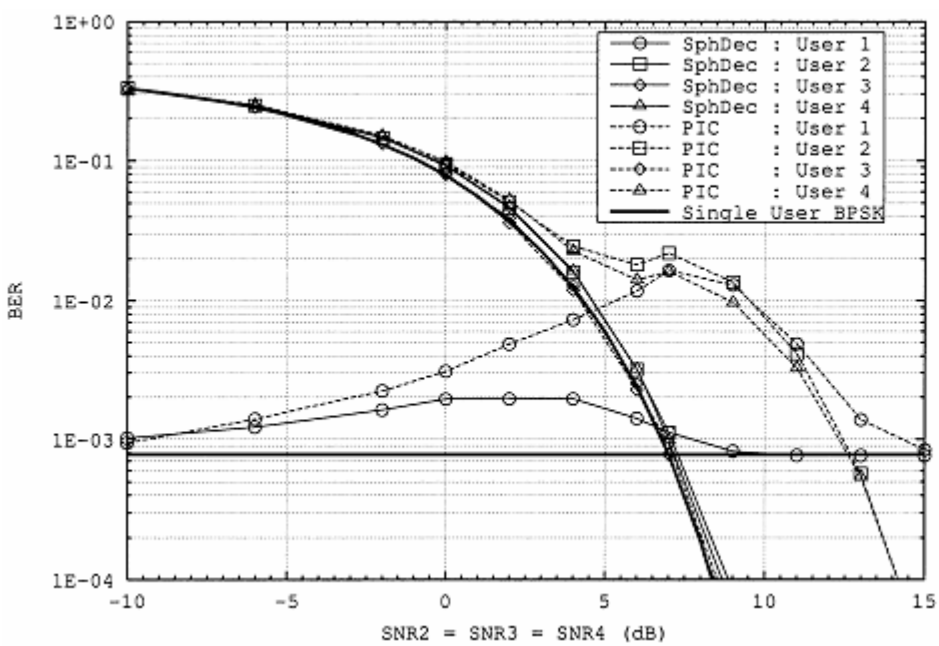

Figure 12: Performance of SD and PIC detectors with 4 users [11].

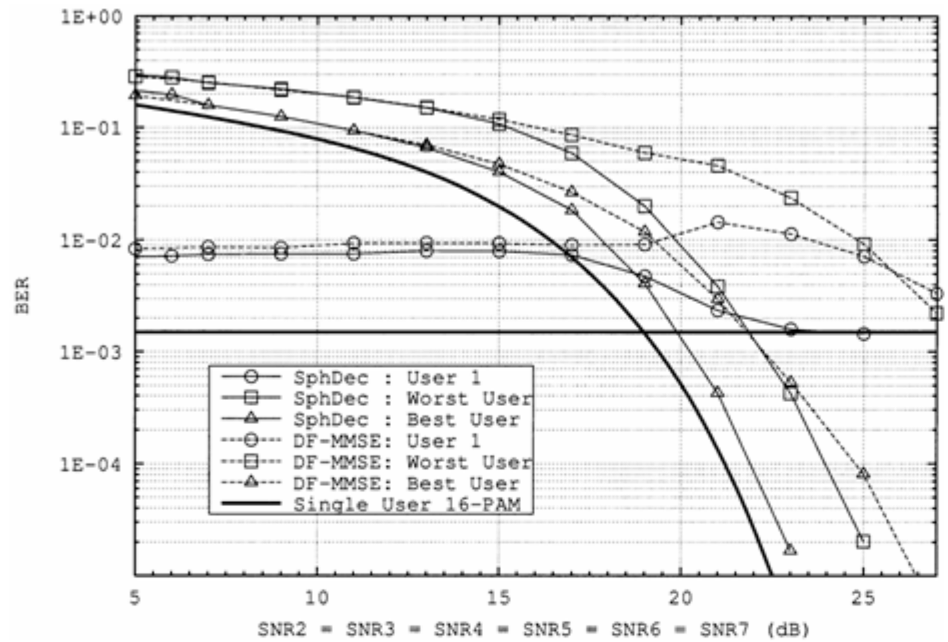

Figure 13: Performance of SD and DF detectors with 7 users [11].

Sphere Decoder

K	Additions per user	Multiplications per user	Divisions per user	Square Roots per user	Total per user	Total for all K users
4	111	68	14	14	208	832
7	480	332	49	49	910	6371

Full ML Search

K	Additions per user	Multiplications per user	Total per user	Total for all K users
4	10^5	10^5	2.10^5	8.10^5
7	6.10^8	6.10^8	12.10^8	8.10^9

Table 1: Complexity estimates of sphere decoders and full ML search in 4 and 7 user systems [11].

Relaxation and Heuristics

Unlike SD that decreases the search space, relaxations increase the search space by dropping certain constraints so that the new search problem is easier to solve. One of the simplest relaxations is unconstrained relaxation [13] where the constraint on alphabet is dropped and the search is done over the real numbers. Then, taking the sign of the vector over R, a decision on the original user signals can be made. The unconstrained relaxation in MUD can be written as:

$$\begin{aligned}\mathbf{z}_{opt} &= \arg\max_{\mathbf{z}\in\{+1,-1\}^K} 2z^* A\mathbf{y} - \mathbf{z}^* H\mathbf{z} \\ &= \arg\max_{\mathbf{z}\in\{+1,-1\}^K} J(\mathbf{z})\end{aligned} \quad \Longrightarrow \quad \begin{aligned}&\mathbf{r}_{opt} = \arg\max_{\mathbf{r}\in\Re^K} J(\mathbf{r}) \\ &\Rightarrow \mathbf{r}_{opt} = H^{-1}A\mathbf{y} \qquad \hat{\mathbf{z}} = \operatorname{sgn}\left(R^{-1}\mathbf{y}\right)\end{aligned}$$

A modification of this relaxation is to penalize the decisions that have large magnitudes [14]. A positive constant γ is used to obtain the following penalized unconstrained relaxation:

$$\begin{aligned}&\mathbf{r}_{opt} = \arg\max_{\mathbf{r}\in\Re^K} J(\mathbf{r}) - \gamma\|\mathbf{r}\|^2 \\ &\Rightarrow \mathbf{r}_{opt} = (H + \gamma I)^{-1} A\mathbf{y} \qquad \hat{\mathbf{z}} = \operatorname{sgn}\left(\left(R + \gamma A^{-2}\right)^{-1}\mathbf{y}\right)\end{aligned}$$

Depending on γ the relaxation can be viewed as a form of linear detection. For example if γ = 0 the relaxation is similar to a decorrelating detector and if γ =No the relaxation will be a MMSE detector.

Another type of relaxation is semi-definite relaxation (SDR) [14]. By adding some variable c = +/-1, the MUD detection problem can be expressed in the following manner:

$$\begin{aligned}\max_{z\in\{+1,-1\}} J(\mathbf{z}) &= \max_{\substack{z\in\{+1,-1\}^K \\ c\in\{+1,-1\}}} J(c\mathbf{z}) = \max_{\substack{z\in\{+1,-1\}^K \\ c\in\{+1,-1\}}} 2c\mathbf{z}^* A\mathbf{y} - \mathbf{z}^* H\mathbf{z} \\ &= \max_{\substack{z\in\{+1,-1\}^K \\ c\in\{+1,-1\}}} \begin{bmatrix}\mathbf{z}^* & c\end{bmatrix} \begin{bmatrix} -H & A\mathbf{y} \\ (A\mathbf{y})^* & 0\end{bmatrix} \begin{bmatrix}\mathbf{z} \\ c\end{bmatrix} \\ &= \max_{\mathbf{x}\in\{+1,-1\}^K} \mathbf{x}^* Q\mathbf{x} \qquad \textit{where } \mathbf{x} = \begin{bmatrix}\mathbf{z} \\ c\end{bmatrix} \\ &= \max \; Trace(\mathbf{X}Q) \qquad s.t \;\; \mathbf{X} = \mathbf{x}\mathbf{x}^*, \;\; \mathbf{x}\in\Re^K, \; X_{ii} = 1\end{aligned}$$

In SDR the rank one constraint on **X=xx*** is dropped and **X** is required to be only symmetric and positive semi-definite (noted by >0). The new relaxed search is given by:

$$\max \ Trace(\mathbf{XQ}) \quad s.t \ \ \mathbf{X} \succ 0, \ X_{ii} = 1$$

A polynomial time algorithm (O(K3.5)) can be found to the above relaxed search. More information on such algorithm can be found in [15]. After such a solution **X** is found a randomization approach can be applied to retain the original decisions (**x**). Decomposing **X** into any square root decomposition (**X**=**V***V**). Then a correspondence between the elements of **x** and the columns of **V** can be observed. The increase in the problem dimensions actually corresponds to the dropping of the rank one constraint.

$$\max_{\mathbf{x} \in \{+1,-1\}^K} \mathbf{x}^* Q \mathbf{x} = \max_{x_i^2 = 1, \ i=1,..K} \sum_{i=1}^{K} \sum_{j=1}^{K} x_i x_j q_{ij} \quad \Longleftrightarrow \quad \max \ Trace(\mathbf{X}Q) = \max_{\|\mathbf{v}_i\|^2 = 1, \ i=1,..K} \sum_{i=1}^{K} \sum_{j=1}^{K} \mathbf{v}_i^* \mathbf{v}_j X_{ij}$$

The randomization process applied to estimate **x** from **V** is given by:

$$\begin{aligned} & for \quad i = 1, ..., M_{rand} \\ & \quad \text{-} \ Randomly \ \ generate \ \ vector \ \ \mathbf{u} \\ & \quad \text{-} \ \tilde{\mathbf{x}}_\mathbf{i} = \mathbf{sgn}(\mathbf{V}^* \mathbf{u}) \\ & \hat{\mathbf{x}}_{SDR} = \arg\max_{i=1,...,M_{rand}} \tilde{\mathbf{x}}_\mathbf{i}^* Q \tilde{\mathbf{x}}_\mathbf{i} \end{aligned}$$

The number of randomization (Mrand) needed to obtain near ML performance is 10 in a 4 user system (see Fig. 14). With this number of randomization, the SDR is compared to other detectors in Fig.15.

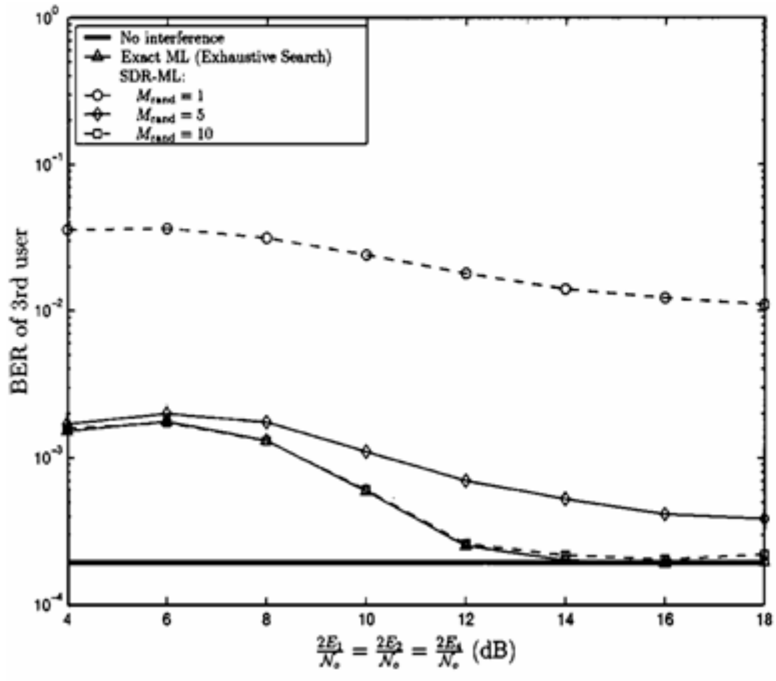

Figure 14: Effect of randomization on SDR performance [14]

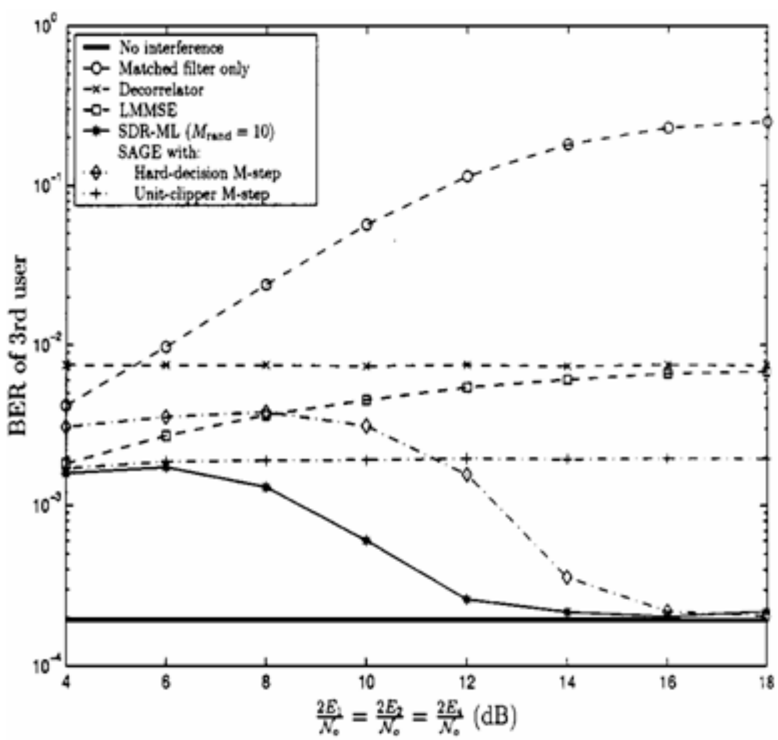

Figure 15: Performance of SDR in a 4 user system where SNR3=11 dB [14].

Probabilistic Data Association

Probabilistic data association (PDA) [16] is a multistage detector. At each stage a belief/probability is associated with the decision of each user. Recall that the received vector for user k is:

$$y_k = a_k z_k + \sum_{l \neq k} a_l z_l \rho_{lk} + w_k$$

The MAI contribution in PDA is treated as a random variables whose statistics are derived from the belief on **z**. Assuming BPSK modulation, PDA is given by:

- Order users in decreasing power
- Belief on the decision of user k at stage i is: $p_{z_k}^{(i)} \overset{\Delta}{=} p\left(z_k^{(i)} = 1\right)$
- At each stage, update the belief of each user by treating the decisions of other user as AWGN with the statistics of a sum of binomial random variables. Therefore the belief update equations are given by:

$$p_k^{(i)} = p\left(z_k^{(i)} = 1 \middle| y, \left\{p_l^{(i,i-1)}\right\}_{l \neq k}\right)$$
$$\sim CN\left(y_k - \sum_{l \neq k} a_l \left(2 p_l^i - 1\right) \sum_{l \neq k} 4 p_l^i \left(1 - p_l^i\right) AA^* + N_o R\right)$$

- Stop when belief converges
- Decide by comparing *p* to 0.5
-

Figure 16 shows that PDA has a very close performance to ML and its complexity is comparable to that of SD [16].

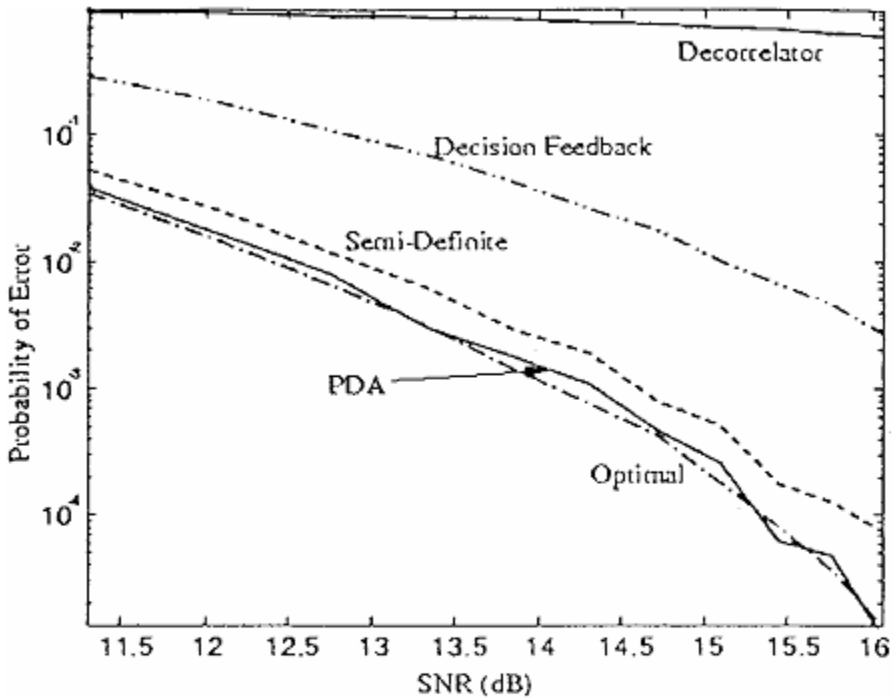

Figure 16: Average BER in a 29 user system [16].

Conclusions and Future Work

Several suboptimal schemes for MUD were presented in this paper. These schemes trade performance for complexity as the optimum MUD is prohibitively complex. The schemes discussed may apply for other detection problems such as MIMO Detection. Another topic of further study is iterative/turbo MUD where soft information on the users decisions from the individual channel decoders or equalizers is iteratively exchanged with the multiuser detector [17]. In such type of detection very low complexity MUD such as decorrelating detector or MMSE detector can be used with the turbo principle significantly improving its performance. Using suboptimal Viterbi decoders for MUD can also be a topic for further study where low complexity Viterbi algorithm with decreased number of states can be applied to MUD [18].

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