Space-Time Coding for Multi-Antenna Systems

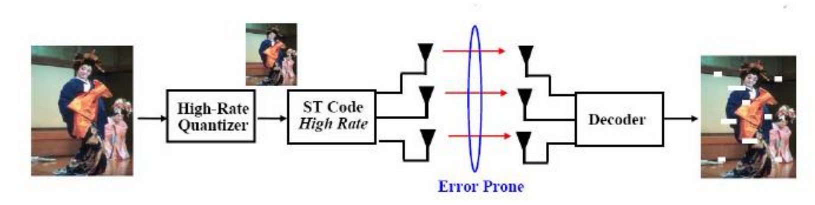
ECE 559VV Class Project

Sreekanth Annapureddy
vannapu2@uiuc.edu

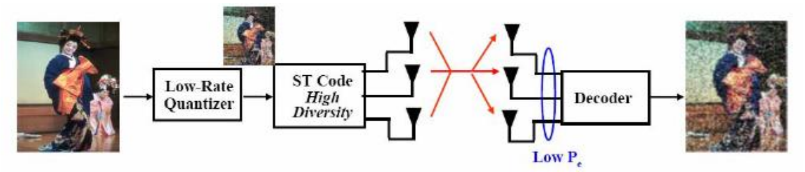
Dec 3rd 2007

MIMO: Diversity vs Multiplexing

Multiplexing

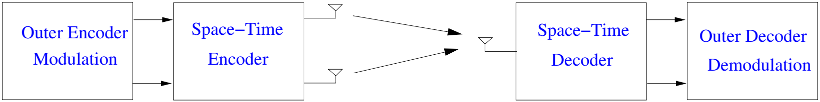


Diversity



Pictures taken from lectures notes on Space Time Coding at Univ. of Waterloo

Multi-Antenna Wireless Channel



$$Y = \sqrt{\frac{\rho}{N_t}} HX + Z$$

Space-Time Coding: Signal Processing Toolbox

Rate vs. Reliability vs. Complexity

Space Time Code: An example

MISO 2x1 Channel:

$$y = \sqrt{\frac{\rho}{2}}[h_1 \;\; h_2]X + z$$

- Channel Capacity: $I(X; y|H) = \log(1 + \frac{\rho}{2}||H||^2)$
- Achievable Distribution: Gaussian i.i.d. across space and time.

Achievable coding strategy: Joint ML decoding

Alamouti's Scheme

$$X = \begin{pmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{pmatrix}$$

- Effective Scalar Channel: $r = \sqrt{\frac{\rho}{2}}||H||s + w$
- Achievable Mutual Information: $\log(1 + \frac{\rho}{2}||H||^2)$

No Performance Loss! Low decoding complexity!

Is it possible to convert every MIMO channel into

- scalar effective channel ? (Low complexity)
- retaining optimality in mutual information sense ?
- if not, in some other weak sense ?

Orthogonal Space Time Block Codes (OSTBC): $N_t > 2$

- Orthogonal designs with $\eta = \frac{N_s}{T} = 1$ exist only for $N_t = 2$
- $N_t = 3$, $\eta = \frac{3}{4}$

$$X = \begin{pmatrix} s_1 & -s_2^* & -s_3^* & 0 \\ s_2 & s_1^* & 0 & s_3^* \\ s_3 & 0 & s_1^* & -s_2^* \end{pmatrix}$$

- $N_t = 4$, $\eta = \frac{3}{4}$

$$X = \begin{pmatrix} s_1 & -s_2^* & -s_3^* & 0 \\ s_2 & s_1^* & 0 & s_3^* \\ s_3 & 0 & s_1^* & -s_2^* \\ 0 & s_3 & -s_2 & -s_1 \end{pmatrix}$$

Properties of OSTBC

Codes exist for all N_t with $\eta = 0.5$

N_t	2	3	4	5	7
η	1	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{5}{8}$

Better efficiencies ?? (Open Research Problem)

Is it possible to convert every MIMO channel into

- scalar effective channel: Yes!
- with out loss in mutual information: No (Unless $N_t = 2, N_r = 1$)
- if not, in some other weak sense: Full diversity gain.

Alternate Space-Time Coding Techniques

- Linear Dispersion (LD) Codes (Hassibbi & Hochwald)
- Bell Labs Layered Space Time Codes (BLAST)

Space Time Coding: Perfect Feedback

Perfect knowledge of channel at transmitter converts the MIMO channel into $D = \min(N_t, N_r)$ parallel channels

$$r_i = \lambda_i s_i + w_i,$$

where $H = U \Lambda V^H \;\; R = U^H Y, S = V^H X, W = U^H Z$

- Improvement in performance
- Scalar channels with linear processing
- No loss in mutual information
- For all values of N_t, N_r

Finite Rate Feedback (B bits): Beamforming

Example of use of finite rate feedback: MISO 4 x 1

- No Feedback: Full diversity with $\eta = \frac{3}{4}$
- Antenna Selection: $B = \log_2(4) = 2$ bits. Index of the antenna with best antenna is fedback. Full diversity with $\eta = 1$
  Mutual information $= \log(1 + \rho \max(|h_i|^2))$

Can we achieve full diversity and full rate with less number of feedback bits?

Full diversity with one bit feedback: Precoded Alamouti

- Group antennas into 2 subgroups i.e., (h_1, h_2) and (h_3, h_4)
- Use the one bit feedback to inform which group is strong
- Transmitter does Alamouti coding on the selected antennas.

Mutual information $= \log\left(1 + \frac{\rho}{2}\max(|h_1|^2 + |h_2|^2, |h_3|^2 + |h_4|^2)\right)$

Optimality w.r.t. mutual information??

Capacity of MIMO with finite rate feedback

$$\text{Capacity} = \max_{\{Q_i\}} \sum_{i=1}^{2^B} \mathrm{E}_{H\in\mathcal{H}_i} \log |I + HQ_iH^H|$$

- Select 2^B transmit covariance matrices $\{Q_i\}$.
- Partition $\mathcal{H}$, the space of H, into 2^B regions $\{\mathcal{H}_i\}$ so that $|I + HQ_iH^H|$ is maximized.
- Compute the mutual information with the given $\{Q_i\}$ and $\{\mathcal{H}_i\}$
- Optimize over $\{Q_i\}$

Optimization is non-trivial

OSTBC with feedback achieving mutual information??

Alamouti scheme and beamforming achieves the capacity with and with out feedback respectively. Is there such a simple, yet optimal scheme with finite rate feedback?

Theorem

There exists a adaptive orthogonal space time code achieving the capacity of 2 x 1 MISO system with any number of feedback bits.

Proof.

- For every selection of $\{Q_i\}$, chose the matrix T_i such that $Q_i = T_i T_i^H$
- At the transmitter, perform Alamouti followed by precoding by T_i
- Thus, for every $\{Q_i\}$, there exists a adaptive OSTBC achieving the same mutual information

□

Extension to multiple transmit antennas

Theorem

There exists a adaptive orthogonal space time code using $B + n - 1$ feedback bits achieving the capacity of a 2^nx1 MISO system with B bits of feedback.

Proof.

Use the extra $n - 1$ bits to group the antennas into sub groups of size 2 and then use Alamouti scheme. □

Conjecture

There exists a threshold on B, beyond which adaptive OSTBC can achieve the capacity.

Summary

- Space-Time coding: Signal processing toolbox to transmit information over MIMO channels
- Orthogonal Space Time Block Codes (OSTBC) convert the MIMO channel into a scalar channel
- OSTBC are in general sub-optimal, except for Alamouti scheme and cannot be used for high data rates
- A few bits of feedback can be used to improve performance significantly

Thank You

Questions/Suggestions