ECE559VV Project Report

(Supplementary Notes)

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I. MAX SUM-RATE SCHEDULING: THE UPLINK CASE

We have seen (in the presentation) that, for downlink (broadcast) channels, the strategy maximizing the sum-rate is one which only transmits to the user with best channel reception [1]. We now present a similar result for uplink (multi-access) channels [2].

Consider the single-cell multiuser uplink scenario where the signal received by the base station is as follows:

$$y = \sum_{i=1}^{K} h_i x_i + z,$$

where K is the number of users in the cell, h_i and x_i are gain and information for user i, respectively. We assume that the information sources x_i's are zero-mean, have unit energy, and are mutually uncorrelated. The noise z is a zero-mean Gaussian random variable with variance N_0.

If the channel gains h_i's are deterministic, then this is simply a Gaussian multi-access channel whose capacity region is defined as follows [3].

$$\sum_{i\in\mathcal{S}} R_i \quad < \quad \frac{1}{2}\log\left(1+\sum_{i\in\mathcal{S}}\gamma_i\right), \qquad \forall\ \mathcal{S}\subset\{1,2,\ldots,K\},$$

where R_i and $\gamma_i = |h_i|^2/N_0$ are the information rate and received SNR of the user i, respectively. Therefore, the sum-rate is

$$\sum_{i=1}^{K} R_i \quad < \quad \frac{1}{2}\log\left(1+\sum_{i=2}^{K}\gamma_i\right).$$

Now, if we assume *frequency flat Rayleigh fading*, then h_i has a Rayleigh distribution, and in turn, γ_i has the following exponential distribution

$$p_{\gamma_i}(x) \quad = \quad \frac{1}{\gamma_{si}}\exp\left(\frac{x}{\gamma_{si}}\right)\mathcal{I}_{x\geq 0},$$

where γ_{si} is the average received SNR for user i.

We would like to find a power control law $\mu_i(\gamma)$ for user i, with $\gamma = [\gamma_1, \gamma_2, \ldots, \gamma_K]$, which takes the instantaneous received SNR of other users into account. This can be done by introducing some feedback between the base station and users. The goal is to maximize the sum-rate

$$C_{sum} = \frac{1}{2}\int\int\cdots\int \log\left(1+\sum_{i=1}^{K}\mu_i(\gamma)\gamma_i\right)p(\gamma)d\gamma$$

subject to the constraints that the average power is one unit, i.e.,

$$\int\int\cdots\int \mu_i(\gamma)p(\gamma)d\gamma = 1, \tag{1}$$

and $\mu_i(\gamma) \geq 0$. This is a standard convex optimization. Introducing the Lagrange multipliers, λ_i, for each constraint, we obtain the following Lagrangian form:

$$\begin{aligned} \max \quad L &= \int\int\cdots\int \log\left(1+\sum_{i=1}^{K}\mu_i\gamma_i\right)p(\gamma)d\gamma - \sum_{i=1}^{K}\lambda_i\left(\int\int\cdots\int \mu_i p(\gamma)d\gamma - 1\right) \\ s.t. \quad & \mu_i \geq 0, \qquad \forall\ i. \end{aligned}$$

The partial derivative is

$$\frac{\partial L}{\partial \mu_i} = \frac{\gamma_i}{1+\sum_{i=1}^{K}\mu_i\gamma_i} - \lambda_i.$$

Thus, the optimality condition yields

$$\begin{aligned} \frac{\gamma_i}{1+\sum_{i=1}^{K}\mu_i\gamma_i} - \lambda_i &= 0 \qquad \text{if } \mu_i > 0, \\ \frac{\gamma_i}{1+\sum_{i=1}^{K}\mu_i\gamma_i} - \lambda_i &< 0 \qquad \text{else}, \end{aligned}$$

and λ_i is chosen so that the average power condition (1) is satisfied. The above optimality condition can be rewritten as:

$$1+\sum_{i=1}^{K}\mu_i\gamma_i \geq \frac{\gamma_i}{\lambda_i}, \qquad \text{with equality iff } \mu_i > 0. \tag{2}$$

Assume that all $\frac{\gamma_i}{\lambda_i}$'s are different. Then we can see from the condition (2) that if $\mu_i > 0$, then $\mu_j = 0,\ \forall j \neq i$ and consequently,

$$\frac{\gamma_i}{\lambda_i} > \frac{\gamma_j}{\lambda_j},\ \forall j \neq i.$$

That is, the only user allowed to transmit at any given time is the one with the largest $\frac{\gamma_i}{\lambda_i}$. Now, suppose that all users have the same average received power. By symmetry, all λ_i's are equal. Then the above result can be interpreted as follows: only the user with largest instantaneous received power is allowed to transmit, and the others must remain quiet until one of them becomes the strongest user.

The power allocation for user i is a *water-filling* solution with the form

$$\mu_i(\gamma) = \begin{cases} \frac{1}{\lambda_i} - \frac{1}{\gamma_i} & \text{if } \gamma_i > \lambda_i \ , \ \gamma_i > \frac{\lambda_i}{\lambda_j}\gamma_j \ , \ j \neq i \\ 0 & \text{else} \end{cases}$$

where λ_i is chosen such that the condition (1) is satisfied.

II. More about Proportional Fair Scheduling (PFS)

A. Additional property of PFS

It has been shown in [4] that, for PFS, if the channel processes have the form $h_i(t) = a_i \cdot b_i(t)$ where a_i is a constant and the $b_i(t)$ processes are i.i.d, then in the long run the fractions of slots allocated to each user are equal. As a special case, in the static environment, PFS will become the equal-time scheduling.

B. Proportional fair with minimum/maximum rate constraints

Note that the PFS algorithm does not provide any guarantees on the service rate provided to any user. For some cases, we may need to introduce the minimum as well as maximum bandwidth to each user. Suppose that for each user i we have a minimum rate T_i^{min} and a maximum rate T_i^{max} and we want the average throughput T_i to satisfy $T_i^{min} \leq T_i \leq T_i^{max}$.

Therefore, the optimization problem becomes

$$\begin{aligned} \max \quad & \sum_i \log T_i \\ s.t. \quad & T_i^{min} \leq T_i \leq T_i^{max}, \ \forall i \\ & \{T_i\} \in \mathcal{C} \end{aligned} \tag{3}$$

where $\mathcal{C}$ denotes the capacity region of the system. An algorithm for this problem has been considered in [5]. The idea is to maintain a token counter $Q_i(t)$ for each user i, and it is updated according to the following rule:

$$Q_i(t+1) = \begin{cases} Q_i(t) + T_i^{token} - R_i(t) & \text{if user } i \text{ is served} \\ Q_i(t) + T_i^{token} & \text{otherwise} \end{cases}$$

where $T_i^{token} = T_i^{min}$ if $Q_i(t) \geq 0$ and $T_i^{token} = T_i^{max}$ if $Q_i(t) < 0$. Note that the update rule of $T_i(t)$ is the same as in the original PFS:

$$T_i(t+1) = \begin{cases} \left(1 - \frac{1}{W}\right) T_i(t) + \frac{1}{W} R_i(t) & \text{if user } i \text{ is served} \\ \left(1 - \frac{1}{W}\right) T_i(t) & \text{otherwise.} \end{cases}$$

At time t, the algorithm serves the user

$$i^*(t) = \arg\max_i \frac{R_i(t)}{T_i(t)} e^{\alpha_i Q_i(t)},$$

where α_i is a parameter that determines the timescale over which the rate constraints are satisfied.

The basic idea of the token counter is that if the average service rate to user i is less than R_i^{min} then $Q_i(t)$ is positive and then we are more likely to serve user i. Also, if the average service rate to user i is larger than R_i^{max} then $Q_i(t)$ is negative and then we are less likely to serve user i. Finally, it has been shown in [5] that the algorithm is asymptotically optimal with respect to the optimization (3) when W goes to infinity.

C. *Other questions*

1) What is the intuition behind the throughput updating rule in PFS?

   We can think of $T_i(t)$ as an estimate of the actual throughput for user i. The larger the window size W, the more accurate the estimate $T_i(t)$. Note that we cannot calculate the exact throughput on-the-fly: throughput is a long-term quantity by definition. So this is one way to estimate it.

2) How about the scheduling algorithms based on some function of h_i and $\frac{h_i}{E[|h_i|^2]}$, e.g., to schedule the user only if $|h_i|$ is larger than some threshold?

   This is related to the question I have during the presentation: what if we do not fix the power and find the optimal power allocation across time, e.g., a "water-filling" solution. I do not have an answer for this question.

3) What is the "rate region" achievable using the PFS scheme? How does it compare with "max sum-rate" mechanism?

   We have only one rate region (or capacity region) which is the region achievable by any scheduling scheme. The difference between PFS scheme and "max-sum-rate" scheme is that they get to *different points* in the capacity region: the "max-sum-rate" scheme gives us the point which has maximum total rate $\sum_i T_i$, while the PFS scheme gives the point which maximizes $\sum_i \log T_i$.

4) When is PFS better and when is "max-sum-rate" is better to use?

   I guess it depends on the goal of the system operator. In the user's point of view, PFS is better because it is fairer (at least it asymptotically achieves the proportional fair allocation). In the system's point of view, "max-sum-rate" might be better since it gives the maximum total throughput of the system.

III. Opportunistic beamforming [6]

A. Opportunistic beamforming versus space-time codes

The idea of *opportunistic beamforming* is motivated by the multiuser system point of view: the larger the dynamic range of channel fluctuations, the higher the channel peaks. Hence, in opportunistic beamforming, we use multiple transmit antennas to induce more randomness to channels. On the other hand, the *space-time codes* also use multiple transmit antennas in the point-to-point scenario. So we would like to compare between the opportunistic beamforming scheme and the space-time codes in a multiuser system.

Let us consider a multiuser downlink system with two transmit antennas at the base station. The best known space-time code for this scenario is the Alamouti scheme. We assume that the PFS scheduling scheme is used on top of it.

Consider the slow fading scenario. The Alamouti scheme essentially creates a single channel with effective SNR for user k given by

$$\frac{P(|h_{1k}|^2 + |h_{2k}|^2)}{2N_0}$$

where P is the total transmit power. This effective channel does not change with time in a slow fading environment, and hence, the PFS scheduling becomes the equal-time scheduling (see Section II-A). On the other hand, it has been shown in [6] that under opportunistic beamforming with PFS, for large number of users, the users are also allocated equal time with the following effective SNR:

$$\frac{P(|h_{1k}|^2 + |h_{2k}|^2)}{N_0}.$$

That is, the opportunistic beamforming has 3-dB gain more than the Alamouti scheme. Furthermore, both schemes yield a *diversity gain* of 2. Thus, in a multiuser system with enough users under PFS, the opportunistic beamforming scheme outperforms the Alamouti scheme.

Finally, the Alamouti scheme requires *separate* pilots for each transmit antenna and that receivers need to track the channels from both transmit antennas. However, the opportunistic beamforming does not require any of those. The same signal (pilot and data) goes through both transmit antennas, and the receivers only need to track the overall channel.

B. Other questions

1) If perfect CSI is assumed, the BS can beamform to the direction of the channel of the user to be scheduled, and this strategy seems to perform better than opportunistic beamforming. So what is the advantage of opportunistic beamforming?

The point of opportunistic beamforming is to introduce more randomness to the channels by adding more “dumb” transmit antennas. This addition is transparent to receivers. Comparing to the original system without opportunistic beamforming, the effort to get the channel state information with opportunistic beamforming is the same. But opportunistic beamforming yields better performance. So the perfect CSI assumption is for scheduling and beamforming the signal *before* it comes to the dumb antennas.

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