Capacity Analysis of Cellular CDMA Systems

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Capacity Analysis of Cellular CDMA Systems

- Outline:
  - Introduction
  - Reverse & forward link capacity analysis
  - Erlang Capacity
  - Capacity-Coverage Tradeoff
  - Effect of Soft Handoff
  - Capacity-Coverage Tradeoff with Soft Handoff
  - Capacity of UMTS systems

Introduction

- Capacity of a CDMA system is interference limited
- Assumptions
  - Users are power controlled by the BS
  - All BS's require the same power
  - Power control is exercised by the BS corresponding to maximum pilot signal
  - SIR based admission policy
  - Users are are uniformly distributed in each cell

Reverse Link Capacity

- Single Cell (Single User Detection):
  - SIR seen at the BS:

$$SIR = \frac{S}{(N-1)S + \eta}$$

where:
S: power of the received signal per user
N: number of users in the cell
η: Background noise

  - Equivalent to:

$$\frac{E_b}{N_0} = \frac{S/R}{(N-1)S/W + \eta/W}$$

Reverse Link Capacity

- Single Cell Capacity:

$$N = 1 + \frac{W/R}{E_b/N_0} - \frac{\eta}{S}$$

- For multi-cell systems, BS suffers from intra-cell as well as inter-cell interference

$$\frac{E_b}{N_0} = \frac{W/R}{(N-1) + I/S + \eta/s}$$

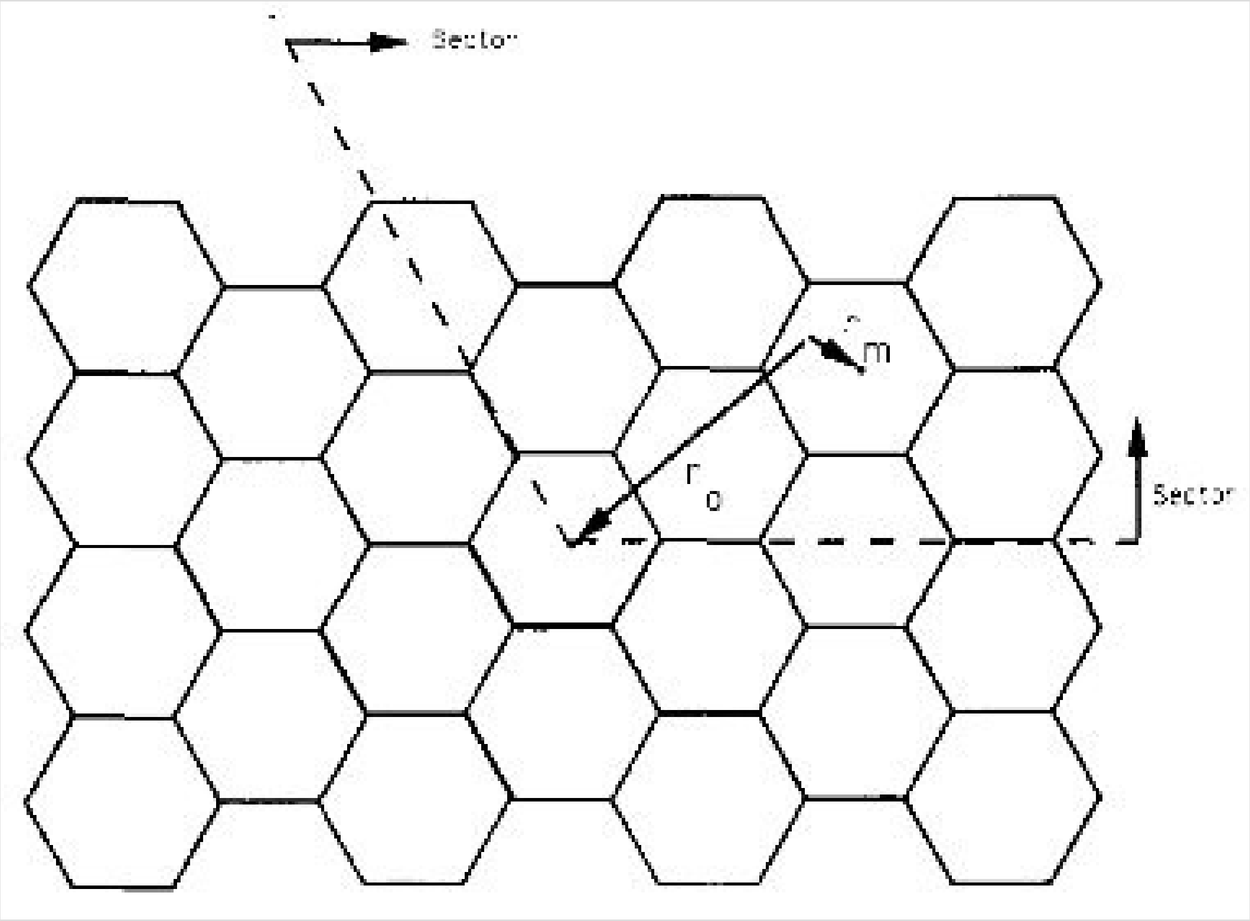
where, I: intra-cell interference (stochastic)

Reverse Link Capacity

- To find the capacity we need the distribution of I
- Depends on the attenuation due to large scale variations (path loss and shadow fading)
- G = $10^{(\xi/10)} r^{-4}$, $\xi : N(0, \sigma^2)$
- For a user at distance r_m from his BS and r_0 from the BS under consideration:

$$\frac{I}{S} = \left(\frac{10^{(\xi_0/10)}}{r_0^4}\right) * \left(\frac{r_m^4}{10^{(\xi_m/10)}}\right) = \left(\frac{r_m}{r_0}\right)^4 * 10^{((\xi_0 - \xi_m)/10)} \leq 1$$

Reverse Link Capacity



Glihousen et al.: On the capacity of a cellular CDMA system

Reverse Link Capacity

- Utilizing the voice activity:

$$\frac{E_b}{N_0} = \frac{W/R}{\sum_{i=1}^{N_s-1} \nu_i + I/S + \eta/s}$$

where ν_i is Bernoulli(ρ)

- Calculate the capacity based on BER for adequate performance: P(BER<10^-3)

Reverse Link Capacity

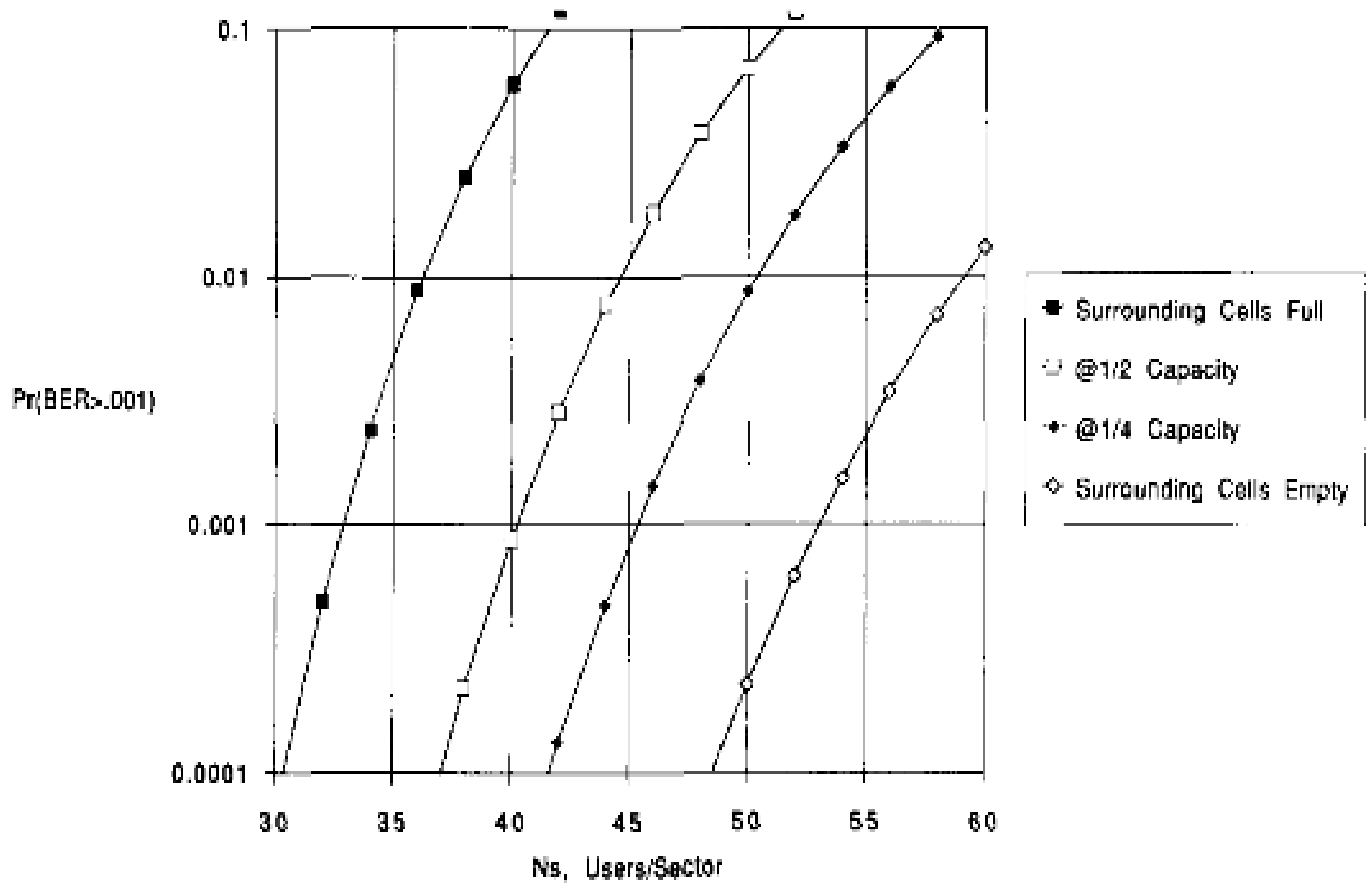


Fig. 3. Reverse link capacity/sector. (W = 1.25 MHz, R = 8 kb/s, voice activity = 3/8).

Glihousen et al.: On the capacity of a cellular CDMA system

Forward Link Capacity

- In most systems, the reverse link capacity is the limiting factor due to the limited power available for the subscribers
- Power control is also exercised in the forward link: Subscriber sends the power received from its BS and the total interference

Forward Link Capacity

- The ith subscriber SNR can be lower bounded by

$$\left(\frac{E_b}{N_0}\right)_i \geq \beta \phi_i \frac{S_{(T_1)}/R}{\left[\left(\sum_{j=1}^{k} S_{(T_j)}\right)_i + \eta\right]/W}$$

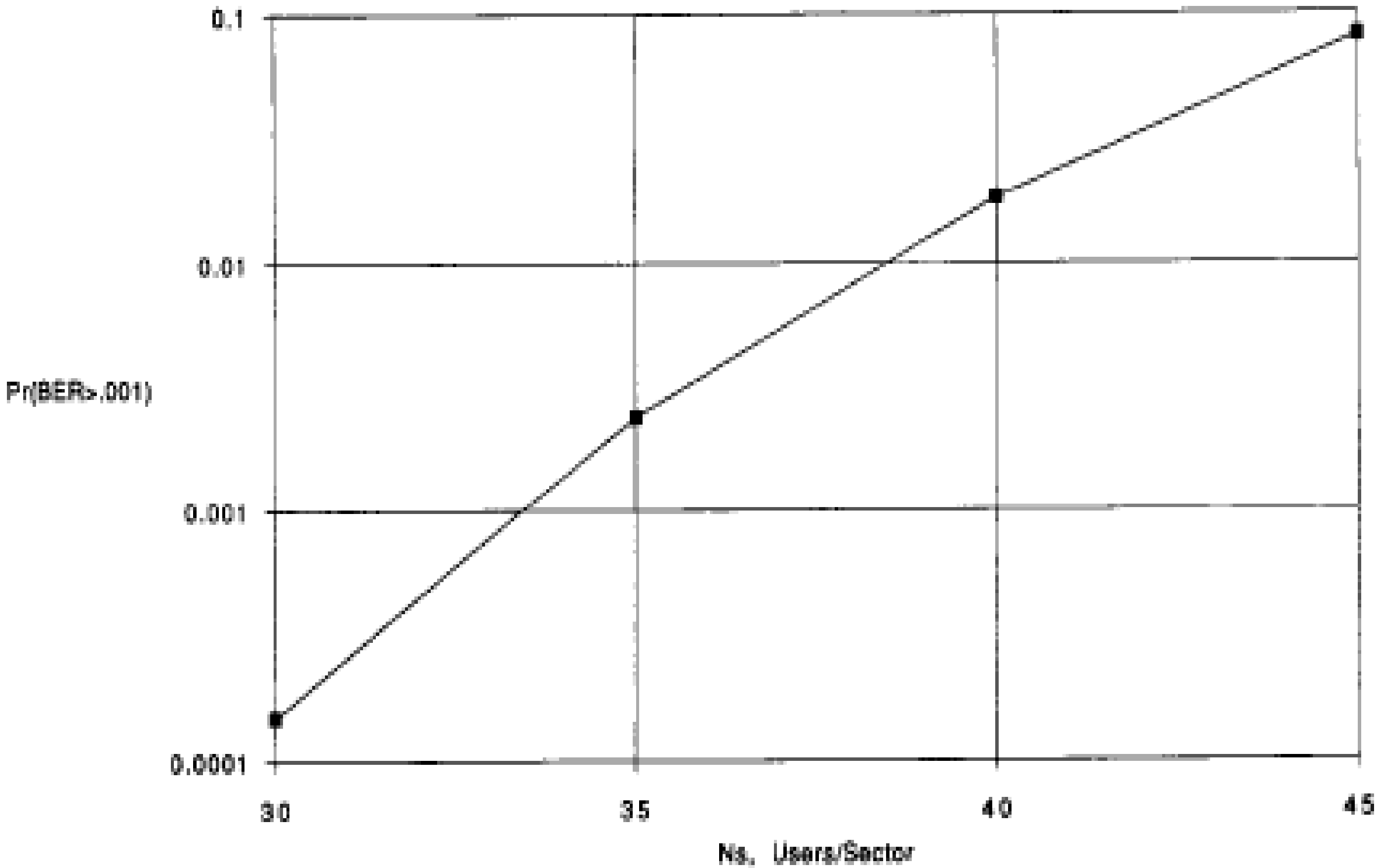
where:

β is the fraction of the total site power devoted to users (excluding pilot)

Φ_i is the fraction of power devoted to the i^{th} subscriber

S_{T_1} is the total power available from BS under consideration

Forward Link Capacity



Forward link capacity/sector. (W = 1.25 MHz, R = 8 kb/s, voice activity = 3/8, pilot power = 20 %).

Glihousen et al.: On the capacity of a cellular CDMA system

Erlang Capacity

- Def: The average traffic load in terms of average number of users requesting service resulting in a certain blocking probability
- Blocking Probability: the probability that a new user will find all channels busy and hence be denied service
- Condition: P($I_0/N_0 > 10$)<0.01

Reverse Link Erlang Capacity

- Simple Case:

a) constant number of users N_U in every sector,

b) each user transmits continuously,

c) users require the same E_b/I_0

- Condition for no blocking:

$$N_u E_b R(1+f) + N_0 W \leq I_0 W$$

$$N_u \leq \frac{(W/R)}{(E_b/I_0)} \cdot \frac{(1-\eta)}{(1+f)}$$

f: ratio of intra-cell interference to inter-cell interference

$\eta = N_0/I_0$

Reverse Link Erlang Capacity

- Practical case:

a) Number of active calls is a Poisson random variable with mean λ/μ

b) each user is gated on with probability ρ and off with probability $1-\rho$ (voice activity)

c) each user's received energy-to-interference ratio is varied according to propagation conditions

Reverse Link Erlang Capacity

- Condition for no blocking:

$$\sum_{i=1}^{k} v_i * E_{bi} * R + \sum_{j}^{other cells} \sum_{i=1}^{k} v_{i(j)} * E_{bi(j)} * R + N_0 * W \leq I_0 * W$$

and so:

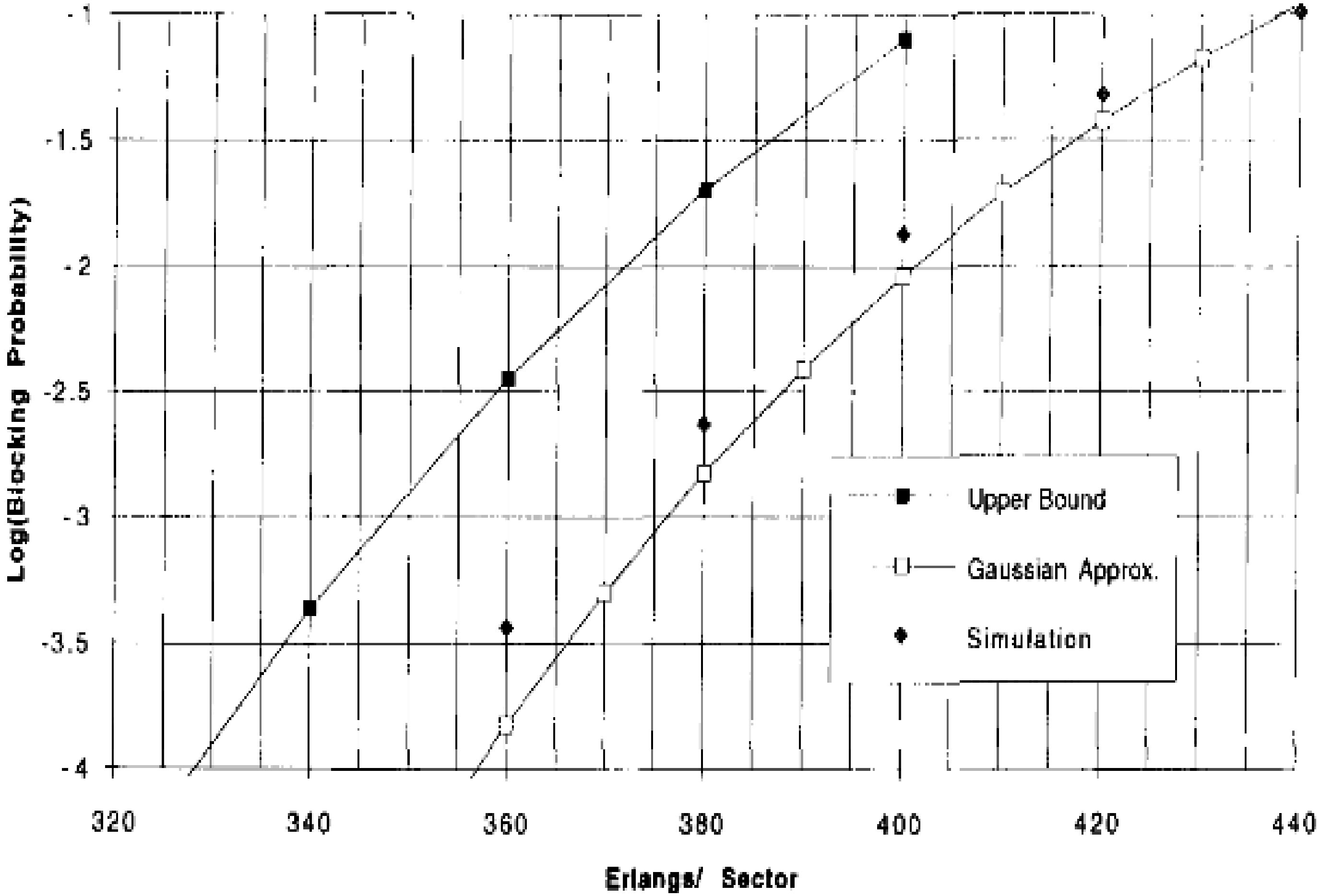
$$P\left\{Z = \sum_{i=1}^{k} v_i * \epsilon_i + \sum_{j}^{other cells} \sum_{i=1}^{k} v_i^{(j)} * \epsilon_i^{(j)} > \frac{W/R}{1-\eta}\right\} = P_{blocking}$$

where $\epsilon_i = E_{bi} / I_0$ (stochastic)

Reverse Link Erlang Capacity

- The statistics of ϵ_i depends on the power control mechanism
- Field trials with all cells fully loaded show that ϵ_i is well modeled as log-normal
- Chernoff pound for the outage probability can't be obtained because the moment generating function of ϵ_i doesn't converge

Reverse Link Erlang Capacity



Blocking probabilities for single cell interference (CDMA parameters: $W/R = 1280$; voice act. $= 0.4$; $I_0/N_0 = 10$ dB; median $E_b/I_0 = 7$ dB; sigma $= 2.5$ dB).

Viterbi & Viterbi: Erlang Capacity of Power Controlled CDMA System

Reverse Link Erlang Capacity

- Using Central Limit theorem for Z we get:

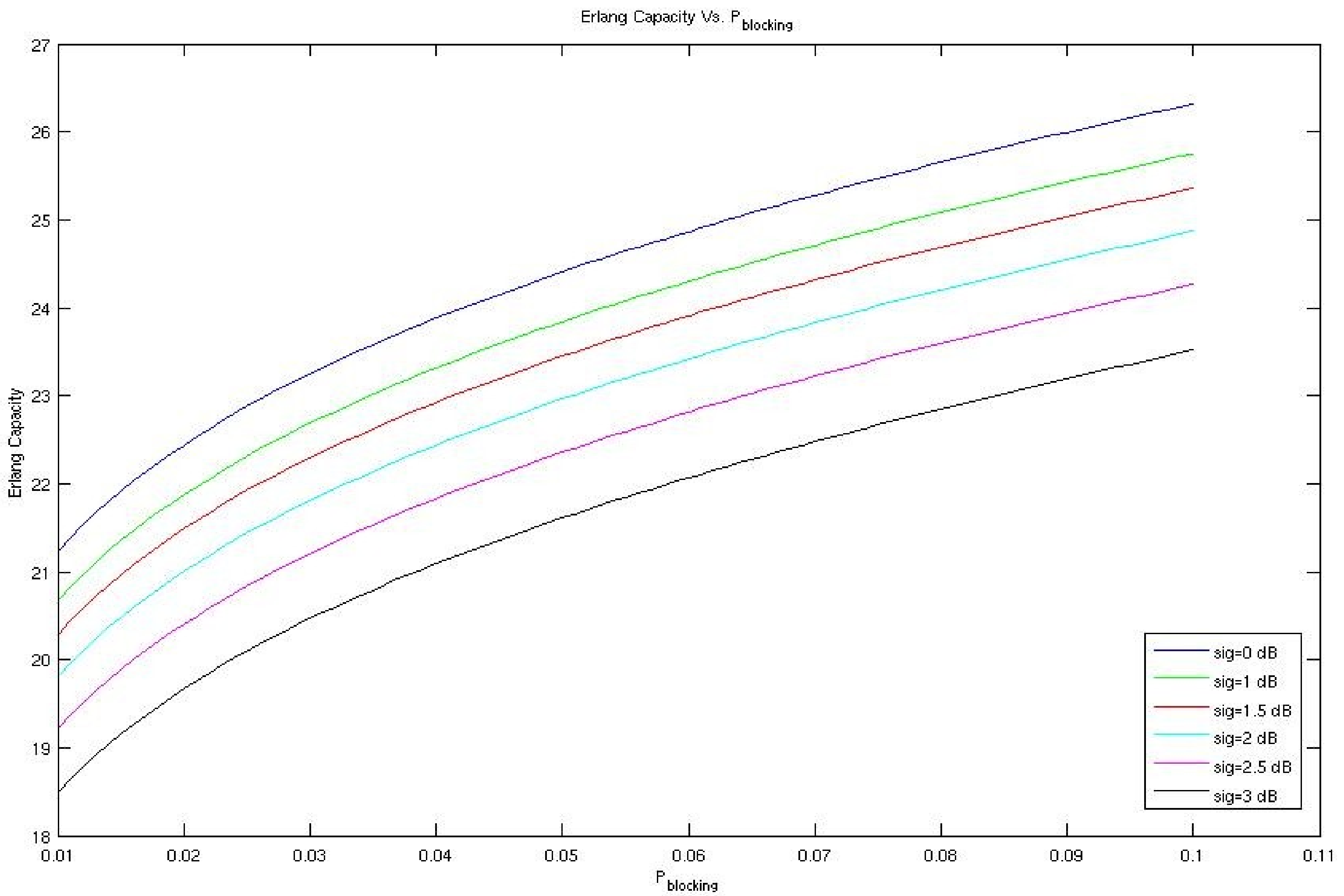
$$P\{Z=\sum_{i=1}^{k} \nu_i * \epsilon_i + \sum_{j}^{other\,cells} \sum_{i=1}^{k} \nu_i^{(j)} * \epsilon_i^{(j)} > \frac{W/R}{1-\eta}\} = P_{blocking}$$

$$P_{blocking} \approx Q[\frac{A-E(Z)}{\sqrt{Var(Z)}}]$$

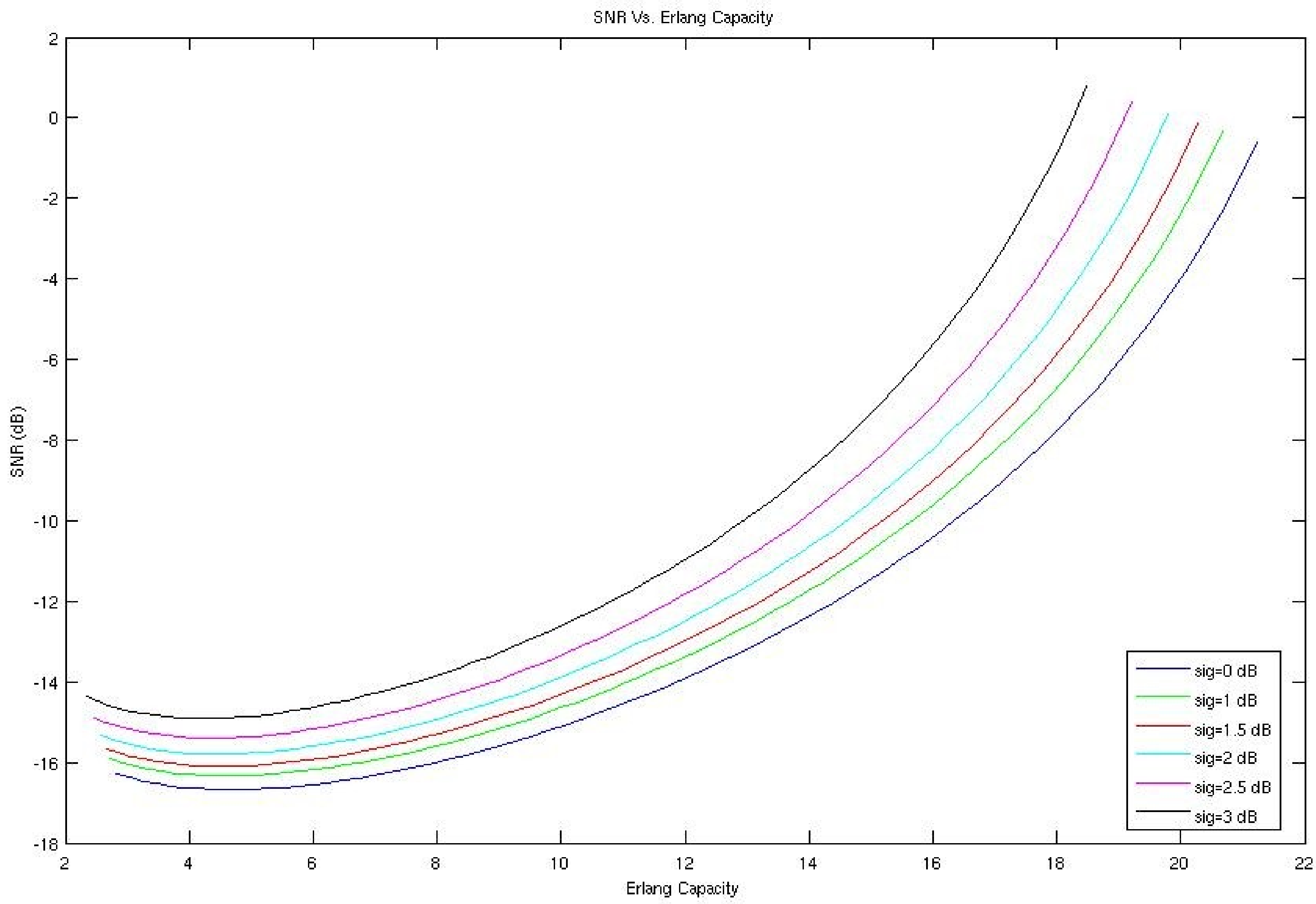
$$\frac{\lambda}{\mu} = \frac{(1-\eta)(W/R)F(B,\sigma)}{\rho(1+f)\exp(\beta m)}$$

$$B = \frac{Q^{-1}(P_{blocking})^2 \exp(\beta m)}{A}$$

Reverse Link Erlang Capacity



Reverse Link Erlang Capacity



Capacity-Coverage Tradeoff

- Cell Coverage: maximum distance that a given user of interest can be from the base station and still have a reliable received signal strength at the base station
- An accurate prediction of cell coverage as a function of user capacity is essential in CDMA network design and deployment

Capacity-Coverage Tradeoff

- As the number of users in the cell increases, the interference seen by each user increases
- Each user has to increase his transmitted power in order to acheive the desired SNR
- For a given upper limit on the transmit power, the coverage of a cell is inversely proportional to the number of users in it

Capacity-Coverage Tradeoff

- Analysis:
  - Case I: Deterministic number of users in the cell
  - Case II: Random number of users in the cell

Capacity-Coverage Tradeoff I

- To account for coverage, we need to include the probability that the power required from the subscriber to achieve a certain SNR is greater than the maximum power possible (power limited)

P(outage) = P(blocking) +

P(req power>Smax|no blocking)

Capacity-Coverage Tradeoff I

- Outage occurs when a user's SNR is less than the minimum required by the BS for a certain amount of time resulting in service degradation and call drop

$$P_{block} = P\left\{ \frac{\hat{S}_j / R}{\sum_{i:i \neq j} \frac{\nu_i \hat{S}_i}{W} + N_0 + I} < \hat{\epsilon}_j^x \right\} = P(A_{out})$$

where

$\hat{\epsilon}_j^x$ is the SNR required by the BS for the jth user

and $\hat{\epsilon}_j^x = \epsilon_j^{t\hat{a}rget} \delta_j^{\hat{\epsilon}}$

Capacity-Coverage Tradeoff I

- Let S_j^x be the required received power to obtain $\hat{\epsilon}_j^x$

So, we have

$$\hat{\epsilon}_j^x = \frac{\hat{S}_j^x / R}{\sum_{i:i \neq j} \frac{\nu_i \hat{S}_i^x}{W} + N_0 + I}$$

- The above equation has feasible solutions when

$$\sum_{i=1}^{k} \frac{R \hat{\epsilon}_i^x \nu_i}{W + R \hat{\epsilon}_i^x \nu_i} < 1$$ and $$P(A_{out}) = P\left\{\sum_{i=1}^{k} \frac{R \hat{\epsilon}_i^x \nu_i}{W + R \hat{\epsilon}_i^x \nu_i} \geq 1\right\}$$

Capacity-Coverage Tradeoff I

- With no limit on the maximum transmitted power, the maximum number of users admitted in the cell is called Pole capacity (k_{pole})

Capacity-Coverage Tradeoff I

- Let B_{out} be the event that the power control equations have feasible solutions but greater than the maximum possible transmitted power
- P(out) = P(A_{out}) + (1 - P(A_{out})) P(B_{out}|A_{out}')

$$P(B_{out})=P(S_{trans}>S_{max})$$

$$S_{trans}=S_1+PL(d)+Z_1$$

$$P_{out}=P(A_{out})+[1-P(A_{out})]P(S_1^x+PL(d)+Z_1>S_{max} l A_{out}^c)$$

Capacity-Coverage Tradeoff I

- The maximum outage probability occurs at the edge of the cell, so:

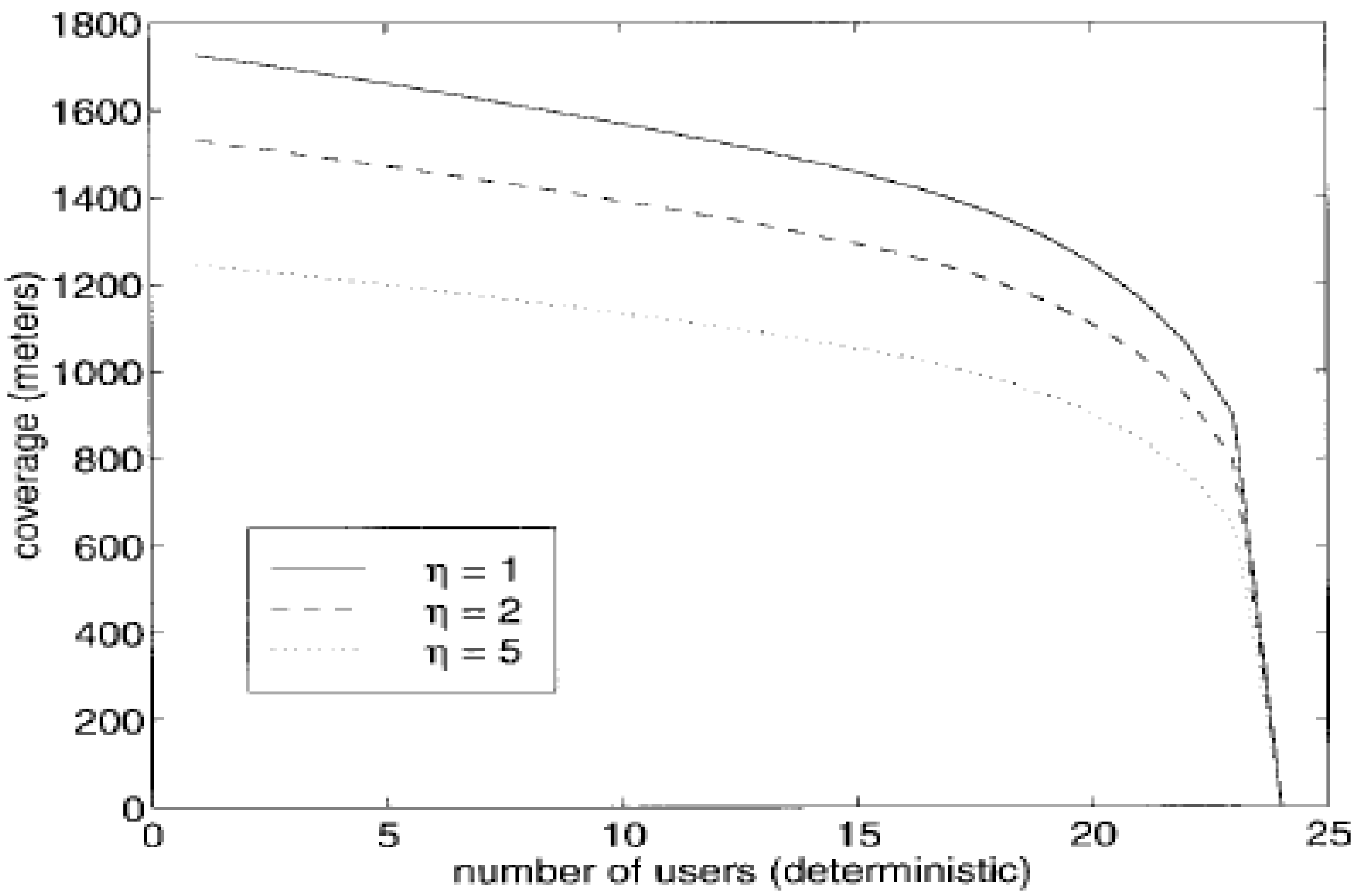
$$p_m = P(A_{out}) + [1 - P(A_{out})] P(S_1^x + PL(R_{cell}) + Z_1 > S_{max} \, l \, A_{out}^c)$$

- After some approximations and computations:

$$log R_{cell} = \frac{1}{K_2} [S_{max} - K_1 - m_S(k) - \sqrt{\sigma_S^2(k) + \sigma_z^2} Q^{-1} (\frac{p_m - P_A(k)}{1 - P_A(k)})]$$

where $PL(d) = K_1 + K_2 \log(d)$

Capacity-Coverage Tradeoff I



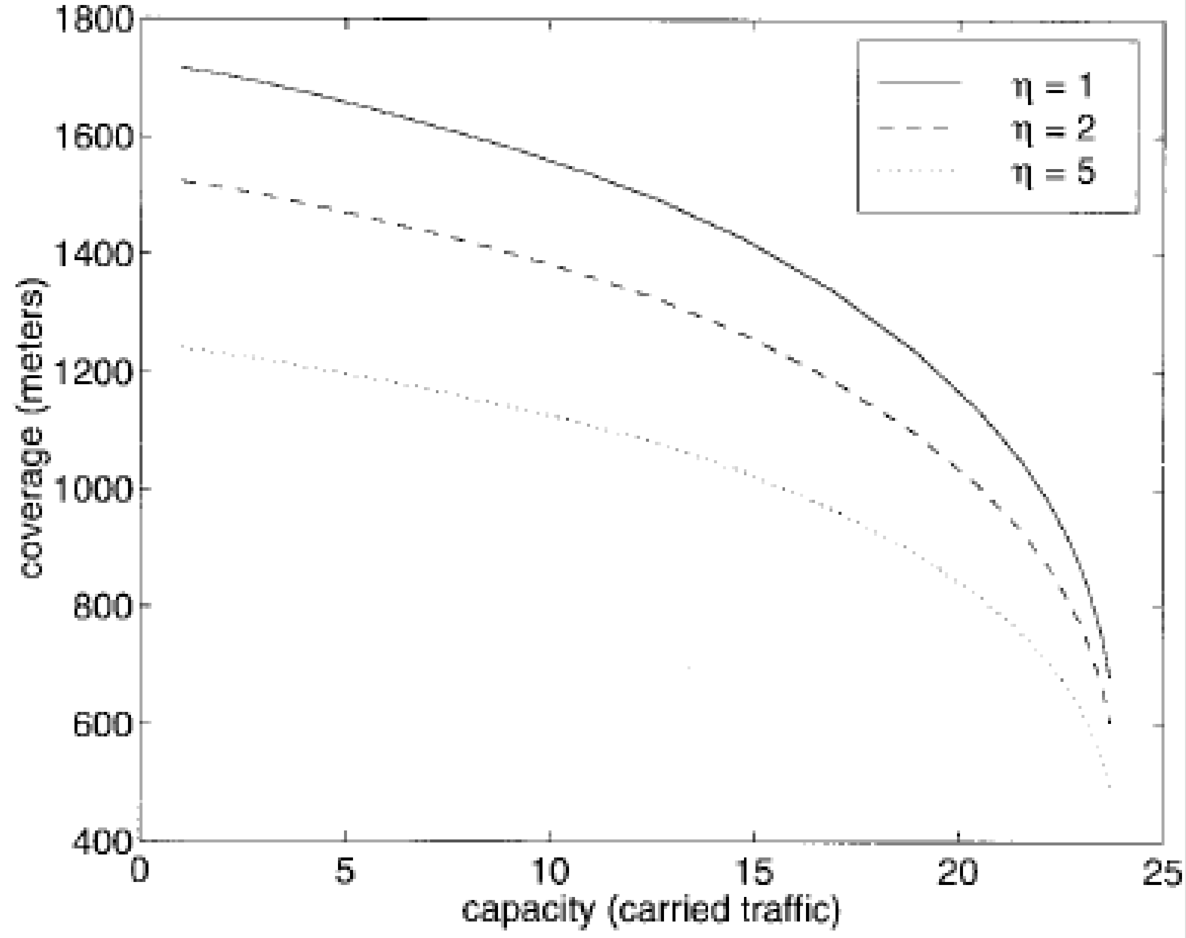
Coverage versus number of users: sensitivity to variations in other-cell interference. The parameter η denotes the ratio I/N_0.

Veeravalli & Sendonaris: Coverage-Capcity tradeoff in cellular CDMA systems

Capacity-Coverage Tradeoff II

- To design cell coverages and capacities to match projected traffic densities in the network, it will be reasonable to model the number of users requesting service as a random variable depending on the admission policy and offered traffic
- For number of users modeled as Poisson, we get the following tradeoff curve

Capacity-Coverage Tradeoff II



Coverage versus capacity for a truncated Poisson user distribution. The parameter η denotes the ratio I/N_0.

Veeravalli & Sendonaris: Coverage-Capcity tradeoff in cellular CDMA systems

Soft Handoff

- Soft Handoff: a technique whereby mobile units in transition between one cell and its neighbor transmit to and receive the same signal from both base stations simultaneously (two-cell handoff)
- Soft handoff increases cell coverage and reverse link capacity compared to hard handoff

Soft Handoff

- Coverage:
  - For hard handoff:

    $$P(B_{out} l A_{out}^c) = P(10^{\zeta_0/10} r_0^{-4} > 1/\gamma)$$

  where γ is the power added by the user to over path loss
  - For soft handoff:

    $$P(B_{out} l A_{out}^c) = P(min(10^{\zeta_0/10} r_0^{-4}, 10^{\zeta_1/10} r_1^{-4}) > 1/\gamma)$$

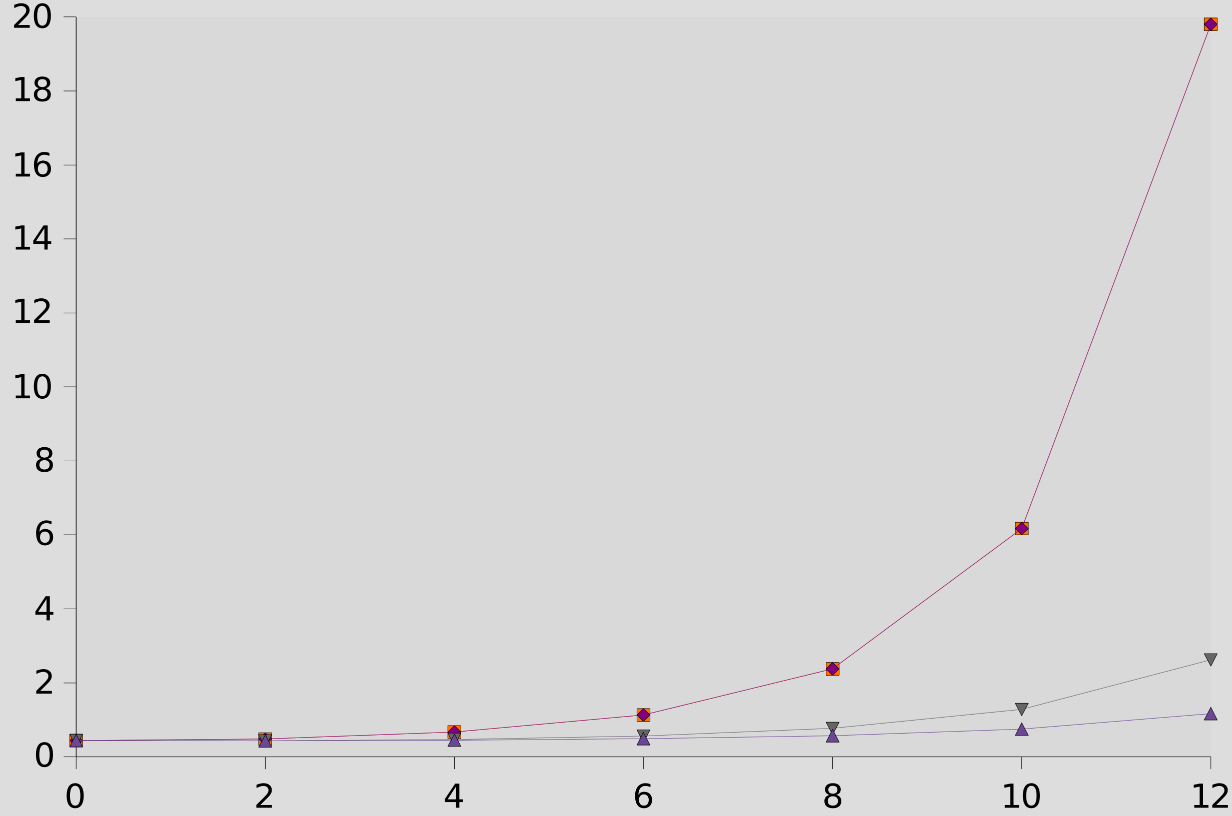
Soft Handoff

Relative Distance Beyond Cell Boundary r_0	Hard Handoff Required Margin γ_{Hard} dB	Relative Margin $\gamma_{Hard} - \gamma_{soft}$ dB	Relative Coverage Area
1	10.3	4.1	1.6
1.05	11.1	4.9	1.8
1.1	12.0	5.8	2.0
1.15	12.7	6.5	2.1
1.2	13.5	7.3	2.3
1.25	14.2	8.0	2.5

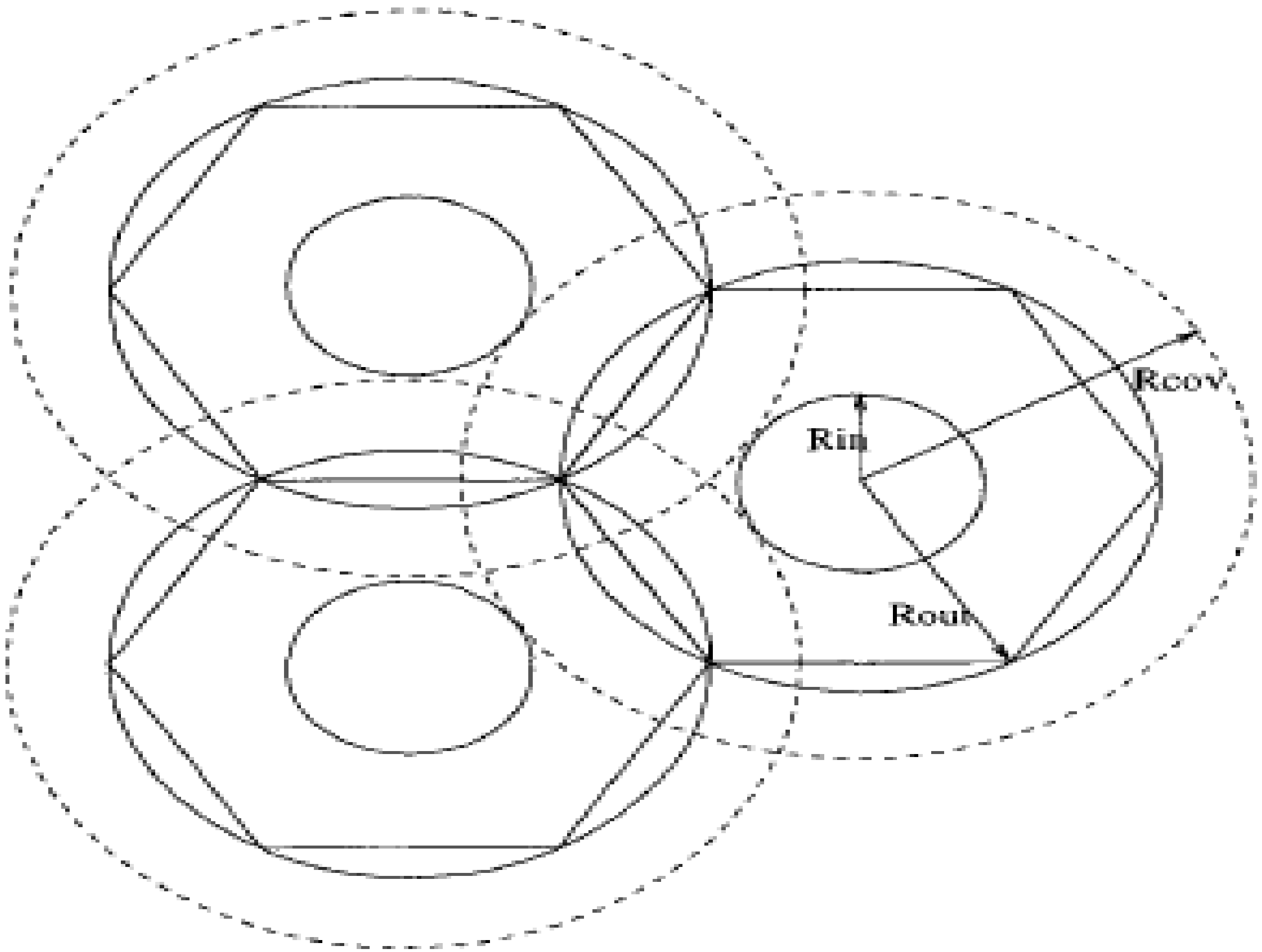
Viterbi et al.: Soft Handoff extends CDMA cell coverage and increases reverse link capacity

Soft Handoff

- Capacity:
  - Analyze the capacity in terms of the ratio f
  - Path loss standard deviation vs f:



Capacity-Coverage Tradeoff with soft handoff



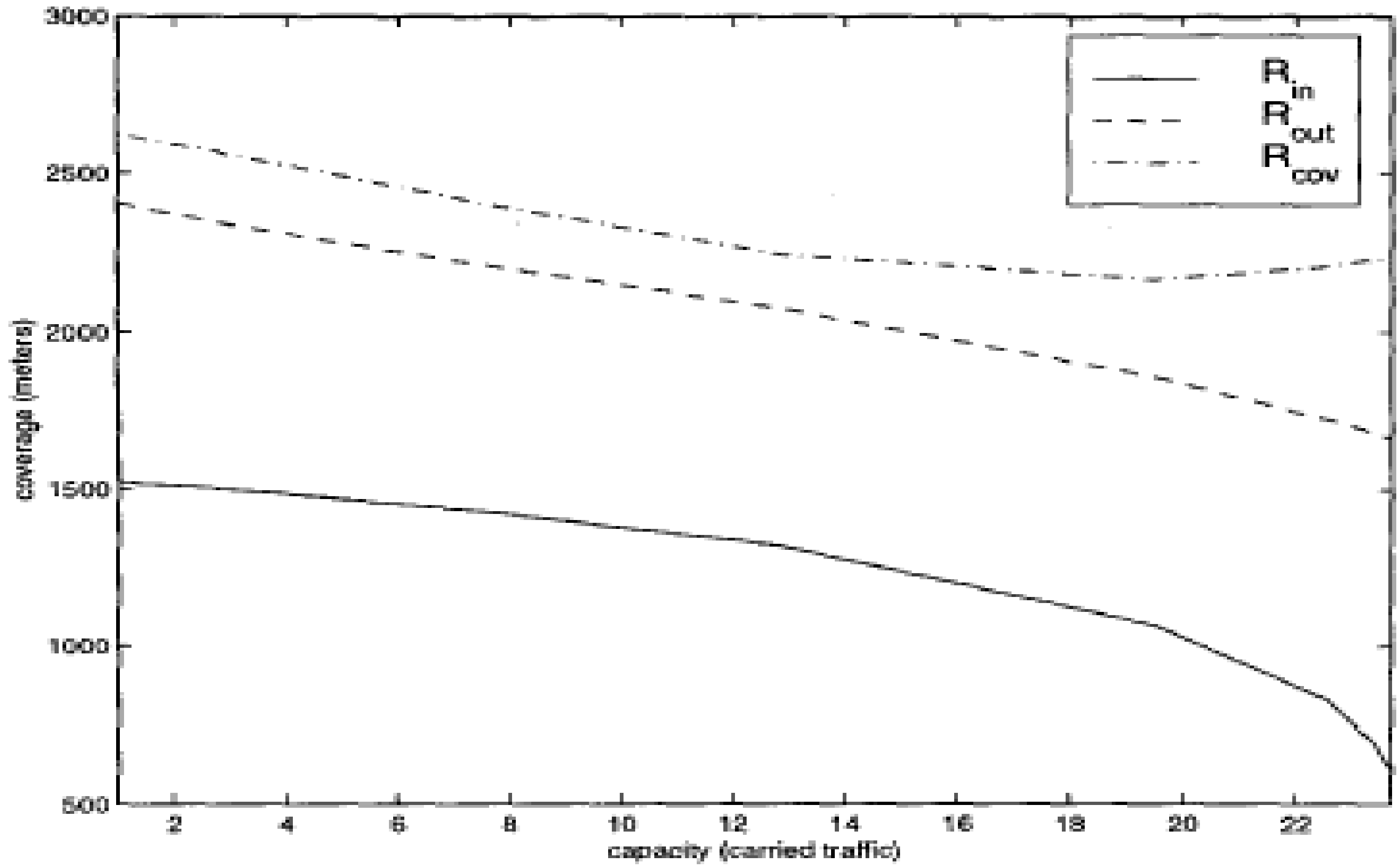
Cell radii and soft handoff regions

Veeravalli & Sendonaris: Coverage-Capcity tradeoff in cellular CDMA systems with soft handoff

Capacity-Coverage Tradeoff with soft handoff

- Similar analysis to soft handoff with:
  - A_{out}: the event that all BSs connected don't have a feasible solution
  - B_{out}: the event that all BSs require power greater than the maximum transmitted

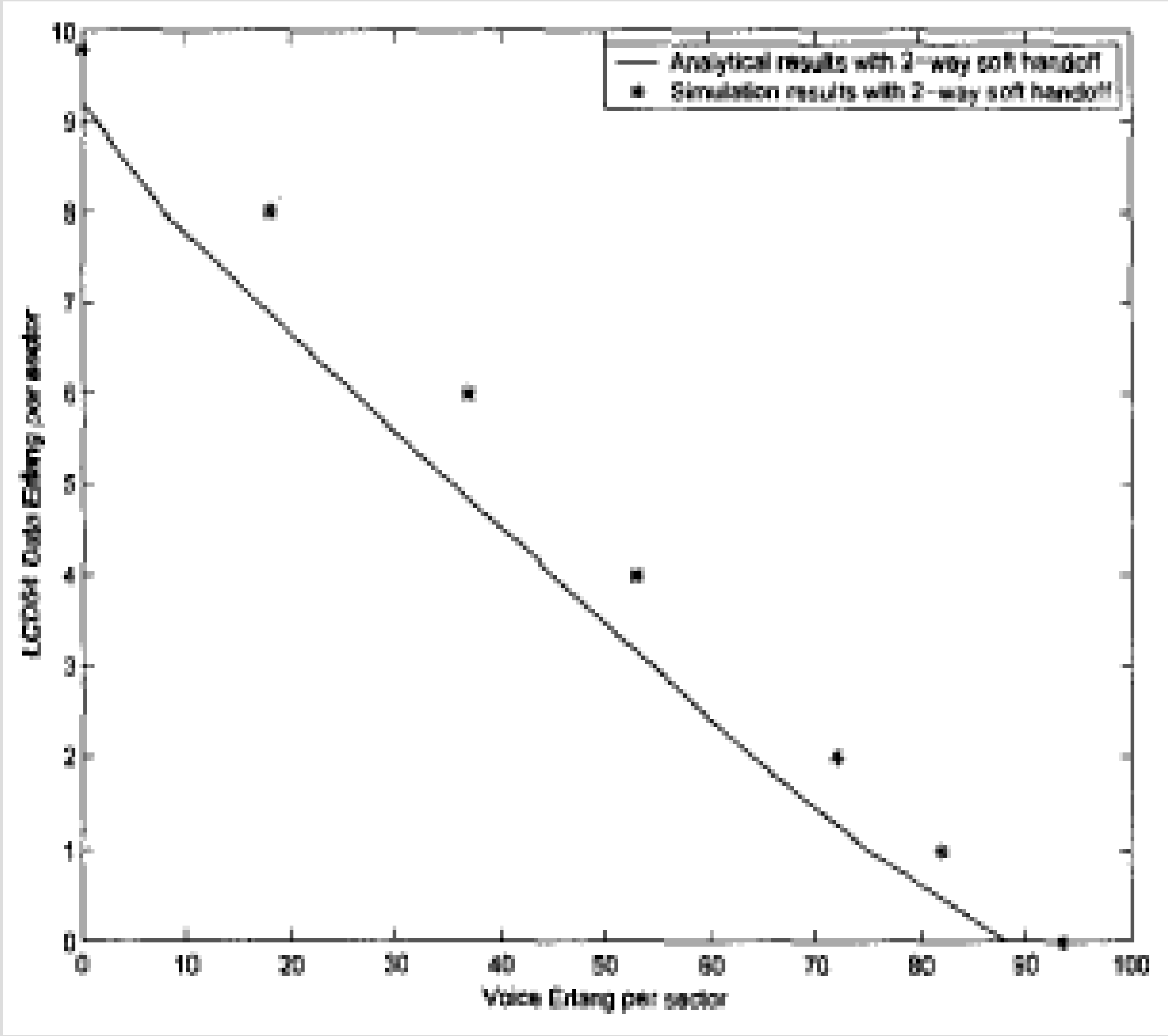
Capacity-Coverage Tradeoff with soft handoff



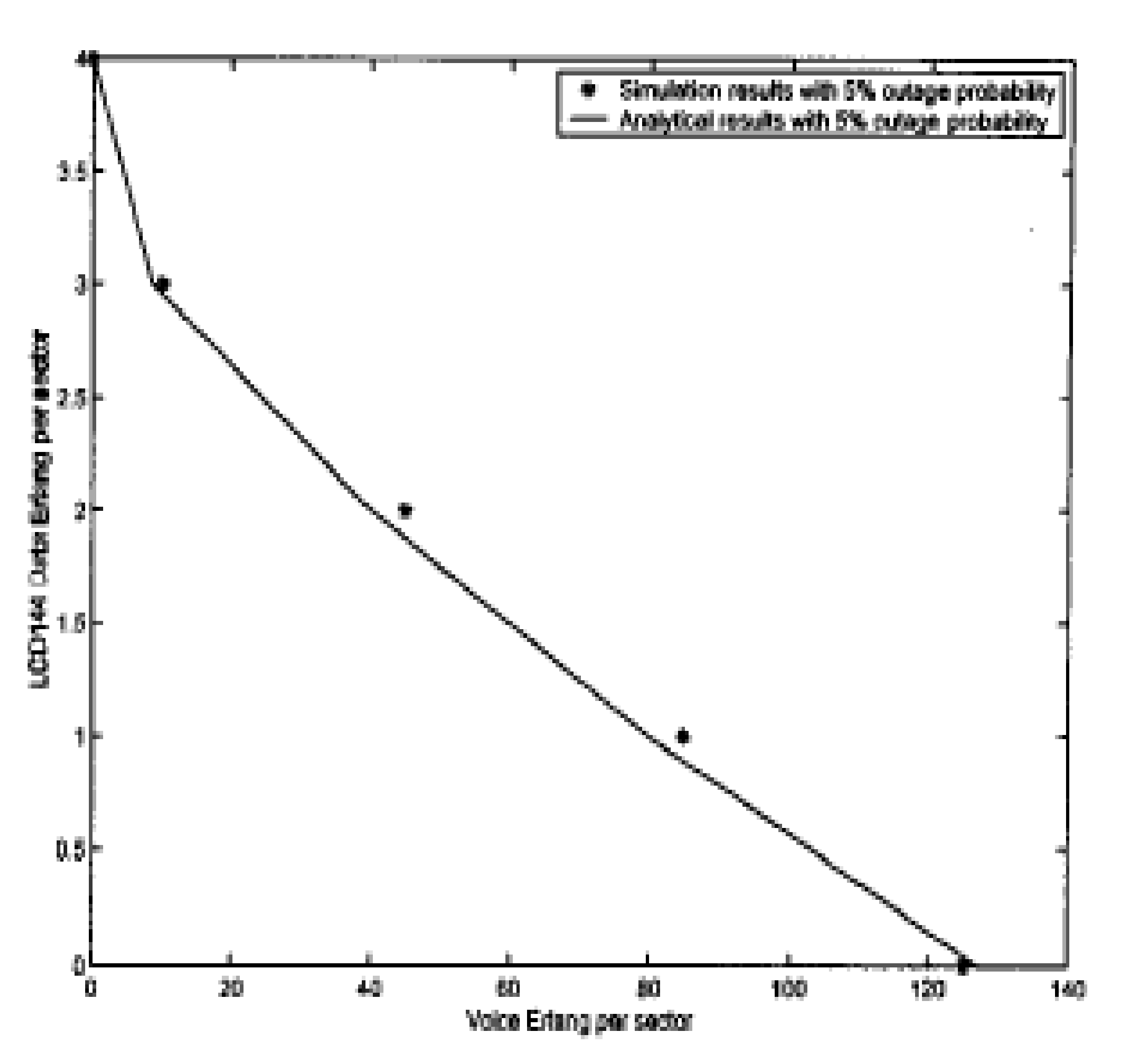
Cell radii versus capacity

Veeravalli & Sendonaris: Coverage-Capcity tradeoff in cellular CDMA systems with soft handoff

Forward link Capacity of UMTS



Reverse link Capacity of UMTS



Conclusions

- Capacity of CDMA systems can be improved by decreasing the interference
- Reverse link is the capacity bottleneck for 2G whereas for 3G it is the forward
- Coverage and capacity are inter-related in cellular CDMA systems
- Soft handoff increases the capacity and coverage compared to hard handoff

Questions?