Capacity Analysis of Cellular CDMA Systems

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Capacity Analysis of Cellular CDMA Systems

- Introduction
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- Erlang Capacity
- Capacity-Coverage Tradeoff
- Effect of Soft Handoff
- Capacity-Coverage Tradeoff with Soft Handoff
- Capacity of UMTS systems
Introduction

- Capacity of a CDMA system is interference limited
- Assumptions
  - Users are power controlled by the BS
  - All BS's require the same power
  - Power control is exercised by the BS corresponding to maximum pilot signal
  - SIR based admission policy
  - Users are uniformly distributed in each cell
Reverse Link Capacity

- Single Cell (Single User Detection):
  - SIR seen at the BS:
    \[ SIR = \frac{S}{(N - 1)S + \eta} \]
    where:
    - S: power of the received signal per user
    - N: number of users in the cell
    - \( \eta \): Background noise
  - Equivalent to:
    \[ \frac{E_b}{N_0} = \frac{S/R}{(N - 1)S/W + \eta/W} \]
Reverse Link Capacity

- Single Cell Capacity:

\[ N = 1 + \frac{W/R}{E_b/N_0} - \frac{\eta}{S} \]

- For multi-cell systems, BS suffers from intra-cell as well as inter-cell interference

\[ \frac{E_b}{N_0} = \frac{W/R}{(N-1) + I/S + \eta/s} \]

where, I: intra-cell interference (stochastic)
Reverse Link Capacity

- To find the capacity we need the distribution of $I$
- Depends on the attenuation due to large scale variations (path loss and shadow fading)
- $G = 10^{(\xi/10)} r^{-4}, \quad \xi \sim N(0, \sigma^2)$
- For a user at distance $r_m$ from his BS and $r_0$ from the BS under consideration:

$$\frac{I}{S} = (\frac{10^{(\xi_0/10)}}{r_0^4}) \ast (\frac{r_m^4}{10^{(\xi_m/10)}}) = (\frac{r_m}{r_0})^4 \ast 10^{((\xi_0-\xi_m)/10)} \leq 1$$
Reverse Link Capacity

Glihousen et al.: On the capacity of a cellular CDMA system
Reverse Link Capacity

- Utilizing the voice activity:

\[
\frac{E_b}{N_0} = \frac{W/R}{\sum_{i=1}^{N_s-1} \nu_i + I/S + \eta/s}
\]

where \( \nu_i \) is Bernoulli(\( \rho \))

- Calculate the capacity based on BER for adequate performance: \( P(\text{BER}<10^{-3}) \)
Reverse Link Capacity

Glihousen et al.: On the capacity of a cellular CDMA system
Forward Link Capacity

- In most systems, the reverse link capacity is the limiting factor due to the limited power available for the subscribers.
- Power control is also exercised in the forward link: Subscriber sends the power received from its BS and the total interference.
Forward Link Capacity

- The $i$th subscriber SNR can be lower bounded by

$$\left(\frac{E_b}{N_0}\right)_i \geq \beta \phi_i \frac{S_{(T_1)}}{R} \frac{S_{(T_1)}}{[\sum_{j=1}^{k} S_{(T_j)}]_i + \eta] / W}$$

where:

- $\beta$ is the fraction of the total site power devoted to users (excluding pilot)
- $\phi_i$ is the fraction of power devoted to the $i$th subscriber
- $S_{T_1}$ is the total power available from BS under consideration
Forward Link Capacity

Glihousen et al.: On the capacity of a cellular CDMA system
Erlang Capacity

- Def: The average traffic load in terms of average number of users requesting service resulting in a certain blocking probability
- Blocking Probability: the probability that a new user will find all channels busy and hence be denied service
- Condition: $P(I_0/N_0 > 10) < 0.01$
Reverse Link Erlang Capacity

- **Simple Case:**
  a) constant number of users $N_U$ in every sector,
  b) each user transmits continuously,
  c) users require the same $E_b/I_0$

- **Condition for no blocking:**
  \[ N_u E_b R (1 + f) + N_0 W \leq I_0 W \]
  \[ N_u \leq \frac{(W/R)}{(E_b/I_0)} \cdot \frac{(1 - \eta)}{(1 + f)} \]

  $f$: ratio of intra-cell interference to inter-cell interference
  \[ \eta = \frac{N_0}{I_0} \]
Reverse Link Erlang Capacity

- Practical case:
  a) Number of active calls is a Poisson random variable with mean $\lambda/\mu$
  b) each user is gated on with probability $\rho$ and off with probability $1-\rho$ (voice activity)
  c) each user's received energy-to-interference ratio is varied according to propagation conditions
Reverse Link Erlang Capacity

- **Condition for no blocking:**

\[
\sum_{i=1}^{k} \nu_i \cdot E_{bi} \cdot R + \sum_{j} \sum_{i=1}^{k} \nu_{i(j)} \cdot E_{bi(j)} \cdot R + N_0 \cdot W \leq I_0 \cdot W
\]

and so:

\[
P\{ Z = \sum_{i=1}^{k} \nu_i \cdot \epsilon_i + \sum_{j} \sum_{i=1}^{k} \nu_{i(j)} \cdot \epsilon_{i(j)} > \frac{W}{R} \frac{1}{1-\eta} \} = P_{\text{blocking}}
\]

where \( \epsilon_i = \frac{E_{bi}}{I_0} \) (stochastic)
Reverse Link Erlang Capacity

- The statistics of $\epsilon_i$ depends on the power control mechanism
- Field trials with all cells fully loaded show that $\epsilon_i$ is well modeled as log-normal
- Chernoff pound for the outage probability can't be obtained because the moment generating function of $\epsilon_i$ doesn't converge
Reverse Link Erlang Capacity

Blocking probabilities for single cell interference (CDMA parameters: $W/R = 1280$; voice act. = 0.4; $I_0/N_0 = 10$ dB; median $E_b/I_0 = 7$ dB; $\sigma = 2.5$ dB).

Viterbi & Viterbi: Erlang Capacity of Power Controlled CDMA System
Reverse Link Erlang Capacity

- Using Central Limit theorem for $Z$ we get:

$$P\{ Z = \sum_{i=1}^{k} \nu_i \epsilon_i + \sum_{j} \sum_{i=1}^{k} \nu_{i}^{(j)} \epsilon_{i}^{(j)} > \frac{W/R}{1-\eta} \} = P_{\text{blocking}}$$

$$P_{\text{blocking}} \approx Q\left[ \frac{A - E(Z)}{\sqrt{\text{Var}(Z)}} \right]$$

$$\lambda = \frac{(1-\eta)(W/R) F(B, \sigma)}{\rho(1+f) \exp(\beta m)}$$

$$\mu = \rho (1+f) \exp(\beta m)$$

$$B = \frac{Q^{-1}(P_{\text{blocking}})^2 \exp(\beta m)}{A}$$
Reverse Link Erlang Capacity

![Graph showing Erlang Capacity vs. P_{blocking} with different slopes.](image)
Reverse Link Erlang Capacity
Capacity-Coverage Tradeoff

- Cell Coverage: maximum distance that a given user of interest can be from the base station and still have a reliable received signal strength at the base station

- An accurate prediction of cell coverage as a function of user capacity is essential in CDMA network design and deployment
Capacity-Coverage Tradeoff

- As the number of users in the cell increases, the interference seen by each user increases.
- Each user has to increase his transmitted power in order to achieve the desired SNR.
- For a given upper limit on the transmit power, the coverage of a cell is inversely proportional to the number of users in it.
Capacity-Coverage Tradeoff

- Analysis:
  - Case I: Deterministic number of users in the cell
  - Case II: Random number of users in the cell
To account for coverage, we need to include the probability that the power required from the subscriber to achieve a certain SNR is greater than the maximum power possible (power limited):

\[ P(\text{outage}) = P(\text{blocking}) + P(\text{req power}>S_{\text{max}}|\text{no blocking}) \]
Capacity-Coverage Tradeoff I

- Outage occurs when a user's SNR is less than the minimum required by the BS for a certain amount of time resulting in service degradation and call drop

\[
P_{block} = P\left\{ \frac{\hat{S}_j}{R} < \hat{\epsilon}_j \right\} = P( A_{out} )
\]

\[
\sum_{i:i\neq j} \frac{\nu_i \hat{S}_i}{W} + N_0 + I
\]

where

\[
\hat{\epsilon}_j
\]

is the SNR required by the BS for the jth user

and

\[
\hat{\epsilon}_j = \epsilon_j^{target} \delta_j
\]
Let $S_j^x$ be the required received power to obtain $\hat{\epsilon}_j^x$. So, we have

$$\hat{\epsilon}_j^x = \frac{\hat{S}_j^x / R}{\sum_{i:i \neq j} \frac{\nu_i \hat{S}_i^x}{W} + N_0 + I}$$

The above equation has feasible solutions when

$$\sum_{i=1}^{k} \frac{R \rho_i^{x} \nu_i}{W + R \rho_i^{x} \nu_i} < 1$$

and

$$P(A_{out}) = P\left\{ \sum_{i=1}^{k} \frac{R \rho_i^{x} \nu_i}{W + R \rho_i^{x} \nu_i} \geq 1 \right\}$$
With no limit on the maximum transmitted power, the maximum number of users admitted in the cell is called Pole capacity ($k_{\text{pole}}$)
Let $B_{\text{out}}$ be the event that the power control equations have feasible solutions but greater than the maximum possible transmitted power.

\[ P(\text{out}) = P(A_{\text{out}}) + (1 - P(A_{\text{out}})) \cdot P(B_{\text{out}}|A_{\text{out}}') \]

\[
P(B_{\text{out}}) = P(S_{\text{trans}} > S_{\text{max}})
\]

\[
S_{\text{trans}} = S_1 + PL(d) + Z_1
\]

\[
P_{\text{out}} = P(A_{\text{out}}) + \left[ 1 - P(A_{\text{out}}) \right] \cdot P(S_1^x + PL(d) + Z_1 > S_{\text{max}} l A_{\text{out}}^c)
\]
Capacity-Coverage Tradeoff I

- The maximum outage probability occurs at the edge of the cell, so:

\[ p_m = P(A_{out}) + [1 - P(A_{out})] P(S^x + PL(R_{cell}) + Z_1 > S_{max} l A_{out}^c) \]

- After some approximations and computations:

\[
\log R_{cell} = \frac{1}{K_2} \left[ S_{max} - K_1 - m_S(k) - \sqrt{\sigma^2_S(k) + \sigma^2_z} Q^{-1} \left( \frac{p_m - P_A(k)}{1 - P_A(k)} \right) \right]
\]

where

\[ PL(d) = K_1 + K_2 \log(d) \]
Veeravalli & Sendonaris: Coverage-Capacity tradeoff in cellular CDMA systems
To design cell coverages and capacities to match projected traffic densities in the network, it will be reasonable to model the number of users requesting service as a random variable depending on the admission policy and offered traffic.

For number of users modeled as Poisson, we get the following tradeoff curve.
Coverage versus capacity for a truncated Poisson user distribution. The parameter \( \eta \) denotes the ratio \( I/N_0 \).
Soft Handoff

• Soft Handoff: a technique whereby mobile units in transition between one cell and its neighbor transmit to and receive the same signal from both base stations simultaneously (two-cell handoff)

• Soft handoff increases cell coverage and reverse link capacity compared to hard handoff
Soft Handoff

- **Coverage:**
  - For hard handoff:
    \[ P(B_{out} lA_{out}^c) = P(10^{\xi_0/10} r_0^{-4} > 1/\gamma) \]
    where \( \gamma \) is the power added by the user to over path loss
  - For soft handoff:
    \[ P(B_{out} lA_{out}^c) = P(min(10^{\xi_0/10} r_0^{-4}, 10^{\xi_1/10} r_1^{-4}) > 1/\gamma) \]
Viterbi et al.: Soft Handoff extends CDMA cell coverage and increases reverse link capacity

<table>
<thead>
<tr>
<th>Relative Distance Beyond Cell Boundary $r_0$</th>
<th>Hard Handoff Required Margin $\gamma_{\text{Hard}}$ dB</th>
<th>Relative Margin $\gamma_{\text{Hard}} - \gamma_{soft}$ dB</th>
<th>Relative Coverage Area</th>
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</tr>
</tbody>
</table>
- Capacity:
  - Analyze the capacity in terms of the ratio $f$
  - Path loss standard deviation vs $f$: 
Capacity-Coverage Tradeoff with soft handoff

Veeravalli & Sendonaris: Coverage-Capcity tradeoff in cellular CDMA systems with soft handoff
Similar analysis to soft handoff with:

- $A_{out}$: the event that all BSs connected don't have a feasible solution
- $B_{out}$: the event that all BSs require power greater than the maximum transmitted
Capacity-Coverage Tradeoff with soft handoff

Veeravalli & Sendonaris: Coverage-Capcity tradeoff in cellular CDMA systems with soft handoff
Forward link Capacity of UMTS
Reverse link Capacity of UMTS
Conclusions

- Capacity of CDMA systems can be improved by decreasing the interference
- Reverse link is the capacity bottleneck for 2G whereas for 3G it is the forward
- Coverage and capacity are inter-related in cellular CDMA systems
- Soft handoff increases the capacity and coverage compared to hard handoff
Questions?