Centralized Wireless Data Networks: Performance Analysis

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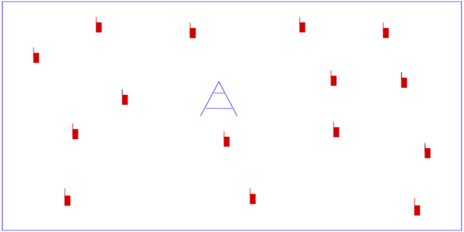
Centralized Wireless Data Networks



Voice networks	Data networks
Inelastic traffic	Elastic traffic
Drop packets	Queue packets
Smaller data rates	Larger data rates

Aim: Obtain a convenient approximate characterization of the network performance in terms of network parameters

Network parameters and Performance metrics

Network parameters

- Traffic
  - No. of users with packets
  - Size of packets
- Physical channel (Fading, Bandwidth, Noise)
- Power used in communicating the packets
- Medium access scheme

Performance Metrics

- Total time in communicating a packet
- Denial of service
- Fairness

Aim: Performance metric $\simeq$ f(network parameters)

Model for Network Parameters

Traffic model: Dynamic-user model

- Users arrive according to a Poisson process of rate $\alpha\lambda$
- Each user has to communicate αS bits, where S is a random variable

Medium access scheme: Symmetric schemes

- If the same u users are in the queue for an arbitrarily long time the average capacity is the same and equal to a deterministic constant $\frac{\psi(u)}{u}$ for each user.
- Mathematically:

$$\frac{1}{t}\int_{T}^{t+T} l_j(\tau;u)d\tau \to \frac{\psi(u)}{u}.$$

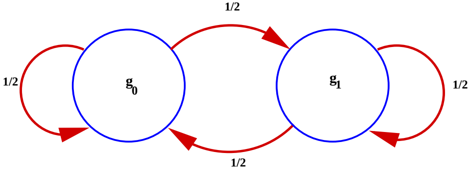
$l_j(t;u)$: Rate of j^{th} at time t with total u users.

  - Random process
  - What determines the statistics of $l_j(t;u)$?

Example: TDMA in Simple Correlated-Block Fading Model

Channel

- Fading for a user in each slot (duration 1) is a constant. User j slot k is $H_{j,k}$
- Channel across users is independent
- $H_{j,k}$ follows the Glibert-Elliot Markov model



If in state g_i and a total of u users use power $P(g_i)$

Example: TDMA in Simple Correlated-Block Fading Model

- Medium access scheme is TDMA
- User 1 will communicate only during $[k, k+\frac{1}{u}]$ and will not communicate in $[k+\frac{1}{u}, k+1]$ => Bursty servicing
- For $t \in [k, k+1]$,

$$I_j(t;u) = \begin{cases} W \log\left(1 + \frac{\|H_{j,k}\|^2 P(H_{j,k})}{N_0}\right) & \text{when } t \in [k+\frac{j-1}{u}, k+\frac{j}{u}]. \\ 0 & \text{otherwise} \end{cases}$$

- Since H is random, so is $I_j(t;u)$.
- Using some simple arguments

$$\frac{\psi(u)}{u} = \frac{W}{u}\left[\log\left(1 + \|g_1\|^2 P(g_1)\right) + \log\left(1 + \|g_0\|^2 P(g_0)\right)\right]$$

Mathematical Problem

Let $\pi(u)$ be the stationary distribution for the no. of users in the system, U.

- Average Delay $= \frac{\mathsf{E}(U)}{\alpha\lambda}$ Little's law
- Block probability $= \text{Prob}\,(U > U_T)$ PASTA property
- Fairness: Medium access scheme ensures this

Problem: Determine $\pi(u)$ as a function of distribution of S, statistics of $I(u,t)$, and λ.

First Order Approximation—Intuitive Arguments

- Users come very rarely $\alpha\lambda$, $\alpha \rightarrow 0$.
- Each user has large file $\frac{S}{\alpha}$

If we send $\alpha \rightarrow 0$ 'fast enough', each user will see an average of $\frac{\psi(u)}{u}$ bits/sec on an average. Recall,

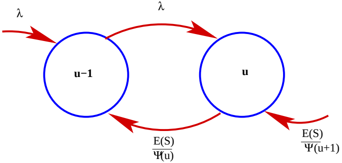
$$\frac{1}{t}\int_{T}^{t+T} I_j(\tau;u)d\tau \rightarrow \frac{\psi(u)}{u}.$$

This leads to the following simplification:

- Users arrive at rate λ (very small)
- Each user has a random file size of size S that is large
- The capacity seen by each user is non-random, non-time-varying $\frac{\psi(u)}{u}$.

First Order Approximation—The Solution

Example: S is exponentially distributed.



$\pi(u)$ is the stationary distribution of this CTMC!

$$\pi(u) \simeq \frac{1}{K} \frac{(\lambda \mathsf{E}(S))^u}{\Pi_{i=1}^{u} \psi(i)}.$$

- As the file size goes to ∞ and arrival rate to 0 approximation is correct
- True for general distributions of S
- Convergence $O(\alpha)$

Second Order Approximation: Intuition

Assume

$$\frac{\left(\int_t^{T+t} I^j(\tau)d\tau - \frac{\psi(u)}{u}\right)^2}{\sqrt{t}} \rightarrow \mathcal{N}(\psi, \sigma^2(u))$$

We can include a correction term

$$\pi(u) \simeq \frac{1}{K}\frac{(\lambda \mathrm{E}(S))^u}{\Pi_{i=1}^u \psi(i)} - \frac{1}{2}\alpha\sigma^2(u).$$

- Accuracy is $o(\alpha)$
- Thus more the 'variance' in the service the average service rate is going to be lower
- Intuition: Assume $I(t;u) = \phi(u) + \sigma^2(u) \times$ White noise
- CTMC with 'noisy rates'

Network Layer Application

- Physical layer and MAC layer has been 'abstracted out' into $\pi(u)$.
- Network layer problem: Fix W, g_0 and g_1.
    - D is delay. Function of u.
    - P is total power used per slot by all users $\sum_{i=1}^{u} P_i$.

$$\min_{P_i} \quad E(D(U))$$

$$E\left(\sum_{i=1}^{U} P_i\right) < \bar{P}$$

- Solution uses dynamic programming

Conclusion

- Obtained a simple and useful approximation for the network performance
- Discussed an example network layer application

Thanks!