The Coding–Spreading Tradeoff in CDMA Systems

Venugopal V. Veeravalli, *Senior Member, IEEE*, and Ashok Mantravadi, *Student Member, IEEE*

***Abstract*—General definitions of spreading and coding are given based on the notion of Shannon bandwidth introduced by Massey (1994), with the goal of distinguishing these operations for signaling with bandwidth redundancy. These definitions are shown to lead to a separation result: every bandwidth redundancy scheme can be expressed as a concatenation of coding followed by spreading. The coding-spreading tradeoff problem is then studied for a code division multiple access (CDMA) system in which the receiver processes the received signal by using a user-separating front-end, which feeds into autonomous single-user decoders. Under the single-user decoding setting, it is established that the linear minimum mean square error (LMMSE) front-end multiuser detector is optimum among all front-ends that are constrained to use only spreading information. Also, conditions are given for the single-user decoders to ignore spreading information without losing optimality. An example illustrating the coding-spreading tradeoff optimization for a direct sequence CDMA system with random spreading is given. Single-cell and multicell scenarios are considered in the optimization, and a comparison is made of the spectral efficiencies that can be achieved with the conventional matched filter and LMMSE front-ends.**

***Index Terms*—Channel coding, code division multiaccess, land mobile radio cellular systems, least mean square methods, matched filters, signal detection, spectral efficiency, spread spectrum communication.**

I. Introduction

In SPREAD spectrum code division multiple access (CDMA) systems, the spectral (Fourier) bandwidth of each user in the system is increased to fill up the entire "available" bandwidth. Such bandwidth expansion is known to facilitate multiple access with many desirable features, particularly in the context of wireless cellular systems [2]. In a CDMA system, each user's transmitted signal has a large time-bandwidth product, i.e., the spectral bandwidth[1] W (in Hz) occupied by the signal is considerably larger than the information rate R (in bits/s). If we consider a hypothetical baseline system where we construct the user's transmitted signal as an uncoded binary phase shift-keying (BPSK) signal with Nyquist sinc pulse shaping, then the bandwidth of this baseline system equals $R/2$ Hz. We, hence, need to introduce bandwidth redundancy into this signal if it is to occupy a spectral bandwidth of $W \gg R$.

In this paper, we use the notion of Shannon bandwidth introduced by Massey [1] to distinguish between two components of a bandwidth redundancy scheme, namely, *spreading* and *coding*. Our goal is then to study the coding-spreading tradeoff in CDMA systems. This tradeoff problem has been considered for specific CDMA systems in previous work (see, e.g., [3] and [4] for single-user detection results, and [5] and [6] for multiuser detection results). One of the contributions of this paper is in formally establishing that the tradeoff problem is well-defined. In addition, we give a general approach to optimizing the tradeoff, and explore the details and implications of this optimization in the context of cellular CDMA systems.

In Section II, we establish a useful separation result that every bandwidth redundancy scheme can be written as a concatenation of coding followed by spreading. This separation result leads naturally to the question of how a fixed bandwidth expansion should be allocated between coding and spreading. The answer to this question depends crucially on the channel and the receiver structure.

In Sections III and IV, we pose the coding-spreading tradeoff problem for the interesting special case of a CDMA system with single-user decoding. Here, the receiver processes the sum of the user's signals, corrupted by white Gaussian noise, by using a user-separating front-end detector that is followed by autonomous single-user decoders. For such a receiver, we first show that among all linear front-end detectors, the one that minimizes the mean square error (MSE) at the input of the decoders does not depend on the codebooks of the users. Furthermore, under the single-user decoding restriction, we give arguments supporting the asymptotic optimality of the linear minimum mean square error (LMMSE) front-end, similar to previous results in [7]. We also give conditions under which the single-user decoders may operate without knowledge of the users' spreading functions and not sacrifice optimality, and give examples of when these conditions are met. We, hence, establish a form of separation between detection and decoding at the receiver.

In Section V, we study the coding-spreading tradeoff in detail for the specific example of a direct sequence CDMA system with ideal single-user coding and decoding, and random spreading. The spectral efficiency of the system is the criterion used for studying the tradeoff. We consider the two extreme cases where the codesymbols are constrained to be binary and where they are unconstrained. Both single-cell and multicell

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V. V. Veeravalli is with the ECE Department and the Coordinated Science Laboratory, 128 CSL, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: vvv@uiuc.edu, http://www.comm.csl.uiuc.edu/~vvv).

A. Mantravadi is at the School of ECE, Cornell University, Ithaca, NY 14853 USA and also with the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: mantrava@uiuc.edu).

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[1]For ease of exposition of the results, we assume real-valued baseband signals and systems throughout the paper. A spectral bandwidth of W denotes that the signal energy is contained in range of frequencies $[-W, W]$. The analysis and results are straightforwardly extended to complex baseband models that represent carrier modulated signals and systems.

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scenarios are considered in the tradeoff study. Using recent results on large system analysis of linear multiuser detectors [6], [8], we obtain expressions for the coding-spreading tradeoff curves for the (near optimum) LMMSE and the conventional matched filter (MF) front-ends.

In Section VI, we compute the tradeoff curves numerically for specific examples, and provide simulation results that confirm the accuracy of the large system analysis for practical system sizes. Based on the peak spectral efficiencies that can be achieved by these two front-ends, we then draw interesting conclusions regarding the applicability of multiuser detection techniques in cellular CDMA systems.

II. Separation of Coding and Spreading

A. Fourier and Shannon Bandwidths

Consider the space of real-valued functions that are time-limited to the interval $[0, T]$ and approximately bandlimited to a baseband bandwidth of W Hz, i.e., most of the energy is contained in the range of frequencies $[-W, W]$. Denote this space by $\mathcal{F}(T;W)$, and note that $\mathcal{F}(T;W)$ has dimension D_F given by [see e.g., ([9, p. 294])]

$$D_F \approx 2WT.$$

We refer to D_F as the *Fourier dimension* of the signal space. The prolate spheroidal wave functions (PSWFs) form the best basis for $\mathcal{F}(T;W)$ in the sense described in [10]. But if $T \gg 1/W$ and we ignore edge effects, we have the following simpler set of orthonormal basis functions that are formed by translations of sinc pulses:

$$\phi_i(t) = \sqrt{2W}\,\mathrm{sinc}(2Wt - i), \quad i = 1, 2, \ldots, D_F \tag{1}$$

where $\mathrm{sinc}(x) = (\sin \pi x)/(\pi x)$.

Consider the following single-user communications problem. Suppose our goal is to transmit information at the rate of R bits/s, i.e., we wish to send one of $M_T = 2^{RT}$ possible symbols (equivalently a sequence of RT bits) in time $[0, T]$, using one of the signals from the set $\{s_1(t), s_2(t), \ldots s_{M_T}(t), t \in [0, T]\}$. We do not restrict the form of signals in any way except that the signal set occupies a Fourier bandwidth of W, i.e., $[-W, W]$ is the smallest range of frequencies that encompasses the essential bandwidths of all the signals in the set.

It is clear that each $s_m(t)$ belongs to $\mathcal{F}(T;W)$, and can, hence, be represented by a D_F-dim vector $\boldsymbol{s}_m = [s_{m,1} \; s_{m,2} \; \cdots \; s_{m,D_F}]^\top$, with

$$s_m(t) = \sum_{i=1}^{D_F} s_{m,i}\phi_i(t). \tag{2}$$

We denote the signal set $\{\boldsymbol{s}_1, \ldots, \boldsymbol{s}_{M_T}\}$ by $\mathcal{S}$. We will abuse notation slightly, and use $\mathcal{F}(T;W)$ to denote both the space of functions $s(t)$ and their corresponding D_F-dim vector representations. Thus, we can consider $\mathcal{S}$ to be a subset of $\mathcal{F}(T;W)$. Without loss of generality we may assume that $D_F \leq M_T$, since this will hold for sufficiently large T.

Before we proceed we define a baseline signal set that will be used in the remainder of the paper. We will use the superscript prime to denote the parameters of the baseline signal set.

Definition 1: The *baseline signal set* for signaling at rate R bits/s on $[0, T]$ is given by $\mathcal{S}' = \{-1, +1\}^{RT}$, i.e., the information bits are modulated using BPSK with Nyquist sinc pulse shaping.

Note that the Fourier bandwidth of the baseline system $W' = R/2$, and $\mathcal{S}' \subset \mathcal{F}(T;W')$. This motivates the following definition.

Definition 2: The *bandwidth expansion factor* Ω of a general signaling scheme with Fourier bandwidth W is given by

$$\Omega = \frac{W}{W'} = \frac{2W}{R}. \tag{3}$$

For CDMA signals Ω is usually $\gg$1 for each user.

Definition 3: A signaling scheme is said to be a *bandwidth redundancy scheme* if $\Omega > 1$.

We now note that the signal set $\mathcal{S} \subset \mathcal{F}(T;W)$ may have a span whose dimension is smaller than D_F. This leads to the following definition of Shannon bandwidth that was introduced by Massey in [1].

Definition 4: The dimension of $\mathrm{Span}(\mathcal{S})$ is called the *Shannon dimension* of the signal set, and is denoted by D_S. The ratio $D_S/2T$, which represents half the number of dimensions per second occupied by the signal set, is called the *Shannon bandwidth* of the signal set, and is denoted by B.

Clearly $D_S \leq D_F$ and $B \leq W$. The notion of Shannon bandwidth was used by Massey to precisely define a spread-spectrum system as one for which $B < W$ (actually $B \ll W$), with spreading factor N defined as

$$N = \frac{W}{B}. \tag{4}$$

Note that B may be smaller or larger than W', and hence, Ω may be smaller or larger than N, depending on the alphabet size chosen for signaling. If the signaling is constrained to be *binary*, then it is easy to see that $B \geq W'$ and, hence, that $\Omega \geq N$. Also, for the baseline signal set of Definition 1, it is clear that $B' = W' = R/2$.

Based on the definition of a spread spectrum system as one for which $N \gg 1$, Massey gives several examples in [1] that illustrate the difference between spread-spectrum and nonspread-spectrum systems. In Section II-B, we use the notion of Shannon bandwidth to define the operation of *spreading*, and more importantly to distinguish it from the operation of *coding*.

B. Spreading as a Linear Mapping in Signal Space

The process of converting the sequence of RT bits into a signal $\boldsymbol{s} \in \mathcal{S} \subset \mathcal{F}(T;W)$ is generally referred to as *coding*. The mapping of the set of all sequences of RT bits to the set $\mathcal{S}$, can equivalently be considered to be a mapping $\mathcal{X}$ from the baseline signal set $\mathcal{S}' \subset \mathcal{F}(T;W')$ to $\mathcal{S}$. Assuming that $\Omega = W/W' > 1$, this mapping $\mathcal{X}$ is a bandwidth redundancy scheme (see Definition 3).

The above general definition of coding encompasses the case when $B < W$, i.e., the case of spread-spectrum signaling. In fact, standard direct sequence spreading used in CDMA systems can be considered as equivalent to repetition coding. However, as noted in [1], coding and spreading should be considered to be fundamentally different components of the bandwidth redundancy mapping $\mathcal{X}$. Our goal is to identify these components so that we can study the coding-spreading tradeoff problem.

If we constrain $\mathcal{X}$ so that $\mathcal{S}$ has Shannon bandwidth B, then the maximum rate achievable on a single-user additive white Gaussian noise (AWGN) channel with Fourier bandwidth W, is given by [1]

$$\mathcal{R} = B\log_2\left(1+\frac{P}{N_0B}\right) \le W\log_2\left(1+\frac{P}{N_0W}\right) = \mathcal{C} \quad (5)$$

where P is the signal power, $N_0/2$ is the two-sided noise power spectral density, and $\mathcal{C}$ is the capacity of the channel. Since $\mathcal{R}$ is monotonically increasing in B, the capacity $\mathcal{C}$ is achieved only if $B = W$. Thus, any mapping $\mathcal{X}$ that achieves capacity must satisfy $B = W$. For spread-spectrum signaling, the mapping $\mathcal{X}$ necessarily results in $B < W$. Since the maximum achievable rate is determined by B, spreading could be considered to be providing zero coding gain for single-user communications on an AWGN channel.

Based on Massey's definition of a spread spectrum system, it is tempting to define the operation of spreading as any bandwidth redundancy scheme that increases the Fourier bandwidth, while preserving the Shannon bandwidth. However, such a definition may not be consistent with the notion that spreading provides zero coding gain in AWGN, as the following example illustrates.

Example 1: Consider a signal set $\mathcal{S}^{(1)} \subset \mathcal{F}(T, W^{(1)})$, with $B^{(1)} = W^{(1)}$, consisting of the signals

$$\boldsymbol{s}_1 = \begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{bmatrix}^\top, \quad \boldsymbol{s}_2 = [0 \quad 1]^\top, \quad \boldsymbol{s}_3 = [1 \quad 0]^\top.$$

Consider the bandwidth redundancy mapping that increases the Fourier dimension to three in such a way that the new signal set $\mathcal{S}^{(2)}$ consists of the signals

$$\begin{aligned}\boldsymbol{s}_1' &= \begin{bmatrix}\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}^\top\\ \boldsymbol{s}_2' &= \begin{bmatrix}-\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0\end{bmatrix}^\top\\ \boldsymbol{s}_3' &= \begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\end{bmatrix}^\top.\end{aligned}$$

Clearly, the Shannon dimension (and bandwidth) of $\mathcal{S}^{(2)}$ are the same as those of $\mathcal{S}^{(1)}$. However, the mapping increases the Euclidean distance between the signals while preserving their energies and, hence, provides coding gain in AWGN.

It is easy to check that the mapping from $\mathcal{S}^{(1)}$ to $\mathcal{S}^{(2)}$ in Example 1 is not linear. Now consider a linear mapping from $\mathcal{S}^{(1)} \subset \mathcal{F}(T, W^{(1)})$ to $\mathcal{S}^{(2)} \subset \mathcal{F}(T, W^{(2)})$, with $W^{(2)} > W^{(1)}$, and let this mapping be defined by matrix $\boldsymbol{L}$. If $\boldsymbol{L}^\top\boldsymbol{L}$ is nonsingular, then this mapping clearly preserves the Shannon bandwidth. This might motivate the definition of spreading as an energy preserving, nonsingular, linear mapping. However, the following simple example illustrates that even such a definition is not specialized enough.

Example 2: Consider $\mathcal{S}^{(1)} = \{[1/\sqrt{2}\ 1/\sqrt{2}]^\top, [1\ 0]^\top\}$, and the linear mapping defined by

$$\boldsymbol{L} = \begin{bmatrix}1 & -1\\ 0 & \sqrt{2}\end{bmatrix}.$$

This mapping takes $\mathcal{S}^{(1)}$ to $\mathcal{S}^{(2)} = \{[0\ \ 1], [1\ \ 0]\}$. Hence, $\boldsymbol{L}$ is energy preserving, but it increases the distance between the signals and provides coding gain in AWGN.

If we further restrict the linear mapping $\boldsymbol{L}$ in such a way that it not only preserves the energy of the signals in $\mathcal{S}^{(1)}$, but also the distances between them, then it is easy to see that $\boldsymbol{L}$ leaves performance unchanged (provides zero coding gain) for single-user communication on an AWGN channel. A sufficient condition on $\boldsymbol{L}$ that guarantees this property is that it is *unitary*, i.e., $\boldsymbol{L}^\top\boldsymbol{L} = \boldsymbol{I}$. This motivates the following definition.

Definition 5: A bandwidth redundancy mapping from $\mathcal{S}^{(1)} \subset \mathcal{F}(T, W^{(1)})$ to $\mathcal{S}^{(2)} \subset \mathcal{F}(T, W^{(2)})$, with $W^{(2)} > W^{(1)}$, is called *spreading* if it can be expressed as a unitary linear mapping.

While such a definition may appear to be restrictive, it generalizes the two standard ways of spectrum spreading: direct sequence spreading and frequency hopping.

Example 3: Direct Sequence Spreading: Consider a M_T-ary signal set $\mathcal{S}^{(1)} \subset \mathcal{F}(T; W^{(1)})$. Using the basis functions of (1) with $W = W^{(1)}$, we have $\boldsymbol{s}_m^{(1)} = [s_{m,1}^{(1)}\ s_{m,2}^{(1)}\ \cdots\ s_{m,D_F}^{(1)}]^\top$, $m = 1, 2, \ldots, M_T$, where $D_F = 2W^{(1)}T$.

Direct sequence spreading by factor N, with $W^{(2)} = NW^{(1)}$, involves replacing $\phi_i(t)$ in (2) by the chip signal $c_i(t)$ that is given by

$$c_i(t) = \sum_{n=1}^{N} c_{i,n}\psi_{(i-1)N+n}(t)$$

where

$$\psi_j(t) = \sqrt{2W^{(2)}}\,\mathrm{sinc}\left(2W^{(2)}t - j\right), \quad j = 1, \ldots, ND_F$$

are basis functions for $\mathcal{F}(T, W^{(2)})$. Thus, the spread signal is given by

$$s_m^{(2)}(t) = \sum_{i=1}^{D_F} s_{m,i}^{(1)}c_i(t) = \sum_{i=1}^{D_F}\sum_{n=1}^{N} c_{i,n}s_{m,i}^{(1)}\psi_{(i-1)N+n}(t).$$

Of course, since $s_m^{(2)}(t)$ belongs to $\mathcal{F}(T, W^{(2)})$, we can write

$$s_m^{(2)}(t) = \sum_{j=1}^{ND_F} s_{m,j}^{(2)}\psi_j(t).$$

Now, if we define $\boldsymbol{s}_m^{(2)} = [s_{m,1}^{(2)}\ s_{m,2}^{(2)}\ \cdots\ s_{m,ND_F}^{(2)}]^\top$ and $\boldsymbol{c}_i = [c_{i,1}\ c_{i,2}\ \cdots\ c_{i,N}]^\top$, then

$$\boldsymbol{s}_m^{(2)} = \boldsymbol{L}\boldsymbol{s}_m^{(1)}, \quad m = 1, 2, \ldots, M_T$$

where

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{c}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{c}_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{c}_{D_F} \end{bmatrix}.$$

If we normalize the spreading sequences so that $\boldsymbol{c}_i^\top \boldsymbol{c}_i = 1$, it is easy to see that $\boldsymbol{L}^\top \boldsymbol{L} = \boldsymbol{I}$.

Example 4: Fast Frequency Hopping: Consider the same M_T-ary signal set on $\mathcal{S}^{(1)}$ as in Example 3. Frequency hopping involves replacing $\phi_i(t)$ in (2) by

$$\psi_i(t) = f\left(t - i/2W^{(1)}; j(i)\right)$$

where

$$f(t;p) = 2\sqrt{W^{(1)}} \cos\left[2\pi W^{(1)} pt\right] \operatorname{sinc}\left(2W^{(1)} t\right)$$

and where $j(i) \in \{1, 2, \ldots, N\}$ is the hopping sequence. It is easy to show that the mapping that describes frequency hopping is also linear with

$$\boldsymbol{L} = \begin{bmatrix} \boldsymbol{e}_{j(1)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \boldsymbol{e}_{j(2)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{0} & \boldsymbol{e}_{j(D_F)} \end{bmatrix}$$

where $\boldsymbol{e}_j$ denotes the unit length N vector with a one in the jth position.

It is easily checked that other forms of spreading based on multicarrier approaches are also special cases of Definition 5.

C. Separation Result

To complete our description of the dichotomy between coding and spreading, we give a narrower definition of coding in terms of Shannon and Fourier bandwidths.

Definition 6: Let $\mathcal{S}^{(1)} \subset \mathcal{F}(T, W^{(1)})$ be a signal set with $B^{(1)} = W^{(1)}$. A mapping from $\mathcal{S}^{(1)}$ to $\mathcal{S}^{(2)} \subset \mathcal{F}(T, W^{(2)})$, is called *coding* if $B^{(2)} = W^{(2)}$ as well.

Note that coding does not necessarily expand bandwidth, i.e., $W^{(2)}$ could be less than $W^{(1)}$. It is clear from (5) that the coding that achieves the capacity of a single-user AWGN channel satisfies Definition 6. The same is true of ideal, capacity achieving codes for other channels that we consider in Section V. Also, as argued in [1], for large block lengths any nontrivial coding scheme will satisfy Definition 6.

Definitions 5 and 6 lead naturally to the following separation result.

Proposition 1: Separation of Coding and Spreading: Consider the baseline signal set $\mathcal{S}' \subset \mathcal{F}(T; W')$, which has $W' = B' = R/2$. Consider any bandwidth redundancy mapping $\mathcal{X}$ that increases the Fourier bandwidth of the signal set to $W > W'$, and changes the Shannon bandwidth to B. This scheme can be written as a concatenation of coding from $\mathcal{S}'$ to $\tilde{\mathcal{S}} \subset \mathcal{F}(T; \tilde{W})$, followed by spreading from $\tilde{\mathcal{S}}$ to $\mathcal{S} \subset \mathcal{F}(T; W)$, where $\tilde{W} = B$.

Proof: Let $D_F = 2WT$, $D_S = 2BT$ and $D_F' = 2W'T$. The M_T-ary signal set can be represented by the D_F-dim vectors $\{\boldsymbol{s}_1, \boldsymbol{s}_2, \ldots, \boldsymbol{s}_{M_T}\}$, the D_F'-dim vectors $\{\boldsymbol{s}_1', \boldsymbol{s}_2', \ldots, \boldsymbol{s}_{M_T}'\}$ in $\mathcal{F}(T; W)$, and $\mathcal{F}(T; W')$, respectively. Then

$$\boldsymbol{s}_m = \mathcal{X}(\boldsymbol{s}_m'), \quad \text{for } m = 1, 2, \ldots, M_T.$$

Note that the signals $\{\boldsymbol{s}_m\}$ are D_F-dim vectors whose span has dimension D_S. Thus, we may identify a set of D_S orthonormal vectors $\{\boldsymbol{l}_j\}$, and write

$$\boldsymbol{s}_m = \sum_{j=1}^{D_S} \tilde{s}_{m,j} \boldsymbol{l}_j. \tag{6}$$

If we set $\boldsymbol{L} = [\boldsymbol{l}_1 \ \cdots \ \boldsymbol{l}_{D_S}]$, then it is clear that $\boldsymbol{L}^\top \boldsymbol{L} = \boldsymbol{I}$.

Associated with each $\boldsymbol{s}_m$ is a D_S-dim vector $\tilde{\boldsymbol{s}}_m = [\tilde{s}_{1,m} \ \cdots \ \tilde{s}_{D_S,m}]^\top$. Using (6) and the fact that $\boldsymbol{L}^\top \boldsymbol{L} = \boldsymbol{I}$, we can write

$$\tilde{\boldsymbol{s}}_m = \boldsymbol{L}^\top \boldsymbol{s}_m = \boldsymbol{L}^\top \mathcal{X}(\boldsymbol{s}_m') =: g(\boldsymbol{s}_m'), \qquad \text{for } m = 1, 2, \ldots, M_T.$$

Now, g is a mapping from $\mathcal{S}'$ to $\tilde{\mathcal{S}} \subset \mathcal{F}(T; \tilde{W})$, with $\tilde{W} = B$. Also, it is clear that $\tilde{B} = \dim[\mathrm{span}(\tilde{\mathcal{S}})] = B$ as well. Thus, g is *coding* according to Definition 6. The *spreading* part is obviously described by the mapping $\boldsymbol{L}$, since it is unitary and

$$\boldsymbol{s}_m = \boldsymbol{L} \tilde{\boldsymbol{s}}_m, \quad \text{for } m = 1, 2, \ldots, M_T.$$

□

III. The Tradeoff Problem for CDMA Systems

Given any bandwidth redundancy scheme, Proposition 1 allows us to identify and separate the coding and spreading components of this scheme. It is also clear that coding and spreading can contribute differently to system performance. The natural question that arises then is how a fixed bandwidth expansion factor should be allocated between coding and spreading.

For illustration, consider the tradeoff problem for a single-user, AWGN communication system. Since spreading cannot increase channel capacity, the capacity maximizing solution puts all of the bandwidth expansion into coding. However, as noted in [1], spreading need not reduce capacity too much, and if practical constraints such as decoding complexity are taken into account, using a significant fraction of the bandwidth expansion for spreading may be justified. Other motivations for spreading could come from the low probability of interception (LPI) of a spread spectrum signal and its immunity to multipath fading in a wireless environment.

Now consider the coding-spreading tradeoff in the context of CDMA systems. For simplicity of presentation, consider the symmetric situation where each one of K users independently needs to send information at the rate of R bits/s in interval $[0, T]$. We may construct baseline signal sets for each of the users as per Definition 1, with $B' = W' = R/2$. The baseline signal set for user k is denoted by $\mathcal{S}_k' = \{\boldsymbol{s}_{k,1}', \ldots, \boldsymbol{s}_{k,M_T}'\}$.

Suppose all K signals are coded by a factor B/W' and then spread by a factor W/B, as discussed in Proposition 1. For user k, let g_k denote the mapping (codebook) that defines the coding,

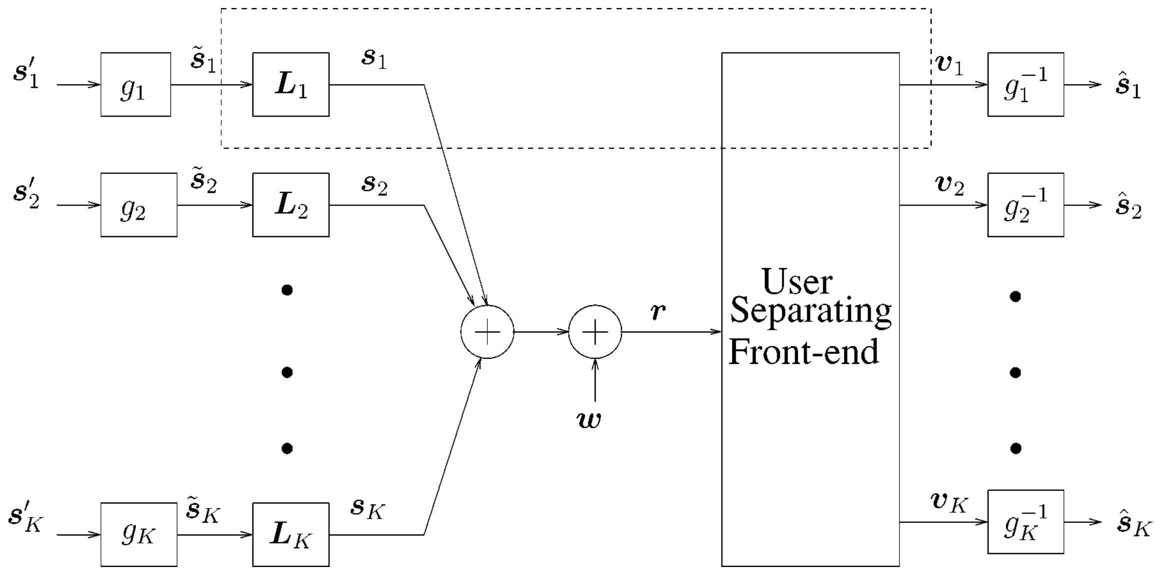

Fig. 1. Single-user decoding paradigm.

and let $\boldsymbol{L}_k$ denote the linear mapping that defines the spreading. Then the coding and spreading operations at the transmitters are, respectively, described by

$$\tilde{\boldsymbol{s}}_{k,m} = g_k(\boldsymbol{s}'_{k,m}), \quad \text{and} \quad \boldsymbol{s}_{k,m} = \boldsymbol{L}_k \tilde{\boldsymbol{s}}_{k,m}. \tag{7}$$

The mappings $\{g_k\}$ and $\{\boldsymbol{L}_k\}$ could be different across the users, although in practical CDMA systems, such as the one based on the IS-95 standard [11], the codebooks are identical. The transmitted signal corresponding to the mth symbol of user k is, thus, $s_{k,m}(t)$.

The received signal obviously depends on the effect of the channel on the users' signals. For wireless channels, typical effects include addition and attenuation of transmitted signals, propagation delays, multiple resolvable paths, additive noise, etc. In general, if we assume a channel model where the signals undergo linear distortion and the additive noise at the receiver is white Gaussian, the received signal can be written as

$$\boldsymbol{r} = \sum_{k=1}^{K} \breve{\boldsymbol{L}}_k \tilde{\boldsymbol{s}}_k + \boldsymbol{w} \tag{8}$$

where $\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{0}, (N_0/2)\boldsymbol{I})$, and $\breve{\boldsymbol{L}}_k$ depends on $\boldsymbol{L}_k$ and the channel model. In particular, when the users are synchronous and there is no multipath fading, we have $\breve{\boldsymbol{L}}_k = \boldsymbol{L}_k$. We will consider this case in greater detail in Section V. In addition, though most of the discussion in this section holds for general $\breve{\boldsymbol{L}}_k$, we use $\boldsymbol{L}_k$ for simplicity in notation.

For an AWGN multiaccess channel, the solution that maximizes the sum capacity again favors all coding [6]. Of course, this solution assumes an optimum joint decoding scheme that forms estimates of the information symbols of all users jointly from $\boldsymbol{r}$, using a decoder that exploits information about both the spreading and coding components of all users' signals, as well as the channel state information. For synchronous users with spreading factor N equal to number of users K, it is possible to orthogonalize users so as to not incur any loss in capacity. Even when $K > N$ it is possible to find spreading sequences that incur no loss in capacity [12], [13].

The tradeoff problem is much more interesting in practical CDMA systems, where complexity constraints favor adopting a receiver that consists of a front-end multiuser detector followed by autonomous single-user decoders. The front-end produces K outputs $\{\boldsymbol{v}_k\}_{k=1}^{K}$, with $\boldsymbol{v}_k$ being a "good" estimate of the encoder output $\tilde{\boldsymbol{s}}_k$ of user k, based on $\boldsymbol{r}$. The estimates $\{\boldsymbol{v}_k\}$ are then fed to autonomous single-user decoders (see Fig. 1). We formally state below the single-user decoding assumption that will be used in the remainder of the paper.

Assumption 1: The decoder for a particular user does not depend on the codebooks of the interferers.

IV. LMMSE Front-End and Separation at the Receiver

Under the single-user decoding restriction, we will establish a separation between detection and decoding in the following sections. We first show that the LMMSE front-end does not benefit from knowledge of the codebooks of the users. Further, we establish that the LMMSE front-end is the optimum front-end among all front-ends (linear and nonlinear) that do not use knowledge of the codebooks of the users. Finally, we give conditions under which the single-user decoders that follow the LMMSE front-end do not benefit from knowledge of the spreading matrices of the users.

A. LMMSE Front-End Does not Require Codebooks

Suppose we use the MMSE criterion for producing the estimates $\{\boldsymbol{v}_k\}$ from $\boldsymbol{r}$. The goal is then to pick the $\{\boldsymbol{v}_k\}$ to minimize the MSE

$$\text{MSE} = E\left[\sum_{k=1}^{K} \|\boldsymbol{v}_k - \tilde{\boldsymbol{s}}_k\|^2\right]. \tag{9}$$

The expectation in (9) is over the distribution of the noise and the prior distribution on the symbols of the users.

Note that the probability distribution of the code-vector $\{\tilde{\boldsymbol{s}}_k\}$ is a function of the prior distribution on the information symbols and the codebooks $\{g_k\}$. Since the users send independent pieces of information, the code-vectors $\{\tilde{\boldsymbol{s}}_k\}$ can be assumed to be mutually independent across users. The individual components of each code-vector $\tilde{\boldsymbol{s}}_k$ are necessarily dependent (for a nontrivial code). However, it is reasonable to assume that the components are uncorrelated.

Assumption 2: The code-vectors $\{\tilde{\boldsymbol{s}}_k\}$ are mutually independent. Furthermore, the components of each $\tilde{\boldsymbol{s}}_k$ are zero-mean, identically distributed, and uncorrelated, i.e.,

$$E[\tilde{\boldsymbol{s}}_k] = \boldsymbol{0}, \quad \text{and} \quad E\left[\tilde{\boldsymbol{s}}_k \tilde{\boldsymbol{s}}_k^\top\right] = \frac{\mathcal{E}_k}{D_S}\boldsymbol{I}$$

where $\mathcal{E}_k$ is the average energy of $\tilde{\boldsymbol{s}}_k$. We further assume that $\mathcal{E}_k$ is not a function of g_k.

While Assumption 2 appears to be restrictive, it holds for most coding schemes of interest.

Claim 1: Suppose g is constructed from a (D_S, D_F) error control code $\mathcal{C}$ that is linear over a finite field $\mathrm{GF}(q)$, by mapping the symbols from $\mathrm{GF}(q)$ to the real line $\mathbb{R}$. Suppose further that this mapping is designed to yield average energy $\mathcal{E}/D_S$ per symbol, while maximizing the distance between constellation points on $\mathbb{R}$. Then Assumption 2 holds.

Proof: This construction obviously requires the mild condition that $M_T = q^{D_F}$. Let $\boldsymbol{c}$ denote a generic codeword of $\mathcal{C}$. If we consider a particular position, say the ith, it is easy to show from linearity over $\mathrm{GF}(q)$ that the ith codesymbol c_i takes on each value in $\mathrm{GF}(q)$ the same number of times if we look across codebook $\mathcal{C}$. (This assumes of course that we eliminate code symbols that are zero for all codewords.) To construct g from $\mathcal{C}$, we simply map the elements of $\mathrm{GF}(q)$ to $\mathbb{R}$. We note that to maximize distance under given average power constraints, the mapping to $\mathbb{R}$ must be symmetric around the origin. With the assumption that the information symbols are equally likely, the claim follows. □

The MMSE front-end could in general use knowledge of both the codebooks $\{g_k\}$ and the spreading matrices $\{\boldsymbol{L}_k\}$ to minimize the MSE of (9). If we restrict the front-end to be linear, we have the following result.

Proposition 2: Under Assumption 2, the LMMSE front-end depends only on the spreading matrices of the various users, and not their codebooks.

Proof: Any linear front-end can be described by a set of matrices $\{\boldsymbol{H}_k\}_{k=1}^K$ that map $\boldsymbol{r}$ of (8) to the vectors $\{\boldsymbol{v}_k\}$, i.e.,

$$\boldsymbol{v}_k = \boldsymbol{H}_k \boldsymbol{r}, \quad k = 1, \ldots, K.$$

Under Assumption 2, it is easy to show [see, e.g., ([14, Chapter 6])] that the LMMSE solution for user k is given by

$$\boldsymbol{H}_k = \left(\frac{N_0}{2}\boldsymbol{I} + \frac{\mathcal{E}_k}{D_S}\boldsymbol{L}_k^\top \boldsymbol{Q}^{-1}\boldsymbol{L}_k\right)^{-1} \boldsymbol{L}_k^\top \boldsymbol{Q}^{-1} \tag{10}$$

where

$$\boldsymbol{Q} = \sum_{j \neq k} \frac{\mathcal{E}_j}{D_S}\boldsymbol{L}_j \boldsymbol{L}_j^\top + \frac{N_0}{2}\boldsymbol{I}. \tag{11}$$

The result follows. □

Proposition 2 illustrates a partial separation of coding and spreading at the receiver, i.e., coding does not help with linear signal separation at the front-end under the MMSE criterion. On the other hand, a nonlinear front-end can make use of knowledge of the codebooks to improve performance. An example of such a nonlinear front-end is an interference cancellation scheme that uses the code-books of the interferers to reconstruct their signals for cancellation. In Section IV-B, we consider the LMMSE front-end from a different viewpoint, and argue that it is optimal among all (linear and nonlinear) front-ends that are constrained to use only spreading information.

B. Optimality of the LMMSE Front-End

In the remainder of this section, we assume that in addition to Assumptions 1 and 2, the following assumption holds.

Assumption 3: The front-end is not allowed to use the codebooks of any user.

Consider the received signal (8) again

$$\boldsymbol{r} = \boldsymbol{L}_k \tilde{\boldsymbol{s}}_k + \sum_{j \neq k} \boldsymbol{L}_j \tilde{\boldsymbol{s}}_j + \boldsymbol{w} \tag{12}$$

where we have separated out the signal of user k. Since neither the front-end nor the decoder of user k is allowed to use codebook information of interfering users, we can interpret (12) as a single-user vector channel between $\tilde{\boldsymbol{s}}_k$ and $\boldsymbol{r}$. The interferers are, thus, treated as part of the additive noise, i.e.,

$$\boldsymbol{r} = \boldsymbol{L}_k \tilde{\boldsymbol{s}}_k + \boldsymbol{w}_I \tag{13}$$

where $\boldsymbol{w}_I = \sum_{j \neq k} \boldsymbol{L}_j \tilde{\boldsymbol{s}}_j + \boldsymbol{w}$. Using Assumption 2, it follows that the noise vector $\boldsymbol{w}_I$ has the covariance matrix

$$E\left[\boldsymbol{w}_I \boldsymbol{w}_I^\top\right] = \sum_{j \neq k} \frac{\mathcal{E}_j}{D_S}\boldsymbol{L}_j \boldsymbol{L}_j^\top + \frac{N_0}{2}\boldsymbol{I} = \boldsymbol{Q}.$$

The connection between $\tilde{\boldsymbol{s}}_k$ and $\boldsymbol{v}_k$ can be considered to be the effective single-user (ESU) channel for user k (see Fig. 1). The maximum information rate for user k is determined by the capacity of this ESU channel. If we assume for now that the decoder for user k is allowed to use the spreading information of all the users, then capacity of the ESU channel of user k is determined by the mutual information $I(\tilde{\boldsymbol{s}}_k; \boldsymbol{v}_k \mid \{\boldsymbol{L}_j\}_{j=1}^K)$. This motivates the following definition.

Definition 7: A front-end is *optimum* for single-user decoding of user k if it maximizes $I(\tilde{\boldsymbol{s}}_k; \boldsymbol{v}_k \mid \{\boldsymbol{L}_j\}_{j=1}^K)$ for each choice of distribution for $\tilde{\boldsymbol{s}}_k$.

From the data processing inequality [15, Chapter 2] it is clear that

$$I\left(\tilde{\boldsymbol{s}}_k; \boldsymbol{v}_k \;\middle|\; \{\boldsymbol{L}_j\}_{j=1}^K\right) \leq I\left(\tilde{\boldsymbol{s}}_k; \boldsymbol{r} \;\middle|\; \{\boldsymbol{L}_j\}_{j=1}^K\right). \tag{14}$$

Our goal is to show that the LMMSE front-end achieves this upper bound and is hence optimum according to Definition 7.

To proceed we need to make the following additional assumption.

Assumption 4: The noise vector $\boldsymbol{w}_I$ has a *Gaussian* distribution with zero-mean and covariance $\boldsymbol{Q}$.

Clearly, the assumption would hold if the codesymbol vectors of the interferers were themselves Gaussian. For an arbitrary distribution of the symbols, it is possible to show that $\boldsymbol{w}_I$ tends to a Gaussian vector in distribution as $K \to \infty$, under mild conditions on the spreading matrices [16, Section 29]. While this asymptote may justify the Gaussian approximation to some extent, it is of greater interest to let $N \to \infty$ as well with K/N tending to a constant. A rigorous application of the Central Limit Theorem in the latter asymptote is an interesting open problem.

The Gaussian approximation for the interference implies the following result, which follows from reasoning similar to that in [7].

Proposition 3: Under Assumptions 1–4, the LMMSE is optimum for single-user decoding of each user k.

Proof: It follows from the data processing inequality [15, Chapter 2] that mutual information is preserved under invertible transformations of the received signal. Thus

$$\begin{aligned} I\left(\tilde{\boldsymbol{s}}_k; \boldsymbol{r} \mid \{\boldsymbol{L}_j\}_{j=1}^K\right) &\overset{\text{(a)}}{=} I\left(\tilde{\boldsymbol{s}}_k; \boldsymbol{Q}^{-1/2}\boldsymbol{r} \mid \{\boldsymbol{L}_j\}_{j=1}^K\right) \\ &\overset{\text{(b)}}{=} I\left(\tilde{\boldsymbol{s}}_k; \boldsymbol{L}_k^\top \boldsymbol{Q}^{-1}\boldsymbol{r} \mid \{\boldsymbol{L}_j\}_{j=1}^K\right) \\ &\overset{\text{(c)}}{=} I\left(\tilde{\boldsymbol{s}}_k; \boldsymbol{H}_k\boldsymbol{r} \mid \{\boldsymbol{L}_j\}_{j=1}^K\right) \end{aligned}$$

where $\boldsymbol{H}_k$ is the LMMSE front-end. In the above equalities, (a) follows since $\boldsymbol{Q}$ is invertible. To see (b) note that

$$\boldsymbol{Q}^{-1/2}\boldsymbol{r} = \boldsymbol{Q}^{-1/2}\boldsymbol{L}_k\tilde{\boldsymbol{s}}_k + \boldsymbol{Q}^{-1/2}\boldsymbol{w}_I$$

is just a noise-whitening operation and $\boldsymbol{Q}^{-1/2}\boldsymbol{w}_I$ is $\mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$. Therefore, $\boldsymbol{L}_k^\top \boldsymbol{Q}^{-1}\boldsymbol{r}$ is a sufficient statistic, and the mutual information is preserved [15, Chapter 2]. Finally, (c) follows because $\boldsymbol{H}_k$ is related to $\boldsymbol{L}_k^\top \boldsymbol{Q}^{-1}$ through an invertible matrix [see (10)]. Thus, the LMMSE front-end achieves the upper bound in (14), and the proposition follows. □

C. Separation Between Detection and Decoding

Proposition 3 illustrates that the LMMSE front-end that uses only spreading information does not lead to a loss of mutual information. As mentioned earlier, this corresponds to a partial separation of coding and spreading at the receiver. For complete separation, the decoder should use only the codebook of the desired user and no spreading information. To see whether complete separation is possible without a loss in performance, consider the output of the LMMSE detector that is fed to the decoder. Let $\boldsymbol{P}_k = \boldsymbol{L}_k^\top \boldsymbol{Q}^{-1}\boldsymbol{L}_k$. Then, using (10)

$$\begin{aligned} \boldsymbol{v}_k = \boldsymbol{H}_k\boldsymbol{r} &= \left(\frac{N_0}{2}\boldsymbol{I} + \frac{\mathcal{E}_k}{D_S}\boldsymbol{P}_k\right)^{-1} \boldsymbol{L}_k^\top \boldsymbol{Q}^{-1}\boldsymbol{r} \\ &= \left(\frac{N_0}{2}\boldsymbol{I} + \frac{\mathcal{E}_k}{D_S}\boldsymbol{P}_k\right)^{-1} (\boldsymbol{P}_k\tilde{\boldsymbol{s}}_k + \boldsymbol{z}_I) \end{aligned}$$

where $\boldsymbol{z}_I$ is $\mathcal{N}(\boldsymbol{0}, \boldsymbol{P}_k)$. Clearly, the spreading matrices enter the detector output only through the matrix $\boldsymbol{P}_k$. Hence, complete separation between the LMMSE detector and decoder is achieved with no loss of performance if

$$I(\tilde{\boldsymbol{s}}_k; \boldsymbol{v}_k \mid \boldsymbol{P}_k) = I(\tilde{\boldsymbol{s}}_k; \boldsymbol{v}_k). \tag{15}$$

While (15) does not hold in general, there are particular cases in which it is true. A simple example is when we have synchronous users in AWGN and all the users span mutually orthogonal subspaces. In this case, we have $\boldsymbol{P}_k = (2/N_0)\boldsymbol{L}_k^\top \boldsymbol{L}_k = (2/N_0)\boldsymbol{I}$. Another interesting scenario is when the matrices $\{\boldsymbol{L}_j\}_{j=1}^K$ consist of independent, zero-mean, randomly chosen entries, and we consider the large system asymptote where $K, N \to \infty$ with K/N tending to a constant. Then equality in (15) is asymptotically achieved (see, e.g., [17]). A special case of this scenario is described in Section V.

The above discussion justifies the use of LMMSE detector as a benchmark for the front-end detector, under the single-user decoding restriction. It is, hence, of interest to optimize the coding-spreading tradeoff for this front-end. In the following section we discuss an example that illustrates this optimization. We also compare the LMMSE and conventional MF front-ends.

V. Coding-Spreading Tradeoff in Synchronous DS/CDMA System

Consider a CDMA system in which spreading is achieved using a direct sequence approach, where each code symbol is spread by a unit energy chip waveform as in Example 3. *Random* spreading is assumed, i.e., the spreading sequences are independent from codesymbol to codesymbol and across users. Furthermore, the chips within a codesymbol are independent identically distributed (i.i.d.) *binary* zero-mean random variables. In addition, we assume a synchronous system, although similar results will be obtained for asynchronous systems as long as bandwidth restrictions are correctly imposed on the chip waveforms [18], [19]. We first consider a "single-cell" wireless communication system, where all the K users in an isolated cell are received with *equal power* (i.e., they are perfectly power controlled) at the base station and the only interference is from thermal AWGN with a spectral height of $N_0/2$. The approach is easily extended to a multicell scenario, and we demonstrate this through a simple model for the out-of-cell interference in Section VI.

As before, we have that each user sends information at rate of R bits/s of information in some long time interval $[0, T]$. The transmission bandwidth available is W and $\Omega = 2W/R$ is the bandwidth expansion factor. By Proposition 1, we can separate the bandwidth redundancy mapping into a coding component with rate ν bits/symbol followed by a spreading component that leads to an expansion by a factor N. Let $\mathcal{E}_b$, $\mathcal{E}_s$, and $\mathcal{E}_c$ denote the energies per information bit, codesymbol, and chip, respectively. Also, define the corresponding signal-to-noise ratios (SNRs) by $\gamma_b = \mathcal{E}_b/N_0$, $\gamma_s = \mathcal{E}_s/N_0$ and $\gamma_c = \mathcal{E}_c/N_0$. Then

$$\gamma_s = N\gamma_c, \quad \text{and} \quad \gamma_b = \Omega\gamma_c.$$

Also, N and ν are obviously related as

$$\Omega = \frac{N}{\nu} \Rightarrow \gamma_b = \nu\gamma_s. \tag{16}$$

Note that the code rate ν which equals N/Ω can also be interpreted as the *spreading fraction* of the bandwidth expansion.

After projection onto basis functions of the $2WT$-dim signal space (i.e., chip-matched filtering) and normalization, we get the following model for the received vector for one codesymbol interval:

$$\boldsymbol{r} = \sqrt{2\gamma_s}\sum_{j=1}^{K} b_j \boldsymbol{c}_j + \boldsymbol{w}$$

where $\boldsymbol{w} \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{I})$ and $\{b_j\}_{j=1}^{K}$ are the codesymbols, and $\{\boldsymbol{c}_j\}_{j=1}^{K}$ are the binary spreading vectors of length N. The symbols are normalized to have unit energy on average, i.e., $E[b_k^2] = 1$ for all k. The spreading vectors are also normalized so that $\boldsymbol{c}_j^\top \boldsymbol{c}_j = 1$.

Focusing on user k, the received vector can be rewritten as

$$\boldsymbol{r} = \sqrt{2\gamma_s} b_k \boldsymbol{c}_k + \sqrt{2\gamma_s}\boldsymbol{C}\boldsymbol{b} + \boldsymbol{w}$$

where the columns of $\boldsymbol{C}$ are the spreading vectors $\{\boldsymbol{c}_j\}_{j\neq k}$ of the interferers, and $\boldsymbol{b}$ is the vector of the corresponding codesymbols. For a linear front-end, the estimate for the code symbol b_k is based on the scalar

$$v_k = \boldsymbol{h}_k^\top \boldsymbol{r}.$$

We restrict our attention to two linear front-ends, the conventional MF, and the optimum LMMSE[2] front-end from (10)

$$\text{MF: } \boldsymbol{h}_k = \boldsymbol{c}_k$$
$$\text{LMMSE: } \boldsymbol{h}_k = [\boldsymbol{I} + 2\gamma_s \boldsymbol{C}\boldsymbol{C}^\top]^{-1}\boldsymbol{c}_k.$$

For *hard* decisions, the constellation point closest in Euclidean distance to v_k is sent to the decoder for user k. For *soft* decisions, v_k (or some other appropriate function of v_k) is sent to the decoder.

The goal in the coding-spreading tradeoff optimization is to pick the coding and spreading factors to maximize performance. For a given Ω, the code rate (or spreading fraction) ν determines the spreading factor N, so we need to only optimize ν. Since we are constraining the information rates of the users (to one information bit every Ω chips), a reasonable measure of performance is the largest number of users K_m that can transmit their bits reliably on the channel. Clearly K_m is a function of Ω and N (equivalently of Ω and ν). The ratio

$$\kappa(\Omega, \nu) = \frac{K_m(\Omega, \nu)}{\Omega} \tag{17}$$

is then the total *spectral efficiency* of the CDMA system in bits/chip at a spreading fraction ν. The optimum code rate is, hence, given by

$$\nu_{\text{opt}}(\Omega) = \arg\max_{\nu} \kappa(\Omega, \nu)$$

and the peak spectral efficiency is given by

$$\kappa_{\text{peak}}(\Omega) = \kappa(\Omega, \nu_{\text{opt}}).$$

For a practical system, the code would be chosen from a family of multiple rate codes, and K_m would be the largest number of users accommodated with an information bit error rate (BER) of less than some threshold (say 10^{-3}). But we may draw useful conclusions about the coding-spreading tradeoff more easily by assuming ideal coding. Whether we pick practical codes or ideal codes, it is imperative that we verify that they are indeed "codes," in the sense of satisfying Definition 6. Otherwise, the "coding" component could still have a residual "spreading" component and the tradeoff problem is not well-defined. In the discussion below, we will consider capacity achieving codes for various constraints on the input and output alphabets of the ESU channels. As in the case of the single-user AWGN channel discussed in Section II, it can be shown that $\tilde{B} = \tilde{W}$ for these codes as well.

With the assumption of ideal coding and with the above caveat taken into account, K_m is computed as follows. For fixed K and N, we compute the capacity $\mathcal{C}(K,N)$ of the ESU channel corresponding to any one of the users, say user 1. Since the code rate ν must be less than $\mathcal{C}(K,N)$ for reliable transmission, we have[3]

$$K_m(\Omega, \nu) = \text{maximum value of } K \text{ such that } \nu < \mathcal{C}(K,N). \tag{18}$$

We now calculate $\mathcal{C}(K,N)$ for different scenarios. We begin by considering the case of binary signaling for which $b_k \in \{-1,+1\}$. Assume first that the receiver employs hard-decision decoding, i.e., $\hat{b}_k = \text{sgn}(v_k)$. In the single-user decoding paradigm adopted in this paper, the decoder could use knowledge of the spreading sequences but not the codebooks of other users. Hence, for a given realization of the sequences, the effective single-user channel is a binary symmetric channel (BSC) that is time-varying but memoryless by the assumption of random spreading. The instantaneous error probability P_e of the BSC is the average bit error probability over the code bits of the users

$$P_e = \frac{1}{2^{K-1}} \sum_{\boldsymbol{b}\in\{-1,+1\}^{K-1}} Q\left(\sqrt{\frac{2\gamma_s}{\boldsymbol{h}^\top\boldsymbol{h}}}[\boldsymbol{h}^\top\boldsymbol{c}_1 + \boldsymbol{h}^\top\boldsymbol{C}\boldsymbol{b}]\right) \tag{19}$$

where $\boldsymbol{h} = \boldsymbol{c}_1$ and $\boldsymbol{h} = [\boldsymbol{I} + 2\gamma_s\boldsymbol{C}\boldsymbol{C}^\top]^{-1}\boldsymbol{c}_1$ for the MF and LMMSE front-ends, respectively. The corresponding BSC capacity is

$$\mathcal{C}(K,N) = 1 - EH(P_e) \tag{20}$$

where H is the binary entropy function and the expectation is over $\boldsymbol{C}$ and $\boldsymbol{c}_1$.

For binary signaling and soft-decision decoding, the single-user channel is a binary input continuous output channel. If we approximate the conditional probability density function (pdf) of the output by a Gaussian, then [20, p. 273]

$$\mathcal{C}(K,N) = E[\mathcal{C}_{\text{SDG}}(\Gamma)] \tag{21}$$

where

$$\mathcal{C}_{\text{SDG}}(\Gamma) = -\frac{1}{2}\log_2 2\pi e - \int_{-\infty}^{\infty} p(y)\log_2 p(y)\, dy$$

[2]Note that the LMMSE detector here differs from that in (10) by a scalar factor. While the MSE would change with a scaling of the output, the calculations here are based on the output signal-to-interference ratio (SIR) and remain unaffected. Note that the LMMSE detector is also the SIR maximizing detector [14].

[3]The implicit assumption of course is that $\mathcal{C}(K,N)$ is decreasing function of K. This is easily seen to be true for all the cases considered in this paper.

with $p(y)$ being a mixture of $\mathcal{N}(\sqrt{\Gamma},1)$ and $\mathcal{N}(-\sqrt{\Gamma},1)$ with equal weights, and Γ being the SIR at the channel output. It is easy to show that

$$\Gamma = \frac{2\gamma_s(\boldsymbol{h}^\top \boldsymbol{c}_1)^2}{\boldsymbol{h}^\top(\boldsymbol{I}+2\gamma_s\boldsymbol{C}\boldsymbol{C}^\top)\boldsymbol{h}} \tag{22}$$

where $\boldsymbol{h} = \boldsymbol{c}_1$ and $\boldsymbol{h} = [\boldsymbol{I}+2\gamma_s\boldsymbol{C}\boldsymbol{C}^\top]^{-1}\boldsymbol{c}_1$ for the MF and LMMSE front-ends, respectively.

Finally, if we do not constrain either the input or output alphabets of the ESU channel, we can approximate this channel by an AWGN channel for the two front-ends of interest [6]. The SIR of this channel $\mathcal{C}(K,N)$ can be approximated by

$$\mathcal{C}(K,N) \approx \frac{1}{2}E[\log(1+\Gamma)] \tag{23}$$

with Γ being given by (22).

For finite K and N, the $\mathcal{C}(K,N)$ of (20), (21), and (23) are cumbersome to calculate. In the large system asymptotic scenario where $K,N\to\infty$, with the ratio K/N being kept constant, asymptotically exact expressions for Γ and $\mathcal{C}(K,N)$ may be obtained [8], [6], by fixing the code symbol SNR γ_s. In particular, the SIR converges for almost every realization of the sequences to $\tilde{\Gamma}$, where

$$\tilde{\Gamma} = \lim_{\substack{K,N\to\infty,\\ \frac{K}{N}=\beta}} \Gamma = \begin{cases} \dfrac{2\gamma_s}{2\beta\gamma_s+1}, & \text{for MF} \\ 2\gamma_s - \frac{1}{4}\mathcal{F}(2\gamma_s,\beta), & \text{for LMMSE} \end{cases} \tag{24}$$

with

$$\mathcal{F}(x,z) \stackrel{\text{def}}{=} \left(\sqrt{x(1+\sqrt{z})^2+1} - \sqrt{x(1-\sqrt{z})^2+1}\right)^2.$$

Furthermore, $\mathcal{C}(K,N)$ converges in this asymptote as follows:

$$\tilde{\mathcal{C}}(\beta;\gamma_s) = \lim_{\substack{K,N\to\infty,\\ \frac{K}{N}=\beta}} = \begin{cases} 1 - H[Q(\sqrt{\tilde{\Gamma}})], & \text{binary hard} \\ \mathcal{C}_{\text{SDG}}(\tilde{\Gamma}), & \text{binary soft} \\ \frac{1}{2}\log(1+\tilde{\Gamma}), & \text{unconstrained.} \end{cases} \tag{25}$$

Note that, in contrast to [6], we have explicitly introduced the bandwidth expansion factor Ω in setting up the coding-spreading tradeoff problem. Thus, K is not an independent variable (that can be taken to infinity) in our analysis, and the above asymptotics are not directly applicable to our problem. The quantity of interest K_m is determined by Ω and N, and we are, hence, interested in the large system limit of $\kappa(\Omega,\nu)$ where $\Omega,N\to\infty$ with the ratio (coding rate) ν being fixed. Now, it is not clear *a priori* whether the limiting spectral efficiency exists. However, we can show, based on (18) and the results for the large K,N asymptotics described above, that K_m goes to ∞ in such a way that the limit below does exist[4]

$$\lim_{\substack{N,\Omega\to\infty,\\ \frac{N}{\Omega}=\nu}} \frac{K_m(\Omega,\nu)}{\Omega} = \tilde{\kappa}(\nu). \tag{26}$$

[4]A rigorous proof for the limit existence requires the pointwise convergence (in the N,Ω asymptote) of appropriately defined inverse functions of $\mathcal{C}(K,N)$. However, this is more of a technicality and we do not pursue the matter here.

Furthermore, the equation satisfied by $\tilde{\kappa}$ can be obtained by setting the bandwidth expansion constraint to equality in (18) and using (25)

$$\tilde{\mathcal{C}}\left(\frac{\tilde{\kappa}}{\nu};\gamma_b\nu\right) = \nu. \tag{27}$$

Using (27) we can derive the asymptotic spectral efficiency $\tilde{\kappa}$ as a function of ν for a given information bit SNR γ_b.

It is of interest to note that the convergence of $\kappa(\Omega,\nu)$ for the LMMSE detector implies a complete separation of coding and spreading at the receiver in the large system asymptote, as discussed at the end of Section III. Specifically, by Assumption 4 and Proposition 3

$$I(b_k;\boldsymbol{r}\,|\,\boldsymbol{c}_k,\boldsymbol{C}) = I(b_k;v_k\,|\,\boldsymbol{c}_k,\boldsymbol{C}).$$

Moreover, the matrix $\boldsymbol{L}_k^\top\boldsymbol{Q}^{-1}\boldsymbol{L}_k$ that needs to be sent to the decoder is equal to $\boldsymbol{c}_k^\top(\boldsymbol{I}+2\gamma_s\boldsymbol{C}\boldsymbol{C}^\top)^{-1}\boldsymbol{c}_k$. This is just the output SIR for the particular realization of the sequences. If we consider the sequence of systems with K_m users (as $N,\Omega\to\infty$), then from (26) and (24), it is clear that limiting SIR is independent of the realization of the sequences in the limit. Thus

$$\lim_{\substack{N,\Omega\to\infty,\\ \frac{N}{\Omega}=\nu}} I(b_k;v_k\,|\,\boldsymbol{c}_k,\boldsymbol{C}) = \lim_{\substack{N,\Omega\to\infty,\\ \frac{N}{\Omega}=\nu}} I(b_k;v_k).$$

Hence, it follows that:

$$\lim_{\substack{N,\Omega\to\infty,\\ \frac{N}{\Omega}=\nu}} I(b_k;\boldsymbol{r}\,|\,\boldsymbol{c}_k,\boldsymbol{C}) = \lim_{\substack{N,\Omega\to\infty,\\ \frac{N}{\Omega}=\nu}} I(b_k;v_k)$$

which is an asymptotic equality between the mutual information prior to any front-end processing to that with an LMMSE front-end using only the spreading sequences and a decoder for user k that knows only the corresponding codebook g_k. Thus, under the restriction of single-user decoding at the receiver, the structure of an LMMSE front-end followed by a single-user decoder without sequence knowledge is asymptotically optimum.

VI. Numerical Results

We first consider the equal-power, single-cell scenario. Fig. 2 shows the spectral efficiency $\kappa(\Omega,\nu)$ for this case with hard and soft-decision decoding, respectively. The value of the bandwidth expansion factor Ω is set to 64. We note that small values of ν are favored for the MF front-end, while values of ν close to 1 are favored for the LMMSE front-end. This is to be expected since the LMMSE front-end uses linear signal separation through spreading to suppress multiaccess interference while the MF front-end does not. We can also see that coding gives diminishing returns for the MF receiver—a significant portion of the bandwidth expansion can be given to spreading with a marginal loss in spectral efficiency.

For hard decisions, the maximum spectral efficiency (κ_{peak}) for the MF equals 30/64≈0.47 bits/chip, and matches well with the large system, large γ_b asymptotic value of $(\log_2 e)/\pi$ predicted by Hui [3]. For soft decisions, the maximum value is 46/64≈ 0.72 bits/chip, and again matches well the value of $(\log_2 e)/2$ given in [3]. The plots also show the system asymptotic spectral efficiency values obtained by using (24) and (25) in (27), and we see a good match with finite system results.

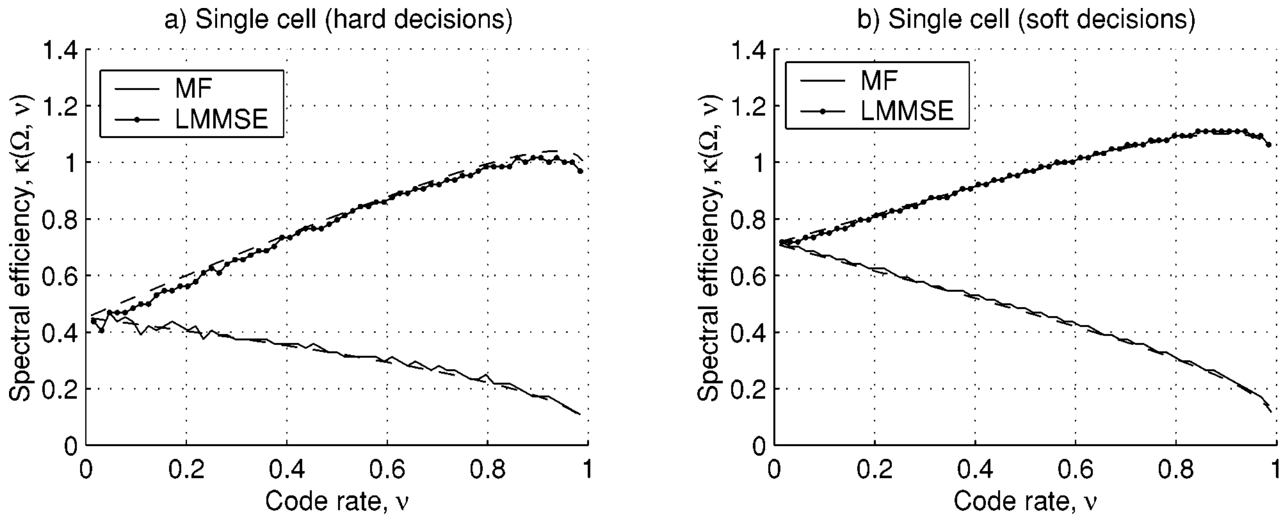

Fig. 2. Spectral efficiency as a function of the spreading factor for a single-cell system and binary signaling. For the simulation results, the bandwidth expansion factor $\Omega = 64$, the information bit SNR $\gamma_b = 18$ dB and the spectral efficiency is averaged over 2000 independent random spreading sequence sets. The dashed lines are the asymptotic spectral efficiencies.

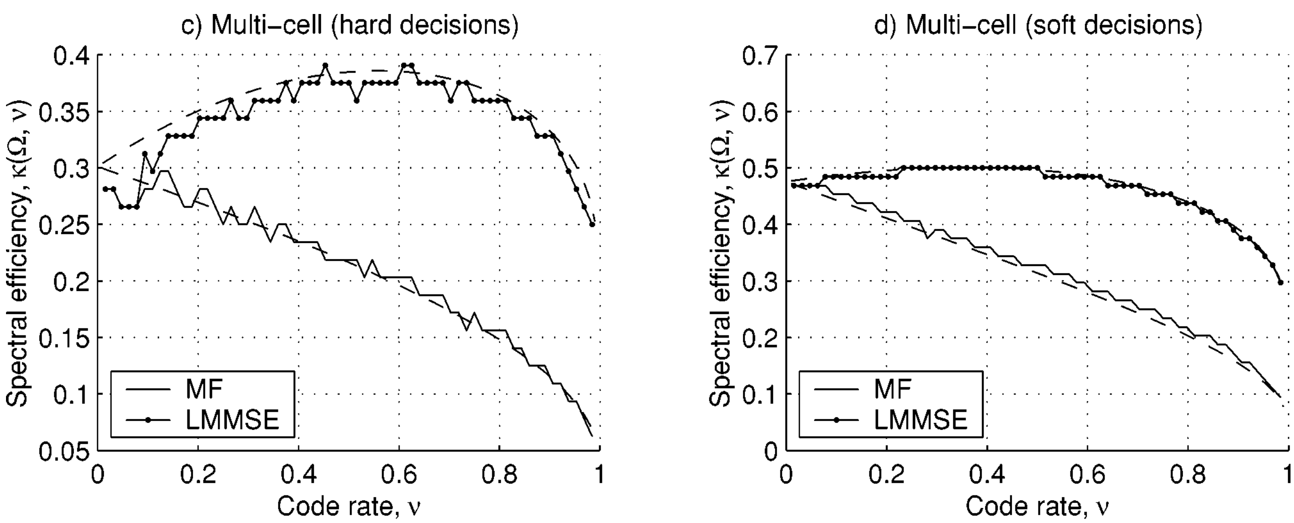

Fig. 3. Spectral efficiency per cell as a function of the spreading factor for binary signaling and a simplified multicell model, with out-of-cell interference having half the power of the in-cell interference; $\Omega = 64, \gamma_b = 18$ dB.

For the LMMSE front-end, κ_{peak} is around 1 bit/chip for hard decisions, and is slightly larger for soft decisions. It is interesting to note that the LMMSE spectral efficiency with random sequences can be higher than the maximum efficiency of 1 bit/chip achievable with orthogonal sequences and $K = N$. This is in contrast to the case without binary constraints where it is known that orthogonalizing users with $K = N$ does not incur any loss in capacity (see, e.g., [6]).

We also give results for a multicell wireless system, in which the base station receives the sum of the in-cell users' signals in the presence of interference from neighboring cells. We assume the following simple model for the other-cell interference. We consider a hexagonal cell structure and consider only the first tier of six interfering cells. We assume that all cells have the same number of users K, and that each other-cell interferer is received at a power equal to one-twelfth of the in-cell user's power. This means that the total power in the other-cell interference equals half the total in-cell power, as described by Viterbi [21]. All $7K$ users in the system are assigned independent random spreading sequences, and the LMMSE receiver uses knowledge of the spreading sequences of all $7K$ users to make its decisions.

We can again obtain asymptotic results in this cellular scenario. For the MF detector, the code symbol SIR in (24) is modified by simply replacing γ_s with $\gamma_{s,\text{eff}}^I$ where

$$\gamma_{s,\text{eff}}^I = \frac{\gamma_s}{1+\beta\gamma_s} \Rightarrow \tilde{\Gamma} = \frac{2\gamma_s}{1+3\beta\gamma_s}. \tag{28}$$

For the LMMSE detector, we obtain an implicit equation for $\tilde{\Gamma}$ by using [8, (4)]

$$\tilde{\Gamma} = \frac{2\gamma_s}{1 + \frac{2\beta\gamma_s}{1+\tilde{\Gamma}} + \frac{\beta\gamma_s}{1+\tilde{\Gamma}/12}} \tag{29}$$

and it can be simplified to yield a cubic equation in $\tilde{\Gamma}$.

The results with uniformly loaded multiple cells are shown in Fig. 3. As expected, the MF spectral efficiency is down by a factor of $2/3$ when compared with single-cell results. It is interesting to see that the gap between the LMMSE and MF

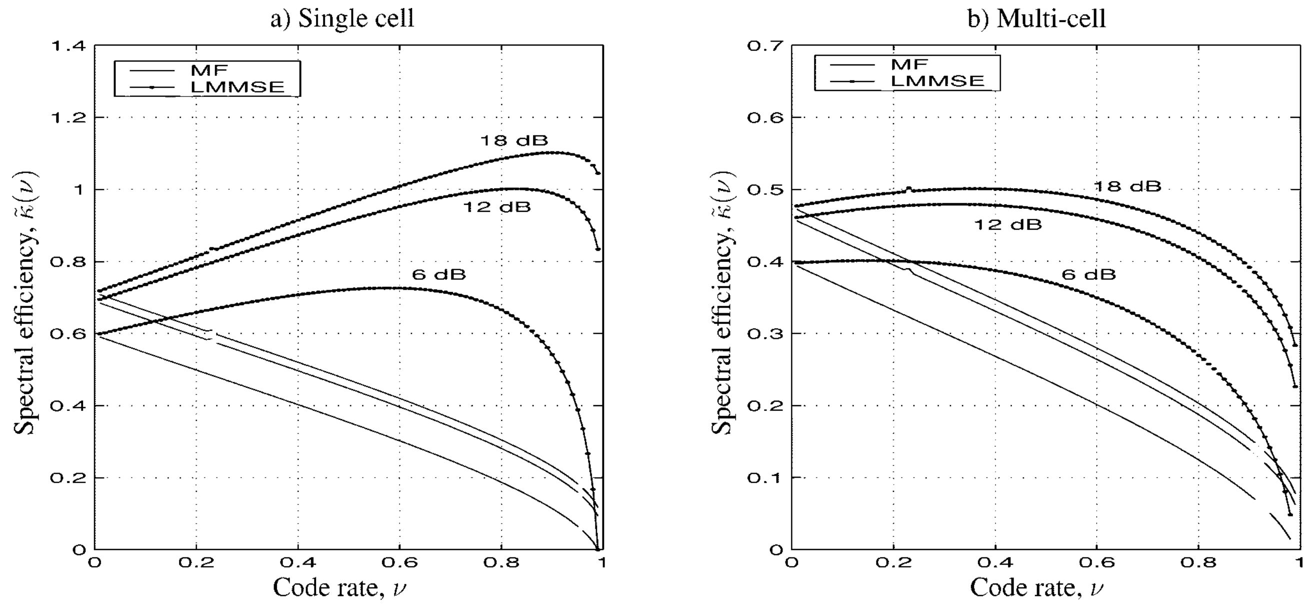

Fig. 4. Asymptotic spectral efficiency per cell with binary signals and soft-decision decoding as a function of ν for single and multicell scenarios; γ_b ={6, 12, 18} dB.

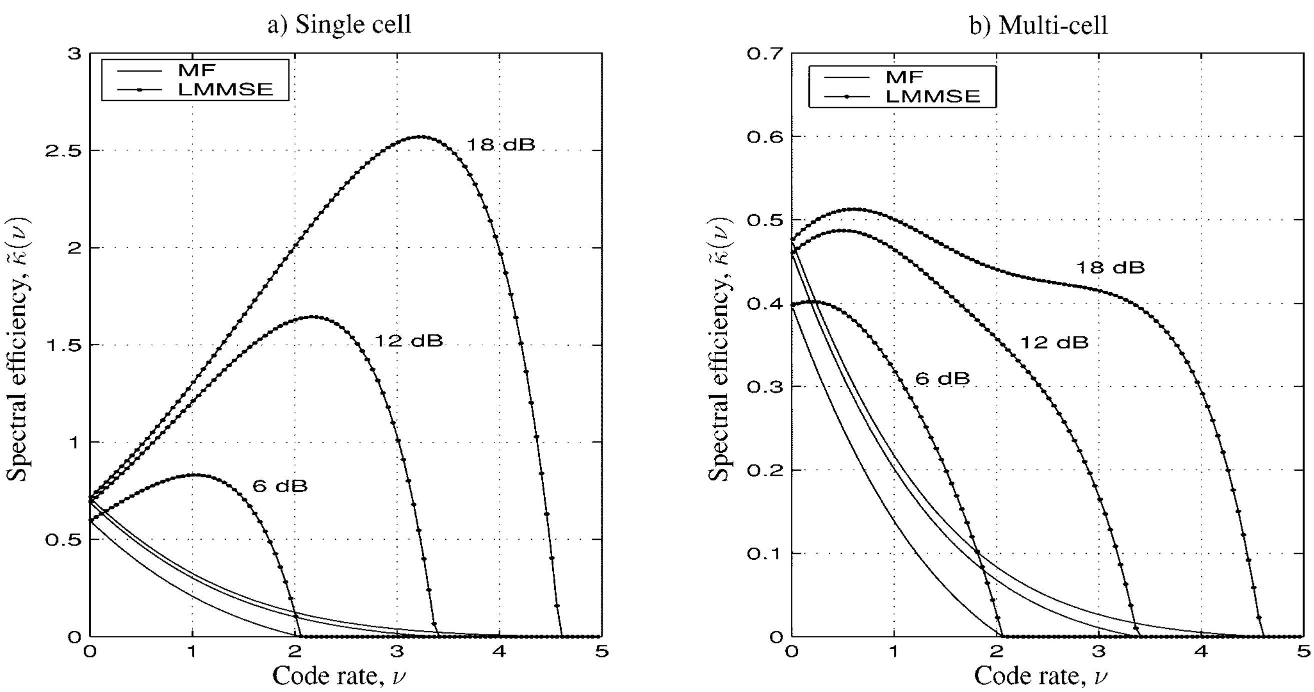

Fig. 5. Asymptotic spectral efficiency per cell with continuous alphabet as a function of ν for single and multicell scenarios; γ_b ={6, 12, 18} dB.

spectral efficiencies is much reduced, particularly in the case of soft-decision decoding.

In Figs. 4 and 5, we provide asymptotic spectral efficiency curves for the binary and continuous alphabet cases, respectively. The results are for the single-cell scenario with γ_b =18 dB and show that, with continuous alphabet, the gap between the LMMSE and MF spectral efficiencies grows without bound as γ_b increases (see, also, [6]). Thus, at γ_b =18 dB, there is significant room for improvement by using an LMMSE receiver and larger constellation sizes. In the multicell scenario, it can again be shown that the gap between the LMMSE and MF spectral efficiency peaks goes to ∞ with γ_b. However, the value of γ_b required to achieve the same gap is much larger than that for the single-cell case. Thus, a question of practical interest is whether the value of γ_b can be large enough to justify constellation sizes larger than binary and/or the more complex LMMSE front end. One perspective on this question is as follows.

The effective bit SNR seen by the LMMSE detector must include both thermal noise and any uncancelled interferers in the system. In particular, suppose that there was a *second* tier of interferers that cannot be suppressed by the LMMSE detector, leading to an effective code symbol SNR of $\gamma_{s,\text{eff}}^{\text{II}}$ and an effective bit SNR of $\gamma_{b,\text{eff}}^{\text{II}}$. Also, suppose that total interference from the second tier is a fraction f of the interference from the first tier. Then

$$\gamma_{s,\text{eff}}^{\text{II}} = \frac{\gamma_s}{1+f\beta\gamma_s} < \frac{1}{f\beta}$$
$$\Rightarrow \gamma_{b,\text{eff}}^{\text{II}} < \frac{1}{f\beta\nu} = \frac{1}{f\tilde{\kappa}(\nu)} \tag{30}$$

where the last equality is obtained using (27). Thus, $\gamma_{b,\text{eff}}^{\text{II}}$ is limited by f and $\tilde{\kappa}$. Clearly, to justify the use of the LMMSE detector over the MF detector, we must consider $\kappa > \kappa_{\text{peak}}^{\text{MF}}$, the maximum spectral efficiency with the MF detector. Analogous to (28), the spectral efficiency values for the MF with two tiers can be obtained by using

$$\tilde{\Gamma} = \frac{2\gamma_s}{1+(3+f)\beta\gamma_s}.$$

It follows that $\tilde{\kappa}(\nu)$ for the MF reaches its maximum as $\nu \to 0$, and the maximum value is:

$$\kappa_{\text{peak}}^{\text{MF}} = \frac{\log_2 e}{f+3}.$$

Hence, the maximum $\gamma_{b,\text{eff}}^{\text{II}}$ seen by the LMMSE detector can be taken to be

$$\gamma_{b,\text{eff}}^{\text{II}} < \frac{1}{f\kappa_{\text{peak}}^{\text{MF}}} = \frac{1}{\log_2 e}(1+3/f).$$

Typical values of f range from about 1 to 0.06, depending on the propagation loss coefficient and the standard deviation of the shadow fading process [22]. The corresponding range of $\gamma_{b,\text{eff}}^{\text{II}}$ is about 4.5 dB–16 dB. Referring to Figs. 4 and 5, we see that the peak spectral efficiency of the LMMSE detector is not significantly higher than that for the MF detector in this range of $\gamma_{b,\text{eff}}^{\text{II}}$.

Finally, we have assumed an LMMSE detector that suppresses the first tier interference in the multicell scenario. The spectral efficiency may be expected to be even smaller if we assume that only in-cell interference is suppressed. The resulting LMMSE detector operates at an effective code symbol SNR of $\gamma_{s,\text{eff}}^{I}$ given in (28). The corresponding spectral efficiency curves are shown in Fig. 6. The main conclusion from Figs. 4–6 can be summarized as follows: under the single-user decoding restriction, in the multicell scenario, not only does binary signaling entail no significant loss in spectral efficiency, it suffices to use the conventional MF detector.

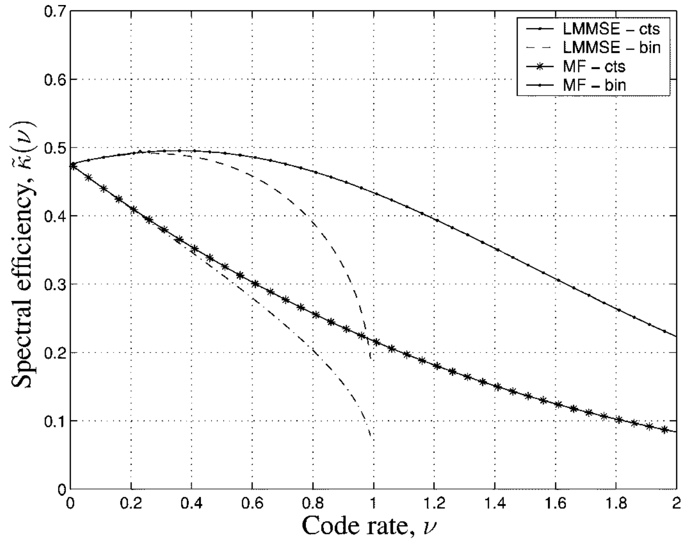

Fig. 6. Asymptotic spectral efficiency for a multicell scenario where the LMMSE detector suppresses only in-cell users (binary and continuous alphabets); γ_b =18 dB.

VII. CONCLUSION

We have given general definitions for coding and spreading, and shown that they lead to an interesting separation result for bandwidth redundancy schemes of the type used in CDMA systems. The separation result makes the coding-spreading tradeoff problem well-defined. Our approach can include more general scenarios than the ones considered in the paper, e.g., signaling across spatial dimensions through the use of multielement antennas.

We have shown through a simple example that optimizing the coding-spreading tradeoff can lead to significant gains in spectral efficiency of CDMA systems. We have also shown that the optimum coding-spreading operating point is a strong function of the type of receiver used. Many analyses comparing multiuser detection schemes with the conventional MF receiver have either ignored coding or used the same code rate for both receivers. We have shown that for a correct comparison of two alternative receiver structures for CDMA, it is important to consider each of them at their optimum coding-spreading operating points.

In particular, the results obtained for a simplified the multicell model lead us to question the applicability of multiuser detection schemes in cellular systems that employ single-user decoding. Of course to draw any concrete conclusions in this direction, we need to consider a more realistic system model. For example, we have assumed ideal coding and, hence, no delay constraints, in this paper. The tradeoff problem with nonideal codes has been considered in [23] and the conclusions are similar. We have also assumed synchronous users in an AWGN channel, perfect channel estimates and perfect power control. The MF detector can be expected to be more robust to channel estimation errors, whereas the LMMSE detector would be more robust to imperfect power control. It would, hence, be interesting to consider the problem with imperfect channel estimation and power control over more realistic channel models, especially on the reverse link. Finally, the results in this paper clearly indicate the limitations of a single-user decoding approach to receiver design, and point to the need for research on low complexity joint decoding approaches like those in [24].

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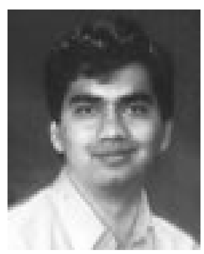
Venugopal V. Veeravalli (S'86–M'92–SM'98) received the Ph.D. degree, in 1992, from the University of Illinois at Urbana-Champaign, the M.S. degree, in 1987, from Carnegie-Mellon University, Pittsburgh, PA, and the B.Tech. degree, in 1985, from the Indian Institute of Technology, Bombay, India (Silver Medal Honors), all in electrical engineering.

He joined the University of Illinois at Urbana-Champaign, in 2000, where he is currently an Associate Professor in the Department of Electrical and Computer Engineering, and a Research Associate Professor in the Coordinated Science Laboratory. During 1996–2000, he was an Assistant Professor at Cornell University, Ithaca, NY. His research interests include mobile and wireless communications, detection and estimation theory, and information theory.

Dr. Veeravalli is currently an Associate Editor for IEEE TRANSACTIONS ON INFORMATION THEORY, and an Editor for *Communications in Information and Systems* (CIS). Among the awards he has received for research and teaching are the IEEE Browder J. Thompson Best Paper Award, in 1996, the National Science Foundation CAREER Award, in 1998, the Presidential Early Career Award for Scientists and Engineers (PECASE), in 1999, and the Michael Tien Excellence in Teaching Award from the College of Engineering, Cornell University, in 1999.



Ashok Mantravadi (S'97) received the B.Tech. degree in electronics and communications engineering from the Indian Institute of Technology, Madras, in 1996, and the M.S. degree from the School of Electrical Engineering, Cornell University, Ithaca, NY, in 1999. Between June and August 1997, he worked as an Engineering Intern with the CDMA system design group at Qualcomm Inc., San Diego, CA. He is currently in the Ph.D. degree program at Cornell University, where he has been a recipient of the John McMullen and Schlumberger Fellowships. Since August 2000, he has also been also a visiting scholar at the University of Illinois, Urbana-Champaign.

His research interests include spread spectrum systems, wireless multiuser communications, and space-time signal processing.