recursive method, nor did we venture to use it for parameter values more extreme than those in Table III.

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Comments on "Decentralized Sequential Detection"

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Abstract—**For the decentralized sequential detection problem studied by Hashemi and Rhodes, it is pointed out that likelihood-ratio tests are not necessarily optimal at the sensors when the decisions made by the sensors are allowed to depend on all their past observations. It is also argued that likelihood-ratio tests are indeed optimal if one restricts the decisions made by each sensor to depend on its present observation and its past decisions.**

Index Terms—**Decentralized sequential detection, likelihood-ratio tests.**

A recent paper[1] studied the following decentralized sequential detection problem. Let there be two hypotheses H_0 and H_1 with known prior probabilities. Each of the local detectors receives independent (in time as well as from detector to detector) observations conditioned on each hypothesis. At any given time (discrete) each of the local detectors sends a binary-valued decision based on *all* its past observations to a supervisor. The supervisor performs a sequential test on the information it receives from the local detectors.

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[1] H. R. Hashemi and I. B. Rhodes, *IEEE Trans. Inform. Theory*, vol. 35, pp. 507–520, May 1989.

The paper[1] analyzed in detail the two-stage version of the previous problem (i.e., one in which the supervisor is forced to stop when each of the local detectors has received two observations) and argued incorrectly that person-by-person optimal (p.b.p.o.) local strategies at both stages are likelihood-ratio tests (LRT's). An example in a paper by Tsitsiklis [1], which predates the Hashemi and Rhodes paper[1], can be used to show that this result is false.

The main objective of this note is to clearly illustrate the erroneous argument made in the paper[1] and to suggest a possible remedy for the solution proposed by the authors. Towards this end, we consider the two-stage problem with two local detectors U and V. Let the hypothesis be denoted by a random variable H that takes on values H_0 and H_1 with the respective prior probabilities. Let $\{X_1, X_2\}$ and $\{Y_1, Y_2\}$ be the observations at detectors U and V, respectively. The local decisions at time 1 and 2 are respectively u_1 and u_2 (v_1 and v_2) at detector U (V). By assumption it follows that

$$u_1 = \phi_1(X_1), \qquad u_2 = \phi_2(X_1, X_2),$$

$$v_1 = \psi_1(Y_1), \qquad v_2 = \psi_2(Y_1, Y_2),$$

where ϕ_1 and ψ_1 are functions mapping R to $\{0, 1\}$ and ϕ_2 and ψ_2 are functions mapping R^2 to $\{0, 1\}$. The supervisor performs a sequential test γ on (u_1, v_1) and (u_2, v_2), and the cost associated with the test is J_γ which is a function of u_1, u_2, v_1, v_2, and H. Also, denote $(\phi_1(X_1), \phi_2(X_1, X_2))$ by $\phi(X^2)$, $(\psi_1(Y_1), \psi_2(Y_1, Y_2))$ by $\psi(Y^2)$, and the triplet (ϕ, ψ, γ) by Γ. The quantity of interest is then given by

$$\min_{\Gamma} E_{X^2, Y^2, H} J_\gamma(\phi(X^2), \psi(Y^2), H).$$

The claim made in the paper[1] is that p.b.p.o. strategies ϕ_1, ϕ_2, ψ_1, and ψ_2 are LRT's. The proof given in the paper[1] for the strategies at time 2 is correct, and indeed p.b.p.o. ϕ_2 and ψ_2 are LRT's with thresholds being functions of u_1 and v_1, respectively. But p.b.p.o. strategies at time 1 are not necessarily LRT's as the following analysis shows.

Fix ϕ_2, ψ, and γ, possibly at the optimum, and consider EJ_γ as a function of ϕ_1. Now, by the conditional independence of Y^2 and X^2 given H, it follows that

$$\begin{aligned} J_{\phi_1} &= E_{X^2, Y^2, H} J_\gamma(\phi(X^2), \psi(Y^2), H) \\ &= E_{X^2, H} E_{v^2 \mid H} J_\gamma(\phi_1(X_1), \phi_2(X_1, X_2), v^2, H) \\ &= E_{X^2, H} \Sigma_2(\phi_1(X_1), \phi_2(X_1, X_2), H), \end{aligned}$$

where the definition of $\Sigma_2(\cdot,\cdot,\cdot)$ is obvious. Similarly, by the conditional independence of X_1 and X_2 given H, it is clear that

$$\begin{aligned} J_{\phi_1} &= E_{X_1} E_{H \mid X_1} E_{X_2 \mid H} \Sigma_2(\phi_1(X_1), \phi_2(X_1, X_2), H) \\ &= E_{X_1} E_{H \mid X_1} \Sigma_1(\phi_1(X_1), X_1, H), \end{aligned}$$

where the definition of $\Sigma_1(\cdot,\cdot,\cdot)$ is obvious from above. We note that Σ_1 depends explicitly on X_1 that contradicts the analysis in the paper[1]. Hence, the test ϕ_1 which minimizes J_{ϕ_1} is not necessarily an LRT.

For the n-stage problem ($n \geq 2$), it is easily seen that p.b.p.o. local strategies at time n alone are LRT's in general. Also, if we restrict the local decisions made by each sensor to depend on its present observation and all its past decisions, then it can easily be shown that p.b.p.o. local strategies are all LRT's with thresholds depending on the past decisions. In this setting, the results claimed in the paper[1] are provable.

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Correction to "Optical Orthogonal Codes: Design, Analysis, and Applications"

Fan R. K. Chung, Jawad A. Salehi, and Victor K. Wei

In the above paper[1] on p. 599, Theorem 2 holds when n is an odd prime. This condition was inadvertently omitted. We do not discuss other cases of n.

Also, the ordering of the first two authors was transposed on the TRANSACTIONS cover.

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