On Performance Analysis for Signaling on Correlated Fading Channels

Venugopal V. Veeravalli

***Abstract*—A general approach is presented for analyzing the performance of digital signaling with multichannel reception on correlated fading channels. The approach is based on: i) exploiting the complex Gaussian model for the joint distribution of the fading on the multiple channels; and ii) applying recent results on the unified performance analysis of digital signaling on fading channels using alternative representations of the $Q(\cdot)$ and related functions. Numerical results that illustrate the effect of correlation on the diversity gain from multichannel reception are also presented.**

***Index Terms*—Correlation, diversity methods, fading channels, multichannel reception.**

I. INTRODUCTION

IN THE performance analysis of diversity schemes for multipath fading channels, it is typically assumed that the fading is independent across the multiple diversity channels. While the independence assumption is usually valid, there are several situations where the fading is correlated among the channels. For example, in multiple antenna systems, physical constraints may not allow the use of antenna spacing that is required for independent fading across antennas. In time diversity schemes, e.g., coding with interleaving, if the interleaver is not long enough, the code symbols undergo correlated fading.

It is well known that correlation in fading across multiple diversity channels results in a degradation of the diversity gain obtained. Initial work on analysis of diversity with correlated fading channels was done by Pierce and Stein in [1], where they studied the special case of Rayleigh fading. Expressions for the error probability for making hard decisions were derived for the two special cases of binary phase shift keying (BPSK) with maximal-ratio combining, and binary frequency shift keying (BFSK) with equal-gain combining. Extending the analysis of [1] to correlated Rician fading is in general difficult. In some special cases, such as BFSK with equal-gain combining, we may use the Laplace transform results of [2] to obtain closed-form expression for hard-decision error probabilities. The results in [2] can also be used to obtain Chernoff *bounds* on error probabilities for coding with imperfect interleaving on Rician fading channels [3], [4] (also see related work in [5]).

Other work on extending the work in [1] has focused on the Nakagami-m fading model for analysis (see, e.g., [6]–[8]). The approach taken in much of this work is to develop a model for the joint distribution of the envelopes at the various channels and to use this model for analysis. While the Nakagami-m model is well justified for describing the first order statistics of the fading envelopes, the joint distribution (correlation) model is not easily justified and is generally quite complicated.

In this letter, we present a general approach to computing the performance for digital signaling on complex Gaussian fading channels with correlated diversity. Our approach makes use of alternative representations of the $Q(\,.\,)$ and related functions function as integrals with finite limits, and with the argument of the function contained in the integrand. These alternative representations have been used recently in several papers to compute error rates for digital modulation on fading channels (see, e.g., [7]–[11] and[12]). In particular, this approach has been used to unify performance results for a wide variety of digital signaling schemes with possible multichannel reception in [10], [11]. We establish that these unified performance analyzes can easily be extended to include correlated diversity for a general complex Gaussian fading channel model. As in these papers, the exact expressions are in the form of a single finite-range integral, with an integrand that is a ratio of polynomials of trigonometric functions, which can hence be easily evaluated numerically. In some special cases, the integral simplifies to a closed form.

II. BPSK WITH MAXIMAL RATIO COMBINING

Let the instantaneous bit signal-to-noise ratio (SNR) on the ℓth channel, $\ell = 1, \ldots, L$, be denoted by γ_ℓ. Then for a general complex Gaussian fading channel we can write

$$\gamma_\ell = |Y_\ell|^2 = Y_{\ell,I}^2 + Y_{\ell,Q}^2 \quad (1)$$

where $Y_\ell = Y_{\ell,I} + jY_{\ell,Q}$ is a nonzero mean proper complex Gaussian (PCG) random variable and $\boldsymbol{Y} = [Y_1\ Y_2\ \cdots\ Y_L]^\top$ is PCG random vector. (See [13] for a summary of the properties of proper complex vectors and processes.) Since $\boldsymbol{Y}$ is a PCG vector, its joint pdf can be written compactly as

$$\begin{aligned} p_{\boldsymbol{Y}}(\boldsymbol{y}) &:= p_{\boldsymbol{Y}_I \boldsymbol{Y}_Q}(\boldsymbol{y}_I, \boldsymbol{y}_Q) \\ &= \frac{1}{\pi^L \det(\boldsymbol{\Sigma})} \exp[-(\boldsymbol{y}-\boldsymbol{m})^\dagger \boldsymbol{\Sigma}^{-1}(\boldsymbol{y}-\boldsymbol{m})] \quad (2) \end{aligned}$$

where $\dagger$ denotes the Hermitian operation, $\boldsymbol{m} = \mathrm{E}[\boldsymbol{Y}]$, and

$$\boldsymbol{\Sigma} = \mathrm{E}[(\boldsymbol{Y}-\boldsymbol{m})(\boldsymbol{Y}-\boldsymbol{m})^\dagger].$$

Note that the complex covariance matrix $\boldsymbol{\Sigma}$ equals $2\boldsymbol{\Sigma}_I + j2\boldsymbol{\Sigma}_{QI}$, where

$$\boldsymbol{\Sigma}_I = \boldsymbol{\Sigma}_Q = \mathrm{E}[(\boldsymbol{Y}_I - \boldsymbol{m}_I)(\boldsymbol{Y}_I - \boldsymbol{m}_I)^\top]$$

$$\boldsymbol{\Sigma}_{QI} = -\boldsymbol{\Sigma}_{IQ} = \mathrm{E}[(\boldsymbol{Y}_I - \boldsymbol{m}_I)(\boldsymbol{Y}_Q - \boldsymbol{m}_Q)^\top].$$

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The author is with the Department of Electrical and Computer Engineering, and the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana, IL 61801 USA (e-mail: vvv@uiuc.edu).

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The envelope on the ℓth channel $|Y_\ell|$ has a Rician distribution [14] with Rice factor κ_ℓ that depends on the mean m_ℓ.

For BPSK with maximal ratio combining (MRC), it is well known [14] that the average bit error rate (BER) is given by

$$\overline{\mathrm{P}}_{\mathrm{b}} = \int_{\boldsymbol{x}} Q\left(\sqrt{2\sum_{\ell=1}^{L} x_\ell}\right) p_{\boldsymbol{\gamma}}(\boldsymbol{x})\, d\boldsymbol{x} \tag{3}$$

where $p_{\boldsymbol{\gamma}}(\boldsymbol{x})$ is the joint pdf of the SNR vector $\boldsymbol{\gamma} = [\gamma_1\ \gamma_2\ \cdots\ \gamma_L]$.

To compute the integral in (3), the approach taken in previous papers has been to use the pdf for the sum $\gamma_t = \sum_\ell \gamma_\ell$ (see, e.g., [1]). For the Nakagami-m fading model, the pdf of γ_t is obtained by first modeling $p_{\boldsymbol{\gamma}}(\cdot)$ in a complicated and somewhat *ad hoc* manner (see, e.g., [6], [7]).

Our approach is to rewrite the integral in (3) in terms of the complex envelope vector $\boldsymbol{Y}$, whose joint distribution is completely characterized by $\boldsymbol{m}$ and $\boldsymbol{\Sigma}$ as in (2). This allows us to arrive at performance results for a general complex Gaussian probability model for the fading on the L channels. Specifically,

$$\overline{\mathrm{P}}_{\mathrm{b}} = \int_{\boldsymbol{y}} Q\left(\sqrt{2\boldsymbol{y}^\dagger \boldsymbol{y}}\right) p_{\boldsymbol{Y}}(\boldsymbol{y})\, d\boldsymbol{y}. \tag{4}$$

We now use the alternative expression for the $Q(\cdot)$ function described in [9]

$$Q(x) = \frac{1}{\pi}\int_0^{\pi/2} \exp\left[-\frac{x^2}{2\sin^2\theta}\right] d\theta \tag{5}$$

to rewrite (4) as

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2}\int_{\boldsymbol{y}} \exp\left[-\frac{\boldsymbol{y}^\dagger\boldsymbol{y}}{\sin^2\theta}\right]\frac{1}{\pi^L \det(\boldsymbol{\Sigma})} \cdot \exp[-(\boldsymbol{y}-\boldsymbol{m})^\dagger \boldsymbol{\Sigma}^{-1}(\boldsymbol{y}-\boldsymbol{m})]\, d\boldsymbol{y}\, d\theta.$$

Completing squares inside the exponential, exploiting the fact that $\boldsymbol{\Sigma}^\dagger = \boldsymbol{\Sigma}$, and using fact that a PCG joint pdf integrates to 1, we get

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2} \frac{1}{\det(\boldsymbol{\Sigma}_1\boldsymbol{\Sigma})} \cdot \exp[-\boldsymbol{m}^\dagger(\boldsymbol{\Sigma}^{-1} - \boldsymbol{\Sigma}^{-1}\boldsymbol{\Sigma}_1^{-1}\boldsymbol{\Sigma}^{-1})\boldsymbol{m}]\, d\theta$$

where

$$\boldsymbol{\Sigma}_1 = \frac{\boldsymbol{I}}{\sin^2\theta} + \boldsymbol{\Sigma}^{-1}.$$

Now, a straightforward application of the matrix inversion lemma [15, pg. 19] yields

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2}\left[\det\left(\frac{\boldsymbol{\Sigma}}{\sin^2\theta} + \boldsymbol{I}\right)\right]^{-1} \cdot \exp[-\boldsymbol{m}^\dagger(\boldsymbol{\Sigma} + \sin^2\theta \boldsymbol{I})^{-1}\boldsymbol{m}]\, d\theta. \tag{6}$$

No further simplification is possible in the general case when $\boldsymbol{m} \neq 0$. However, since (6) is a single finite-range integral, with an integrand that is a ratio of polynomials of trigonometric functions, it can easily be evaluated numerically for any given $\boldsymbol{m}$ and $\boldsymbol{\Sigma}$.

A. Special Case: Rayleigh Fading

In the special case where $\boldsymbol{m} = \boldsymbol{0}$, i.e., the envelopes $|Y_\ell|$ are Rayleigh distributed, (6) simplifies to

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{\pi}\int_0^{\pi/2}\left[\det\left(\frac{\boldsymbol{\Sigma}}{\sin^2\theta} + \boldsymbol{I}\right)\right]^{-1} d\theta. \tag{7}$$

Now suppose $\boldsymbol{\Sigma}$ has distinct eigenvalues $\lambda_1, \ldots, \lambda_L$, then

$$\left[\det\left(\frac{\boldsymbol{\Sigma}}{\sin^2\theta} + \boldsymbol{I}\right)\right]^{-1} = \prod_{\ell=1}^{L}\left(\frac{\lambda_\ell}{\sin^2\theta} + 1\right)^{-1} = \sum_{\ell=1}^{L}\rho_\ell\left(\frac{\lambda_\ell}{\sin^2\theta} + 1\right)^{-1} \tag{8}$$

where ρ_ℓ is the ℓth residue in the partial-fraction expansion, i.e.,

$$\rho_\ell = \prod_{i\neq\ell}\left(1 - \frac{\lambda_i}{\lambda_\ell}\right)^{-1}.$$

Thus (7) can be written as

$$\overline{\mathrm{P}}_{\mathrm{b}} = \sum_{\ell=1}^{L}\rho_\ell \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\lambda_\ell}{\sin^2\theta} + 1\right)^{-1} d\theta. \tag{9}$$

To evaluate the integral in (9), we use the following result from [11, Appendix B]:

$$I_n(c) = \frac{1}{\pi}\int_0^{\pi/2}\left(\frac{\sin^2\theta}{\sin^2\theta + c}\right)^n d\theta = [P(c)]^n \sum_{k=0}^{n-1}\binom{n-1+k}{k}[1-P(c)]^k \tag{10}$$

with $P(c) = (1/2)[1 - \sqrt{c/(1+c)}]$.

If we use (10) with $n = 1$ in (9), we get the expression for the BER given in [1, eq. (52)], i.e.,

$$\overline{\mathrm{P}}_{\mathrm{b}} = \frac{1}{2}\sum_{\ell=1}^{L}\rho_\ell\left[1 - \sqrt{\frac{\lambda_\ell}{1+\lambda_\ell}}\right]. \tag{11}$$

If $\boldsymbol{\Sigma}$ has repeated eigenvalues, then the partial fraction expansion in (8) involves terms of the form

$$\left(\frac{\lambda_\ell}{\sin^2\theta} + 1\right)^{-n}$$

for $n > 1$, and the BER can still be obtained in closed form using (10).

III. Extensions to More General Modulation Schemes

The results obtained for BPSK with MRC in the previous sections can easily be extended to a general class of linear modulation schemes with coherent reception and MRC. This class includes M-ary phase shift keying (M-PSK), M-ary amplitude modulation (M-AM) and M-ary quadrature amplitude modulation (M-QAM). In all these cases, recent work by Alouini *et al.* (see, e.g., [7], [11]), has shown that the probability of symbol error $\mathrm{P_s}$, conditioned on $\boldsymbol{\gamma}$, can be written as a linear combination of terms of the form:

$$F(A, \theta_1, \theta_2, g) = A \int_{\theta_1}^{\theta_2} \exp\left[-\frac{g\gamma_t}{\sin^2\theta}\right] d\theta$$

where $\gamma_t = \sum_{\ell=1}^{L} \gamma_\ell$. For example, for M-PSK, there is only one term with $A = 1/\pi$, $\theta_0 = 0$, $\theta_1 = (M-1)\pi/M$, and $g = \sin^2(\pi/M)$ (see [7, eq. (31)]). Applying the approach used in deriving (4), it is easy to see that $\overline{\mathrm{P}}_\mathrm{b}$ can be written as a linear combination of terms of the form

$$\overline{F}_{A,\theta_1,\theta_2,g} = A \int_{\theta_1}^{\theta_2} \int_{\boldsymbol{y}} \exp\left[-\frac{g\boldsymbol{y}^\dagger \boldsymbol{y}}{\sin^2\theta}\right] \frac{1}{\pi^L \det(\boldsymbol{\Sigma})} \cdot \exp[(\boldsymbol{y}-\boldsymbol{m})^\dagger \boldsymbol{\Sigma}^{-1}(\boldsymbol{y}-\boldsymbol{m})]\, d\boldsymbol{y}\, d\theta.$$

The above integral can be written in closed form in some special cases when the fading is Rayleigh, but in general it can be reduced to a single finite-range integral, with an integrand that is a ratio of polynomials of trigonometric functions.

Extensions to differentially encoded and nonlinear modulation, with differentially coherent and noncoherent multichannel reception (respectively), is also possible. In this case we can usually assume that postdetection equal gain combining (EGC) is performed. Then, based on work by Simon and Alouini [10], it can be shown for a wide class of problems that the bit error probability (conditioned on $\boldsymbol{\gamma}$) can be written in the form

$$\mathrm{P_b}(L, \gamma_t; a, b, \eta) = \frac{\eta^L}{2\pi(1+\eta)^{2L-1}} \int_{-\pi}^{\pi} \frac{f(L;\beta,\eta;\theta)}{1+2\beta\sin\theta+\beta^2} \cdot \exp\left[-\frac{b^2\gamma_t}{2}(1+2\beta\sin\theta+\beta^2)\right] d\theta$$

where a, b, η, and the function f are as defined in [10, eq. (62)], with $\beta = a/b \in (0, 1)$. Here again it is clear that the steps used in simplifying (4) can be applied to obtain an easily computable expression for $\overline{P}_b$.

In the special case of binary FSK with noncoherent reception and EGC, we can use the expression for the BER, conditioned on $\boldsymbol{\gamma}$, given in [1, eq. (55a)], to get a closed-form expression for $\overline{P}_b$ in the general case of Rician fading. (Recall from Section II that $\overline{\mathrm{P}}_\mathrm{b}$ cannot be obtained in closed-form for BPSK with MRC when the fading is Rician.)

IV. Example and Numerical Results

In this section, we numerically evaluate the results of Section II for a BPSK signal subject to correlated fading. While there are several correlated diversity scenarios of interest (the diversity could be across space, time, frequency or a combination thereof), they translate simply into an appropriate choice of the mean vector $\boldsymbol{m}$ and the covariance matrix $\boldsymbol{\Sigma}$ for $\boldsymbol{Y}$ in our model.

For modeling $\boldsymbol{m}$, in the context of correlated fading, it is reasonable to assume that the Rice factor is the same for all L diversity paths:[1] $\kappa_\ell = \kappa$, $\ell = 1, \ldots, L$, and the ℓth component of $\boldsymbol{m}$ is

$$m_\ell = \sqrt{\frac{\kappa\overline{\gamma}_\ell}{\kappa+1}} \tag{12}$$

where $\overline{\gamma}_\ell = \mathrm{E}[\gamma_\ell]$ is the average SNR on the ℓth channel. Note that m_ℓ may in general be complex, but the marginal distribution of γ_ℓ is unaffected by the phase of m_ℓ.

Next, we consider modeling $\boldsymbol{\Sigma}$. When the diffuse component of the fading process is isotropic, the correlation across space (or time) follows a Bessel function [16]. (For illustration, we focus on spatial diversity in the remainder of the letter.) Specifically, the in-phase and quadrature components are uncorrelated and the elements of the resulting real covariance matrix are given by

$$\boldsymbol{\Sigma}(k, \ell) = 2\boldsymbol{\Sigma}_I(k, \ell) = \frac{\sqrt{\overline{\gamma}_k\overline{\gamma}_\ell}}{(\kappa+1)} J_0\left(2\pi\frac{d_{k\ell}}{\lambda_c}\right) \tag{13}$$

where

- $d_{k\ell}$ distance between the kth and ℓth antennas;
- λ_c carrier wavelength;
- $J_0(\cdot)$ zeroth-order Bessel function of the first kind.

We also consider the exponential model suggested in [1]. This model could be representative of the case where the diffuse component of the fading is not isotropic, and the diversity has an underlying equispaced nature. In this case, the elements of $\boldsymbol{\Sigma}$ are given by

$$\boldsymbol{\Sigma}(k, \ell) = \begin{cases} \dfrac{\sqrt{\overline{\gamma}_k\overline{\gamma}_\ell}}{(\kappa+1)} e^{-(\alpha+j\beta)|k-\ell|d}, & k \le \ell \\ \dfrac{\sqrt{\overline{\gamma}_k\overline{\gamma}_\ell}}{(\kappa+1)} e^{-(\alpha-j\beta)|k-\ell|d}, & k > \ell \end{cases} \tag{14}$$

where α and β are (measured) parameters, and d describes the equi-spacing in the model. Note that it is possible to obtain the eigenvalues of $\boldsymbol{\Sigma}$ in closed form with this model [1].

The results with these models are shown in the following figures. In all cases, it is assumed that the average bit SNR on each of the L paths is the same: $\overline{\gamma}_\ell = \overline{\gamma}$, $\ell = 1, \ldots, L$.

First, we consider Rayleigh fading with the Bessel correlation model and $L = 3$. The antennas are assumed to be spaced equally,[2] so that $d_{k\ell} = |d_k - d_\ell| = |k-\ell|d$. A carrier frequency of 900 MHz is assumed, and the corresponding $\lambda_c = 1/3$ m. Fig. 1 shows the variation of $\overline{\mathrm{P}}_\mathrm{b}$ with average bit SNR per path

[1]For receive antenna diversity in particular, correlated fading arises when the antennas are not sufficiently far apart, and hence, all the antennas can be assumed see a specular component of approximately the same strength.

[2]In (13), the first zero of $J_0(2\pi(d_{k\ell}/\lambda_c))$ occurs at $d_{k\ell} \approx 0.383\lambda_c$. If the antenna spacing is $<0.383\lambda_c$, the fading is correlated.

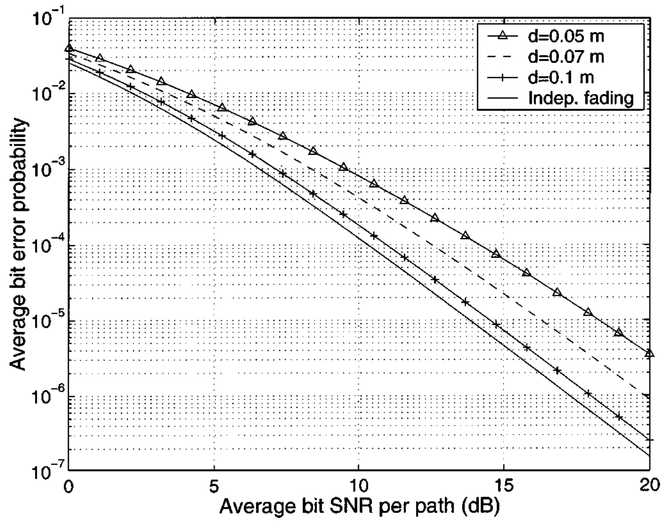

Fig. 1. Rayleigh fading with Bessel correlation model.

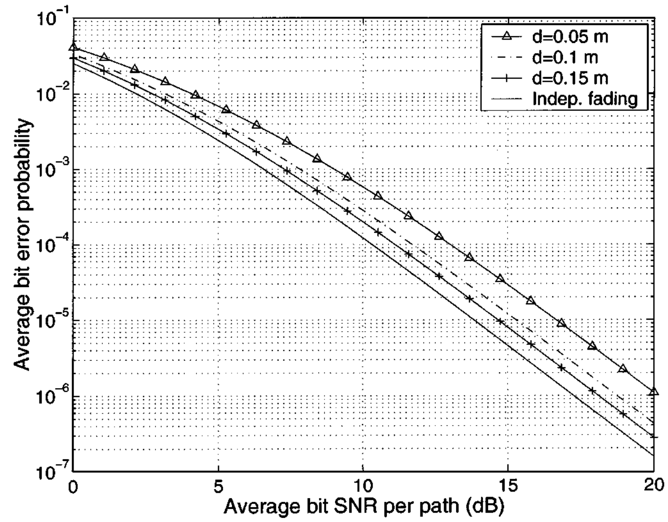

Fig. 3. Rayleigh fading with exponential correlation model.

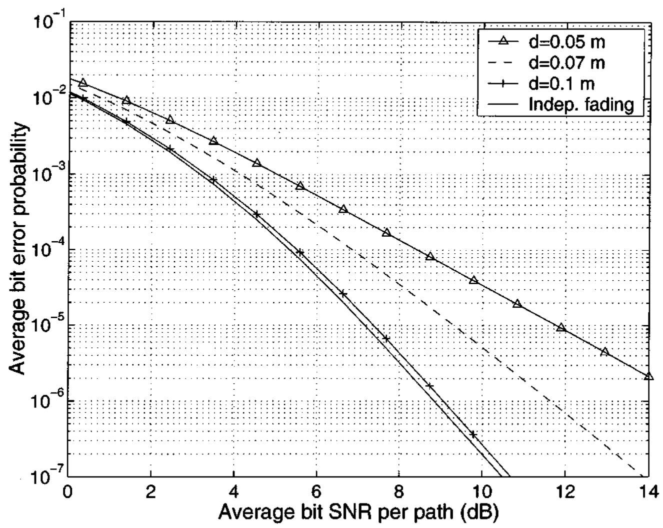

Fig. 2. Ricean Fading with Bessel correlation model, $\kappa = 6$.

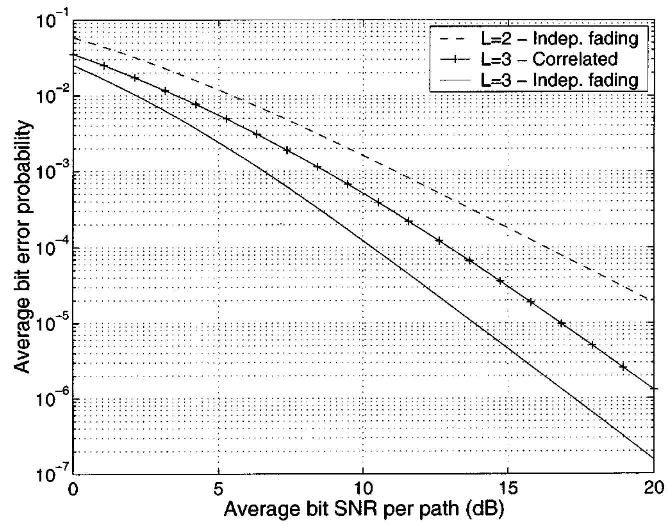

Fig. 4. Comparison of cases with $L = 2$ and $L = 3$.

$\overline{\gamma}$ for different values of the spacing parameter d. Clearly, as d increases, we have greater diversity gain and the curves approach that corresponding to independent fading. It is worth noting that correlation can result in as much as 4 dB loss at a BER of 10^{-4}. The same set of curves is shown in Fig. 2 for Ricean fading with $\kappa = 6$. Along with an expected improvement in overall performance, the loss due to the correlation is reduced due to the presence of the specular component: the loss in SNR at a BER of 10^{-4} reduces to about 1 dB.

Next, we consider the exponential correlation model with Rayleigh fading. To allow comparison with the Bessel model, the parameter β is set to π/λ_c, and α is taken to be 1.5. The results shown in Fig. 3 are similar to those in Fig. 1. Clearly, we may expect a similar comparison to hold with Ricean fading as well.

Finally, we consider a more direct application of the analysis in the letter. Assume we have two antennas spaced to yield independent diversity under the Bessel correlation model (so $d_{12} = 0.383\lambda_c$). The performance of this system is shown in Fig. 4. Now, if we place a third antenna between the two antennas (thereby introducing correlation in the fading), the maximum improvement in performance (obtained with $d_{13} = d_{23} = 0.383\lambda_c/2$) is as shown in Fig. 4. Clearly, the performance is worse than that obtained with $L = 3$ and independent fading, but on the other hand, a performance improvement with respect to $L = 2$ is obtained without an increase in the overall dimension of the array. The system designer could tradeoff this improvement with the cost of adding the third antenna.

V. Conclusion

Recent work (primarily by Simon and Alouini) has shown that it is possible to obtain unified performance results for a wide variety of digital signaling schemes on fading channels. We have established in this letter that this unified performance analysis can be extended to include multichannel reception with correlated diversity for a general complex Gaussian fading channel model. We have also presented examples that illustrate the usefulness of our results in the context of antenna diversity.

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