On Chip-Matched Filtering and Discrete Sufficient Statistics for Asynchronous Band-Limited CDMA Systems

Ashok Mantravadi, *Student Member, IEEE*, and Venugopal V. Veeravalli, *Senior Member, IEEE*

***Abstract*—The problem of generating discrete sufficient statistics for signal processing in code-division multiple-access (CDMA) systems is considered in the context of underlying channel bandwidth restrictions. Discretization schemes are identified for (approximately) band-limited CDMA systems, and a notion of approximate sufficiency is introduced. The role of chip-matched filtering in generating accurate discrete statistics is explored. The impact of approximate sufficiency on performance is studied in three cases: conventional matched filter (MF) detection, minimum mean-squared-error detection, and delay acquisition. It is shown that for waveforms limited to a chip interval, sampling the chip-MF output at the chip rate can lead to a significant degradation in performance. Then, with equal bandwidth and equal rate constraints, the performance with different chip waveforms is compared. In all three cases above, it is demonstrated that multichip waveforms that approximate Nyquist sinc pulses achieve the best performance, with the commonly used rectangular chip pulse being severely inferior. However, the results also indicate that it is possible to approach the best performance with well-designed chip waveforms limited to a chip interval, as long as the chip-MF output is sampled above the Nyquist rate.**

***Index Terms*—Band-limited signals, chip-matched filtering, chip waveform design, code-division multiple access, delay estimation, discrete sufficient statistics, signal detection.**

I. INTRODUCTION

THE continuous-time system model that describes a typical code-division multiple-access (CDMA) system is one where the sum of the transmitted signal waveforms of the users goes through a (possibly time-varying) *band-limited* (BL) channel with additive background noise. It is convenient both for analysis as well as implementation to convert the continuous-time CDMA model into an equivalent discrete model that produces sufficient statistics for decision making at the receiver.

In standard analyses of narrow-band *single-user* systems, the generation of discrete statistics for detection/estimation at the receiver is facilitated by making simplifying assumptions in the BL continuous-time system model [1]. Specifically, the background noise may be idealized by additive white Gaussian noise (AWGN) with infinite bandwidth. Furthermore, in many cases, the performance metric of interest (e.g., bit-error probability) may be independent of the actual spectral shape of the signal and depend merely on its total energy (or the operating SNR). In such cases, it is convenient to use pulses limited to a symbol period to simplify the analysis and exposition of detection operations at the receiver.

Some of the work on CDMA systems, especially that involving joint processing of the received signal (see, e.g., [2]–[4]) has made the same simplifying assumptions, and chip waveforms limited to a chip period (such as the rectangular pulse) are used.[1] We refer to such waveforms as chip-limited (CL) waveforms, as opposed to those that occupy more than one chip, which we refer to as multichip (MC) waveforms. Along with the use of (rectangular) CL waveforms, further simplification is achieved by using a chip-matched filter (chip-MF) to generate discrete statistics at the receiver front-end (see, e.g., [7]–[10]), albeit without rigorous justification in the asynchronous case. Finally, the chip-MF output is often sampled at the chip rate to facilitate analysis.

In this paper, we take a more fundamental approach and consider the generation of sufficient statistics for detection and estimation in band-limited CDMA systems, with particular emphasis on chip-matched filtering. We assume the standard AWGN model for the noise. However, throughout the paper, we assume that the signal has an approximate (or essential) bandwidth of W, where the essential bandwidth is defined in the mean-square sense, and the spillover outside the bandwidth is restricted to be sufficiently small. For each chip waveform that we study, we assume that the chip period is chosen large enough to meet the bandwidth constraint.

We first consider the problem of generating discrete sufficient statistics from the continuous-time received signal observed over the finite time interval $[-T/2, T/2]$. In the special case where the delays of the users are known, it is possible to generate a *finite* set of sufficient statistics (for bit detection, say) by correlating the received signal with the users' signaling waveforms. In general, the number of statistics required for sufficiency is *countably infinite*; these statistics can

[1]There are a few exceptions, however (see, e.g., [5] and [6]). In [5], the performance of single-user detection with square-root raised cosine (SRRC) waveforms is analyzed, and in [6], a joint acquisition scheme with approximately BL multichip waveforms is considered.

Paper approved by U. Madhow, the Editor for Spread Spectrum of the IEEE Communications Society. Manuscript received January 14, 1999; revised January 29, 2000, September 6, 2000, and January 10, 2001. This work was supported by the National Science Foundation under Grant NCR-9523967 and the CAREER/PECASE Award CCR-9733204. This paper was presented in part at the Conference on Information Sciences and Systems (CISS), Baltimore, MD, March 1999, and the Allerton Conference on Communication, Control, and Computing, Monticello, IL, October 1999.

A. Mantravadi is with with the School of Electrical Engineering, Cornell University, Ithaca, NY 14853 USA (e-mail: ashok@ee.cornell.edu).

V. V. Veeravalli is with the Department of Electrical and Computer Engineering and the Coordinated Science Laboratory, University of Illinois at Urbana-Champaign, Urbana IL 61801 USA (e-mail: vvv@uiuc.edu).

Publisher Item Identifier S 0090-6778(01)06942-2.

be generated by projecting the received signal on to the set of Prolate spheroidal wave functions (PSWFs) corresponding to time interval T and bandwidth W [11]. Any finite set of discrete statistics would in general result in a "loss" of sufficiency. We hence introduce a notion of approximate sufficiency, based on signal energy captured by the statistics, to quantify this loss and use it to study chip-matched filtering. We show that sampling the chip-MF at the Nyquist rate produces $2WT$ statistics that are approximately sufficient, with sufficiency loss of the same order as that produced by projection on to a subset of the PSWFs of size $2WT$.

To illustrate the impact of approximate sufficient statistics on system performance, we consider three specific operations at the receiver: matched-filter (MF) detection, linear minimum mean-squared-error (MMSE) multiuser detection, and single-user timing estimation. The performance metric used is the output signal-to-interference ratio (SIR) for detection, and the probability of acquisition error for timing estimation. In general, we have the option of producing a finite set of discrete *sufficient* statistics via correlation, or producing a finite set of approximately sufficient statistics via chip-matched filtering followed by Nyquist sampling. We show that both approaches result in nearly the same performance, thus justifying our claim of approximate sufficiency of chip-matched filtering followed by Nyquist sampling. We also show that if systems with CL waveforms are sampled at the chip rate, there can be a substantial degradation in performance.

Using (approximately) sufficient statistics, we then study the effect of the chip waveform on performance in CDMA systems, specifically for the three receiver operations listed above. Now, for a single user in AWGN, given a fixed bandwidth W, the maximum rate is obtained by using the Nyquist sinc waveform. More commonly, however, some excess bandwidth (and loss in rate) is allowed for and SRRC pulses are used that have a symbol duration $(1+\beta)/2W$, where β is the roll-off factor [12]. On the other hand, for CDMA systems, the processing gain N gives us an additional degree of freedom, so that we have the choice of several chip waveforms without incurring a loss in symbol rate. For example, we could use chip waveforms of any excess bandwidth β and maintain the same symbol rate by keeping $N(1+\beta)$ constant. In addition, the dependence of the performance on the chip waveform is more complicated than in the single-user case. Hence, the problem of optimal chip-waveform selection does not seem to be straightforward. Instead, we consider three candidate chip waveforms in this paper: 1) the CL rectangular pulse that is commonly chosen in the literature to simplify exposition and analysis; 2) an MC waveform that is a truncated version of the (Nyquist) sinc pulse with bandwidth W, which results in the largest processing gain under fixed bandwidth and rate constraints; and 3) the CL time-domain raised cosine pulse (TDRC), which has been identified to have nearly optimal spectral rolloff over all CL waveforms in [13]. For the three receiver operations of MF detection, MMSE detection, and delay acquisition, we show that the sinc MC waveform achieves the best performance, with the commonly used rectangular pulse being much inferior.

The remainder of this paper is organized as follows. In Section II, we discuss sufficient statistics in general and introduce the notion of approximate sufficiency. The CDMA system model under consideration is discussed in Section III. In Section IV, we study the generation of discrete statistics for CDMA and focus on the chip-matched filtering approach. The importance of this understanding is illustrated in Section V through performance studies of single-user and linear MMSE multiuser detection. Single-user acquisition is considered in Section VI. Conclusions are given in Section VII.

II. Preliminaries

Consider the standard problem of parameter estimation involving a continuous-time signal in additive noise

$$Y(t) = s_\theta(t) + n(t), \qquad t \in \left[\frac{-T}{2}, \frac{T}{2}\right] \tag{1}$$

where $\theta \in \Theta$ is the (vector) parameter to be estimated and T is the (finite) observation interval. We assume that the noise is ideal white Gaussian with zero mean and a two-sided power spectral density (PSD) of $N_0/2$.

The likelihood function for the continuous-time function $Y(t)$ is given by the Cameron–Martin formula [14, Ch. VI]:

$$L_\theta(Y) = \frac{2}{N_0}\langle s_\theta(t), Y(t)\rangle_T - \frac{1}{N_0}\langle s_\theta(t), s_\theta(t)\rangle_T \tag{2}$$

where

$$\langle f(t), g(t)\rangle_T = \int_{-T/2}^{T/2} f(u)g(u)du.$$

This is of course the basis of the matched-filtering operation for AWGN channels. If

$$s_\theta(t) = \sum_{k=1}^{\infty} s_{\theta,k} g_k(t) \qquad \forall\, \theta \tag{3}$$

for some countable set of functions $\{g_k(t)\}$, then $\langle s_\theta(t), Y(t)\rangle_T = \sum_{k=1}^{\infty} s_{\theta,k} Y_k$, where $Y_k = \langle g_k(t), Y(t)\rangle_T$. Hence, $\{Y_k\}_{k=1}^{\infty}$ are sufficient statistics for estimation based on the continuous observation $Y(t)$.

While the statistics derived above are sufficient, they would in general be infinite in number, and we would like to have only a finite number in practical applications. There are two cases where this reduction can be achieved with no loss in sufficiency of the statistics. First, if the number of basis functions required to span the signal $s_\theta(t)\ \forall\, \theta \in \Theta$ is finite, say $\{g_k(t)\}_{k=1}^{D}$, then $s_{\theta,k} = 0$, for $k > D$, and it follows that $\{Y_k\}_{k=1}^{D}$ are sufficient statistics. Alternatively, if $|\Theta| < \infty$, we may generate a finite number of sufficient statistics by computing $\langle s_\theta(t), Y(t)\rangle_T$ for each $\theta \in \Theta$.

In the general case where the dimension of the signal space and the size of parameter set is infinite, we cannot reduce the number of statistics to a finite number without losing sufficiency. In such a scenario, we define the notion of approximate sufficiency based on the loss in signal energy when a finite set of functions is used to represent the signal. Let $\hat{s}_\theta(t) = \sum_{k=1}^{D} \hat{s}_{\theta,k} g_k(t)$ be the projection of $s_\theta(t)$ onto the space spanned by $\{g_k(t)\}_{k=1}^{D}$, and let $||g(t)||_T^2 := \langle g(t), g(t)\rangle_T$.

Definition 1: For deterministic θ, the statistics $\{Y_k\}_{k=1}^D$ are said to be δ-*sufficient* if

$$\sup_{\theta\in\Theta} \frac{||s_\theta(t)-\hat{s}_\theta(t)||^2}{||s_\theta(t)||^2} \le \delta.$$

If θ is considered to be a random parameter and $s_\theta(t)$ is a wide-sense stationary process, we modify this definition as follows.

Definition 2: For random θ, the statistics $\{Y_k\}_{k=1}^D$ are said to be δ-*sufficient* if

$$\frac{\mathrm{E}_\theta|s_\theta(t)-\hat{s}_\theta(t)|^2}{\mathrm{E}_\theta|s_\theta(t)|^2} \le \delta.$$

Unless mentioned otherwise, we consider signals that are time-limited to $[-T/2, T/2]$ and approximately confined to a bandwidth W, with bandwidth defined in the mean-square sense. We define this notion of approximate confinement in a manner similar to Definition 1 .

Definition 3: A signal $g(t)\in\mathcal{L}^2[-T/2,T/2]$ is said to have an ϵ-bandwidth W if

$$\frac{||G(f)||_W^2}{||G(f)||^2} = \frac{\int_{-W}^{W}|G(f)|^2df}{\int_{-\infty}^{\infty}|G(f)|^2df} \ge 1-\epsilon$$

where $G(f)$ is the Fourier transform of $g(t)$. Note that the bandwidth is defined in terms of the spillover ϵ outside $[-W, W]$. This is more convenient for our purposes and is in contrast to the standard definition of essential bandwidth in terms of energy within $[-W, W]$ [12]. The special case of $\epsilon = 0$ corresponds to perfectly band-limited signals,[2] which would require the time extent to be infinite. We denote the class of square-integrable functions that are perfectly time-limited to T, and approximately band-limited to an ϵ-bandwidth of W, by $\mathcal{F}_\epsilon(T, W)$.

III. System Model

We wish to apply the results of the previous section to discretization in a CDMA system. We consider a direct-sequence CDMA model with K users, where the received complex baseband signal over an M symbol observation interval is given by

$$r(t) = \sum_m \sum_{k=1}^{K} \sqrt{\frac{\mathcal{E}_k}{N}} b_{k,m} c_k^{(m)}(t-\tau_k)\mathrm{e}^{j\phi_k} + w(t),$$
$$t\in\left[\frac{-T}{2},\frac{T}{2}\right];\; T = MT_s. \tag{4}$$

The notation used is as follows. The term $b_{k,m}$ is symbol m of user k, T_s is the symbol period, and $c_k^{(m)}(t) = \sum_{j=0}^{N-1} c_{k,j}^{(m)}\ \psi(t+T/2-mT_s-jT_c)$ is the corresponding spreading waveform. The term T_c denotes the chip period, $\psi(t)$ is the chip waveform, and $N = T_s/T_c$ is the processing gain of the system. The $T/2$ term appears merely to ensure that the observation interval is $[-T/2, T/2]$. Also, the chip waveform is normalized to have unit energy: $\int_{-\infty}^{\infty}\psi^2(t)dt = 1$. The terms ϕ_k, τ_k and $\mathcal{E}_k$ are, respectively, the carrier phase offset, delay, and the symbol energy of user k. Finally, $w(t)$ is a zero-mean proper complex Gaussian process with two-sided PSD N_0, i.e., $R_w(\tau) = E[w^*(t)w(t+\tau)] = N_0\delta(\tau)$.

In the following sections, we will be concerned primarily with the spectral properties of the transmitted signal. Hence, without loss of generality, we will assume that we are dealing with a binary phase-shift keying (BPSK) system so that $b_{k,m}$, $c_{k,j}\in\{-1,1\}$. Also, for simplicity, we assume that the phases of all users are equal to 0. The latter assumption would imply that all useful information about the signals and the interference is contained in the real part of $r(t)$. This restriction does not affect any of the results that we present in this paper, and our analysis is modified in a straightforward fashion to take nonzero ϕ_k into account. Consequently, the received signal of interest can be expressed as

$$r(t) = \sum_m \sum_{k=1}^{K} A_k b_{k,m} c_k^{(m)}(t-\tau_k) + n(t) \tag{5}$$

where $A_k = \sqrt{\mathcal{E}_k/N}$ and $n(t) = \mathrm{Re}\{w(t)\}$ is a real Gaussian process with two-sided PSD $\sigma^2 = N_0/2$. We allow the chip waveform to span more than one chip period (i.e., $\psi(t)$ can be MC), but impose the restriction that its duration is small enough that at most $M' = (M+2)$ symbols of any user occur in the observation interval of duration MT_s. It is easy to see that this requirement translates to making the chip waveform duration less than the *symbol* duration T_s.

The above CDMA model can then be converted to a problem involving $(M+2)K = M'K = K_e$ effective users by separating the signals corresponding to each bit of each user occurring in the observation interval. For $k = 1\ldots K$, $i = 1\ldots M'$, and $t\in[-T/2, T/2]$, we define

$$\tilde{c}_{M'(k-1)+i}(t) = c_k^{(i-1)}(t-\tau_k),$$
$$\tilde{A}_{M'(k-1)+i} = A_k \quad \text{and} \quad b_{M'(k-1)+i} = b_{k,i}. \tag{6}$$

We then have

$$r(t) = \sum_{k=1}^{K_e} \tilde{A}_k b_k \tilde{c}_k(t) + n(t), \qquad t\in\left[\frac{-T}{2},\frac{T}{2}\right] \tag{7}$$

and this is clearly of the form (1), with θ representing, in general, the unknown delays, powers, and all the K_e bits.

Now, the bandwidth of the CDMA signal $r(t)$ depends on the random model imposed on the bit and chip sequences. For the BPSK model under consideration, we assume that the sequences are independent, identically distributed (i.i.d.) equally likely ± 1 sequences, and are independent across the users. Consequently, the power spectral density of $r(t)$ is proportional to the squared-magnitude spectrum of the chip waveform

$$S_r(f) \propto |\Psi(f)|^2$$

where $\Psi(f) = \int_{-T/2}^{T/2}\psi(t)\mathrm{e}^{-j2\pi ft}dt$, since $\psi(t)$ has a support in $[-T/2, T/2]$. For a fair comparison between different CDMA

[2] Note that this notion of perfect band-limitedness is in the mean-square sense and is weaker than having $G(f) = 0$ for $|f| > W$.

systems using different (time-limited) chip waveforms, we require that the normalized energy spillover of $S_r(f)$ outside the given bandwidth W be the same $(= \epsilon)$, i.e.,

$$\frac{\int_{-W}^{W} S_r(f)df}{\int_{-\infty}^{\infty} S_r(f)df} = \frac{\int_{-W}^{W} |\Psi(f)|^2 df}{\int_{-\infty}^{\infty} |\Psi(f)|^2 df} = \frac{\|\Psi(f)\|_W^2}{\|\Psi(f)\|^2} = 1 - \epsilon. \tag{8}$$

Hence, we require that the chip waveforms have an ϵ-bandwidth of W, and we generally think of ϵ as being a small number, with typical values being 0.01 or 0.001.

IV. Discretization and Chip-Matched Filtering

As seen in Section II, the generation of discrete statistics from the signal $r(t)$ involves the projection of $r(t)$ onto an appropriate set of functions $\{g_n(t)\}$. In general, we would like the number of statistics to be finite, say D. Then

$$y_n = \int_{-T/2}^{T/2} r(t)g_n(t)dt = \langle r(t), g_n(t)\rangle_T, \qquad n = 1, \ldots, D.$$

It is easy to see from (7) that we can represent the resulting vector $\mathbf{y} = [y_1, y_2, \ldots, y_D]^\top$ by the matrix-vector equation

$$\mathbf{y} = \mathbf{CAb} + \mathbf{n} \tag{9}$$

where $\mathbf{C}$ is a $D \times K_e$ matrix with $c_{ij} = \langle g_i(t), \tilde{c}_j(t)\rangle_T$, $\mathbf{n}$ is Gaussian with $n_i = \langle g_i(t), n(t)\rangle_T$, $\mathbf{A} = \mathrm{diag}(\tilde{A}_1 \ldots \tilde{A}_{K_e})$, and $\mathbf{b} = [b_1 \ldots b_{K_e}]^\top$. Equation (9) is then the desired discrete model for the CDMA system, and each discretization scheme corresponds to a particular choice of the functions $\{g_n(t)\}$. In this section, we identify several methods for this discretization and study the loss in sufficiency where applicable.

A. Known Delays

If we assume that the delays and the spreading sequences of all the users are perfectly known, then signal component on the right-hand side of (7) is of the form (3), with $s_\theta(t) = \sum_{k=1}^{D} s_{\theta,k} g_k(t)\ \forall\ \theta$, where $g_k(t) = \tilde{c}_k(t)$, $D = K_e$, and θ represents, in general, the bits, amplitudes (and phases) of all the users. Thus, we have a finite number of statistics obtained from correlations with the spreading waveforms, i.e., $y_k = \langle r(t), \tilde{c}_k(t)\rangle_T$, $k = 1, \ldots, K_e$, which are sufficient for estimating θ. In particular, they are sufficient for detecting the bits of the users. Hence, correlation is useful for the detection problem. However, since knowledge of the delays and spreading sequences is required for generating $\tilde{c}_k(t)$ and $\{y_k\}$, these statistics cannot in general be used for the acquisition problem.

B. Unknown Delays

When the delays are unknown, we need to project $r(t)$ onto a set of functions that do not involve the delays. With approximately band-limited signals, we would, in general, have an infinite number of sufficient statistics, and the reduction to a finite number using $\{g_n(t)\}_{n=1}^{D}$ may lead to a loss in sufficiency, defined by the energy loss in the signal (see Definitions 1 and 2).

Our first step in understanding this loss is to reduce the question of sufficiency from the signal to just the chip waveform, which is motivated by the following lemma. Define $\mathcal{G}$ to be the space spanned by the functions $\{g_n(t)\}_{n=1}^{D}$, and let $\mathcal{P}_g$ denote the operator that takes a signal $x(t) \in \mathcal{L}^2[-T/2, T/2]$ to its projection in $\mathcal{G}$ denoted by the signal $\hat{x}(t)$, i.e., $\hat{x}(t) = \mathcal{P}_g[x(t)]$. Since we are considering projection in the least-square sense, we have

$$\hat{x}(t) = \sum_{n=1}^{D} \hat{x}_n g_n(t), \qquad \text{with } \hat{\mathbf{x}} = \mathbf{R}_g^{-1}\mathbf{x} \tag{10}$$

where $\mathbf{x}$ and $\hat{\mathbf{x}}$ are $D \times 1$ vectors with $x_n = \langle x(t), g_n(t)\rangle_T$ and $\mathbf{R}_g$ is the correlation matrix of the spanning functions: $\mathbf{R}_g(i,j) = \langle g_i(t), g_j(t)\rangle_T$. The lemma can then be stated as follows.

Lemma 1: Let $s(t) = \sum_{n=-N_c}^{N_c} a_n \psi(t - nT_c - \alpha)$, where a_n are i.i.d. zero-mean random variables and $s(t)$ has a support in $[-T/2, T/2]$. Also, assume that the projection operation is *chip-invariant*, i.e., for fixed $\alpha \in [0, T_c]$, $\mathcal{P}_g[\psi(t - nT_c - \alpha)] = \hat{\psi}_\alpha(t - nT_c), \forall n = -N_c, \ldots, N_c$, where $\hat{\psi}_\alpha(t) = \mathcal{P}_g[\psi(t - \alpha)]$. Then

$$\frac{\mathrm{E}|s(t) - \hat{s}(t)|^2}{\mathrm{E}|s(t)|^2} = \frac{\|\psi(t-\alpha) - \hat{\psi}_\alpha(t)\|^2}{\|\psi(t-\alpha)\|^2}.$$

Proof: We have

$$\begin{aligned}\hat{s}(t) &= \mathcal{P}_g[s(t)] = \mathcal{P}_g\left[\sum_{n=-N_c}^{N_c} a_n \psi(t - nT_c - \alpha)\right] \\ &\overset{\text{(a)}}{=} \sum_{n=-N_c}^{N_c} a_n \mathcal{P}_g[\psi(t - nT_c - \alpha)] \\ &\overset{\text{(b)}}{=} \sum_{n=-N_c}^{N_c} a_n \hat{\psi}_\alpha(t - nT_c)\end{aligned}$$

where (a) follows from the linearity of $\mathcal{P}_g$, and (b) follows from the chip-invariance of $\mathcal{P}_g$. Hence

$$s(t) - \hat{s}(t) = \sum_{n=-N_c}^{N_c} a_n \psi_e(t - nT_c)$$

where $\psi_e(t) = \psi(t-\alpha) - \hat{\psi}_\alpha(t)$. Since a_n are zero-mean i.i.d.,

$$\mathrm{E}|s(t) - \hat{s}(t)|^2 = \left(\sum_n \mathrm{E}a_n^2\right) \|\psi_e(t)\|^2$$

and

$$\mathrm{E}|s(t)|^2 = \left(\sum_n \mathrm{E}a_n^2\right) \|\psi(t-\alpha)\|^2.$$

The lemma follows immediately. □

The lemma can be used to study the sufficiency of statistics generated from a single CDMA user with the chip waveform $\psi(t)$ and a fractional delay $\alpha = \tau \bmod T_c$. When we have K independent signals $\{s_k(t)\}_{k=1}^{K}$ with fractional delays $\{\alpha_k\}_{k=1}^{K}$, and if $s(t) = \sum_{k=1}^{K} s_k(t)$ has a support in $[-T/2, T/2]$, it is easy to see that

$$\frac{\mathrm{E}|s(t)-\hat{s}(t)|^2}{\mathrm{E}|s(t)|^2} = \frac{\sum_{k=1}^{K} \|\psi(t-\alpha_k)-\hat{\psi}(t-\alpha_k)\|^2}{K\|\psi(t)\|^2} \le \sup_{\alpha\in[0,T_c]} \frac{\|\psi(t-\alpha)-\hat{\psi}_\alpha(t)\|^2}{\|\psi(t)\|^2}. \quad (11)$$

Using Definition 2, and noting our normalization of $\psi(t)$ to have unit energy, it follows that the D statistics are δ-sufficient if

$$L_\psi := \sup_{\alpha\in[0,T_c]} \|\psi(t-\alpha)-\hat{\psi}_\alpha(t)\|^2 \le \delta. \quad (12)$$

While the above discussion motivates the use of L_ψ as a measure of sufficiency, it requires $\mathcal{P}_g$ to satisfy the chip-invariance assumption of the lemma. Alternatively, note that we could simply *define* L_ψ to be the sufficiency measure for CDMA systems with any projection operator $\mathcal{P}_g$.

We can use the above result to study the loss in sufficiency for different $\{g_n(t)\}$. Now, $\psi(t)$ belongs to $\mathcal{F}_\epsilon(T,W)$, and a complete basis for the $\mathcal{F}_\epsilon(T,W)$ are the PSWFs. We can reduce the number of statistics to a finite number by ignoring components along the PSWFs with negligible energy in $[-W, W]$. More precisely, it follows from the results of [11] that, with $D = \lfloor 2WT \rfloor + 1$, L_ψ is of the order[3] of ϵ for all W and T. Moreover, the D PSWFs are the optimum basis set in the minimax sense, i.e., they lead to the least energy loss for the worst-case waveform in $\mathcal{F}_\epsilon(T,W)$. However, the PSWFs do not have closed-form expressions and are not convenient for analysis or practical implementation. We consider below an alternate approach for discretization based on chip-matched filtering.

Chip-Matched Filtering: The chip-matched filtering approach has been used in much of the recent work on joint acquisition and/or joint detection. The discrete system model can be formed without the knowledge of the delays of the users and is useful for detection as well as acquisition problems. Chip-matched filtering involves passing $r(t)$ through a filter with an impulse response matched to the chip waveform $\psi(t)$, and sampling the output at intervals νT_c, where ν is in general ≤ 1. Equivalently, $r(t)$ is projected onto the set of translated chip pulses

$$g_n(t) = \psi(t-\nu n T_c) = \psi_n(t), \qquad n = 1,\ldots,D \quad (13)$$

where $D = \lceil MN/\nu \rceil$. Correspondingly, the matrix $\mathbf{C}$ in (9) is formed by

$$c_{nk} = \langle \psi_n(t), \tilde{c}_k(t)\rangle_T = \langle \psi_n(t), \sum_j \tilde{c}_{k,j}\psi(t-jT_c-\gamma_k T_c)\rangle_T = \sum_j \tilde{c}_{k,j} R_\psi(j+\gamma_k-\nu n) \quad (14)$$

where $\gamma_k = \tau_k/T_c - \lfloor \tau_k/T_c \rfloor$ is the normalized fractional delay of user k with respect to the timing reference, and $\tilde{c}_{k,j}$ is the spreading sequence corresponding to $\tilde{c}_k(t)$. Furthermore, $R_\psi(u) = \langle \psi(t), \psi(t-uT_c)\rangle_T$ is the autocorrelation of the chip waveform with the argument normalized to T_c.

If $\psi(t)$ is a CL waveform, i.e., time-limited to $[-T_c/2, T_c/2]$, and $\nu = 1$ (chip rate sampling), we have

$$c_{nk} = R_\psi(\gamma_k)\tilde{c}_{k,n} + R_\psi(1-\gamma_k)\tilde{c}_{k,n-1}. \quad (15)$$

In this case, the chip-matched filtered model is convenient for analysis and implementation. Note that the noise vector $\mathbf{n}$ is colored Gaussian with the distribution $\mathcal{N}(0, \sigma^2\mathbf{R}_\psi)$, where $\mathbf{R}_\psi$ is a symmetric Toeplitz matrix with

$$\mathbf{R}_\psi(i,j) = \langle \psi(t-\nu i T_c), \psi(t-\nu j T_c)\rangle_T = R_\psi(\nu(i-j)). \quad (16)$$

Now, the chip-invariance assumption of Lemma 1 is satisfied for chip-matched filtering if $1/\nu$ is an integer, since the relative delays between $\psi(t-nT_c-\alpha)$ from the closest chip-MF basis functions would be the same for all n (ignoring any edge effects). We restrict attention to the case of integral $1/\nu$ without much loss in generality. The loss in sufficiency is measured by L_ψ, the worst-case projection loss of the shifted chip waveform $\psi(t-\alpha)$ onto the space $\mathcal{G}$ over $\alpha \in [0, T_c]$. For fixed α, the projection is given by (10), i.e.,

$$\hat{\psi}_\alpha(t) = \sum_{n=1}^{D} \hat{\psi}_n \psi_n(t), \text{ with } \hat{\boldsymbol{\psi}}_\alpha = \mathbf{R}_\psi^{-1}\boldsymbol{\psi}_\alpha$$

where the components of $\boldsymbol{\psi}_\alpha$ are given by $\psi_n = \langle \psi(t-\nu n T_c), \psi(t-\alpha)\rangle_T = R_\psi(\alpha/T_c - \nu n)$. Since the chip waveform has unit energy, we have

$$\|\psi_e(t)\|^2 = 1 - \hat{\boldsymbol{\psi}}_\alpha^\top \mathbf{R}_\psi \hat{\boldsymbol{\psi}}_\alpha = 1 - \boldsymbol{\psi}_\alpha^\top \mathbf{R}_\psi^{-1}\boldsymbol{\psi}_\alpha.$$

Since $\{\psi_n(t)\}$ have a spacing of νT_c between them, α can be restricted to $[0, \nu T_c]$. Also, by symmetry, it is easy to show that the maximum projection error must occur for $\alpha = \nu T_c/2$, so that

$$L_\psi = \sup_\alpha \|\psi(t-\alpha)-\hat{\psi}_\alpha(t)\|^2 = 1 - \boldsymbol{\psi}_{\nu T_c/2}^\top \mathbf{R}_\psi^{-1} \boldsymbol{\psi}_{\nu T_c/2}. \quad (17)$$

For illustration, if we project $\psi(t-\nu T_c/2)$ onto just the two adjacent chip waveforms $\psi(t)$ and $\psi(t-\nu T_c)$, the loss in energy is given by

$$1 - \begin{pmatrix} R_\psi\left(\frac{\nu}{2}\right) & R_\psi\left(\frac{\nu}{2}\right) \end{pmatrix} \begin{pmatrix} 1 & R_\psi(\nu) \\ R_\psi(\nu) & 1 \end{pmatrix}^{-1} \begin{pmatrix} R_\psi\left(\frac{\nu}{2}\right) \\ R_\psi\left(\frac{\nu}{2}\right) \end{pmatrix} = 1 - \frac{2R_\psi\left(\frac{\nu}{2}\right)^2}{1+R_\psi(\nu)}$$

which goes to zero as $\nu \to 0$. Now, since we know that using $2WT$ PSWFs yields L_ψ to the order of ϵ, we would like to study L_ψ as a function of ϵ with $2WT$ chip-MF samples as well. We must then have

$$\frac{T}{\nu T_c} = 2WT \Rightarrow WT_c = \frac{1}{2\nu}.$$

Correspondingly, the bandwidth constraint (8) gives

$$\int_{-1/2\nu}^{1/2\nu} |\Psi(f)|^2 df = 1 - \epsilon \quad (18)$$

[3]Note that $\epsilon = 0$ corresponds to perfectly band-limited signals, and Nyquist sampling leads to exactly $2WT$ sufficient statistics over time T.

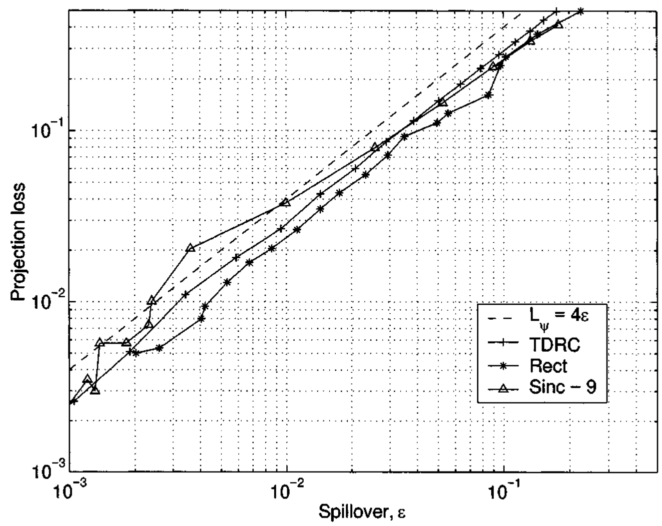


Fig. 1. Projection loss L_ψ with Nyquist sampling versus out-of-band spillover ϵ for different chip waveforms. The sinc pulse is truncated to nine chips.

where T_c has been set to 1 without loss of generality.

Ideally, we would like to compute the worst-case L_ψ for a given ϵ by maximizing it over all possible pulse shapes. While this optimization would yield a good measure of the efficacy of chip-matched filtering, it seems to be a hard problem. Instead, we could numerically evaluate L_ψ and ϵ as a function of the parameter ν using (17) and (18) for each chip waveform of interest and ensure that the chip-MF entails a loss in sufficiency comparable to ϵ. As mentioned in Section I, we consider three candidate chip waveforms in this paper: the CL rectangular pulse, a truncated (Nyquist) sinc pulse with bandwidth W, and the CL TDRC pulse. These waveforms can be written down as

$$\begin{aligned}\psi_{\mathrm{rect}}(t) &= \frac{1}{\sqrt{T_c}}\Pi\left(\frac{t}{T_c}\right)\\ \psi_{\mathrm{sinc}}(t) &= \frac{\kappa}{\sqrt{T_c}}\mathrm{sinc}\left(\frac{t}{T_c}\right)\Pi\left(\frac{t}{mT_c}\right)\\ \psi_{\mathrm{TDRC}}(t) &= \sqrt{\frac{2}{3T_c}}\left(1+\cos\left(\frac{2\pi t}{T_c}\right)\right)\Pi\left(\frac{t}{T_c}\right) \qquad (19)\end{aligned}$$

where $\Pi(t)$ is a unit amplitude rectangular pulse between $[-1/2, 1/2]$. The MC sinc waveform truncated to m chips, with the normalizing factor κ approaching 1 as $m \to \infty$. Fig. 1 shows the variation of L_ψ for ϵ for these three waveforms, along with the line $L_\psi = 4\epsilon$ for comparison. The variation for each chip waveform is quite complicated, but it can be seen that, in all three cases, $2WT$ chip-MF statistics lead to a loss in sufficiency that is of the order of the spillover ϵ in the range of interest. Hence, chip-matched filtering is a useful technique for generating approximate sufficient statistics, *so long as the output is sampled at the Nyquist rate.* Also, while similar values of L_ψ and ϵ are obtained for all waveforms in Fig. 1, the sampling rates required are significantly different. With $\epsilon = 0.01$, the Nyquist rates are approximately 21, 1.02, and 2.8 times the chip rate, respectively. The corresponding value of L_ψ is between 0.02–0.04 for all three waveforms.

Finally, it is of interest to compare the chip-MF approach to discretization with correlation.

Correlation Versus Chip-MF: When the delays are known, the correlation approach generates K_e sufficient statistics, while Nyquist sampling of the chip-MF output produced statistics that are approximately sufficient with δ of the order of ϵ (see Definition 2). Thus, assuming that the loss in sufficiency does not alter performance significantly, the chip-MF approach may lead to lower complexity if $K_e > 2WT$, and correlation may be preferred otherwise. Also note that K_e is independent of W, and the bandwidth restriction does not explicitly enter the correlation calculations. This amounts to assuming that the front end of the *receiver* has infinite bandwidth. On the other hand, sampling the chip-MF at νT_c spacing amounts to assuming a front-end bandwidth of $1/(2\nu T_c)$. As $\nu \to 0$, this bandwidth goes to infinity, the energy loss goes to zero, and the corresponding chip-MF performance may be expected to approach that with correlation.

In the following sections, we study the role of chip-matched filtering on detection and estimation at the receiver. In general, the performance of the receiver operation would depend on the discretization technique, the choice of the chip waveform, and the detector/estimator used. For a given detector/estimator, the generation of sufficient statistics is a prerequisite for a fair comparison across chip waveforms.

V. Detection Performance

In this section, we study the performance of the matched-filter (MF) and minimum mean-squared-error (MMSE) detectors using different chip waveforms. For a fair comparison across the chip waveforms, we impose the following constraints. We require that the ϵ-bandwidths of the chip-waveforms be equal

$$\int_{-\infty}^{\infty}|\Psi(f)|^2 df = 1 \quad \text{and} \quad \int_{-W}^{W}|\Psi(f)|^2 df = 1-\epsilon \qquad (20)$$

where W and ϵ are assumed to be given. Also, for a given W, different chip-waveforms may lead to different values of T_c; we require that the (code) *symbol* rates be the same, i.e.,

$$T_s = NT_c = \text{constant} \;\Rightarrow\; NB = \text{constant} \qquad (21)$$

where $B = WT_c$ is the bandwidth normalized to $1/T_c$. Note that the above equal-rate and bandwidth constraints on the chip waveform can also be found in [13], but the comparison there is restricted to CL waveforms and matched-filter detection. We allow for MC waveforms as well, and require that there must be negligible interchip interference (ICI) when the output of the chip-MF is sampled with perfect synchronization and chip-spacing.

A. Matched Filter (MF) Detection

The conventional detection strategy treats the interfering users as white noise and uses (2) to arrive at the MF detector. With the observation window factor $M = 2p-1$, we have $M' = 2p+1$, and the parameter of interest is the central bit $b_{1,p+1}$. The amplitude and delay of this bit are that of the actual user 1, $A_1 = \sqrt{\mathcal{E}_1}$ and $\tau_1 = \lfloor \tau_1/T_c \rfloor T_c + \alpha_1$, and the effective spreading waveform is $\tilde{c}_{p+1}(t)$. Hence

$$X_{\mathrm{MF}}^{\mathrm{cor}} = \int_{-T/2}^{T/2} r(t)\tilde{c}_{p+1}(t)dt = \langle r(t), \tilde{c}_{p+1}(t)\rangle_T. \qquad (22)$$

Thus, the continuous-time MF involves correlation with the spreading waveform of the bit of interest, with the corresponding delay assumed known.

Alternatively, we could use chip-matched filtering to deal with the detection problem in a discrete framework. Separating out the bit of interest, (9) can be written as

$$\mathbf{y} = A_1 b_{p+1}\mathbf{c}_{p+1} + \mathbf{C}_I\mathbf{A}_I\mathbf{b}_I + \mathbf{n} \tag{23}$$

where the subscript I denotes interference from other bits in the observation interval. Consequently

$$X_{\mathrm{MF}}^{\mathrm{chip}} = \mathbf{c}_{p+1}^\top \mathbf{R}_\psi^{-1}\mathbf{y}$$

where $\mathbf{R}_\psi$ is as defined in (16).

Now, if the fractional delay of the desired user is zero ($\alpha_1 = 0$), it follows that $\tilde{c}_{p+1}(t) = \sum_j \tilde{c}_{p+1,j}\psi(t - jT_c)$, and chip-matched filtering followed by chip rate sampling can be used to generate the matched-filter statistic, as an alternative to direct correlation, i.e.,

$$X_{\mathrm{MF}}^{\mathrm{cor}} = \sum_j \tilde{c}_{p+1,j}\langle r(t), \psi(t-jT_c)\rangle_T = \sum_j \tilde{c}_{p+1,j} y_j = X_{\mathrm{MF}}^{\mathrm{chip}}$$

where the chip-MF functions $\{\psi_n(t)\}$ are chosen with $\nu = 1$, corresponding to chip rate sampling. Thus, chip rate chip-MF statistics are sufficient for matched-filter detection when $\alpha_1 = 0$.

Note that, since the user delay τ_1 is assumed known, we can always set the fractional delay $\alpha_1 = 0$ by redefining the time-axis at the receiver. However, for purely pedagogical reasons, if we let τ_1 (and hence α_1) be known but *arbitrary*, we have that $X_{\mathrm{MF}}^{\mathrm{chip}}$ with chip rate sampling is, in general, not equal to $X_{\mathrm{MF}}^{\mathrm{cor}}$, and the performance would be different. At the same time, from the discussion in Section IV, we expect the chip-MF performance to be close to that with correlation when the output is sampled at the Nyquist rate. We consider below the details of the effect on performance of the MF detector.

The performance metric we use for detection is the signal-to-interference ratio (SIR) at the output of the detector

$$\mathrm{SIR}_{\mathrm{MF}}^{\mathrm{cor}} = \frac{(\mathrm{E}[X_{\mathrm{MF}}^{\mathrm{cor}}|b_{p+1}])^2}{\mathrm{var}[X_{\mathrm{MF}}^{\mathrm{cor}}|b_{p+1}]} \tag{24}$$

where the expectation is taken over the sequences of all the users and the bits and delays of the remaining (effective) interferers. The quantity $\mathrm{SIR}_{\mathrm{MF}}^{\mathrm{chip}}$ is defined similarly. We consider random spreading sequences, with the bits and sequences modeled as i.i.d. equally likely ± 1 random variables. The delays $\{\tau_k\}$ are modeled to be uniform in $[0, T_s]$. It follows that (see [15]):

$$\mathrm{SIR}_{\mathrm{MF}}^{\mathrm{cor}} = \frac{2\mathcal{E}_1}{N_0 + \frac{2\sigma_\psi}{N}\sum_{k=2}^{K}\mathcal{E}_k} \tag{25}$$

where $\sigma_\psi = (1/T_c)\int_{-\infty}^{\infty}|\Psi(f)|^4 df$. For the chip-MF, we have

$$\begin{aligned}\mathrm{E}\left[X_{\mathrm{MF}}^{\mathrm{chip}}\middle|b_{p+1}\right] &= A_1 b_{p+1}\mathbf{c}_{p+1}^\top\mathbf{R}_\psi^{-1}\mathbf{c}_{p+1}\\ \mathrm{var}\left[X_{\mathrm{MF}}^{\mathrm{chip}}\middle|b_{p+1}\right] &= \frac{N_0}{2}\mathrm{E}\left(\mathbf{c}_{p+1}^\top\mathbf{R}_\psi^{-1}\mathbf{c}_{p+1}\right)\\ &\quad + \sum_{k\neq p+1} A_k^2\mathrm{E}(\mathbf{c}_{p+1}^\top\mathbf{R}_\psi^{-1}\mathbf{c}_k)^2.\end{aligned}$$

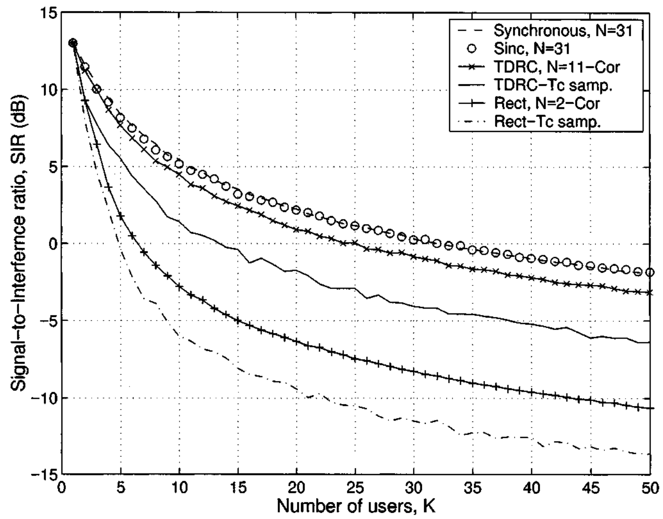


Fig. 2. MF detection—SIR performance with different chip waveforms, and $\mathcal{E}_b/N_0 = 10$ dB. Note the performance degradation of chip-limited waveforms with chip rate sampling. With Nyquist sampling, there was no noticeable degradation from the 'cor' curves above. Also, note that the performance of the asynchronous system with the truncated sinc waveform is identical to that of the synchronous system with $N = 31$.

If the statistics thus obtained are δ-sufficient, it can be easily shown using (12) that

$$\begin{aligned}\left|\mathrm{E}\left[X_{\mathrm{MF}}^{\mathrm{chip}}\middle|b_{p+1}\right] - \mathrm{E}\left[X_{\mathrm{MF}}^{\mathrm{cor}}\middle|b_{p+1}\right]\right|^2 &\leq C_1\delta\\ \left|\mathrm{var}\left[X_{\mathrm{MF}}^{\mathrm{chip}}\middle|b_{p+1}\right] - \mathrm{var}\left[X_{\mathrm{MF}}^{\mathrm{chip}}\middle|b_{p+1}\right]\right| &\leq C_{21}\delta + C_{22}\delta^2\end{aligned}$$

for some finite constants C_1, C_{21}, C_{22} independent of δ. Clearly, as $\nu \to 0$, we have $\delta \to 0$, and it follows that $\mathrm{SIR}_{\mathrm{MF}}^{\mathrm{chip}} \to \mathrm{SIR}_{\mathrm{MF}}^{\mathrm{cor}}$. However, it is also of interest to study the performance with nonzero ν, specifically with chip rate sampling ($\nu = 1$) and Nyquist sampling ($\nu = 1/\lceil 2B\rceil$), as we do later in this section.

Before proceeding further, we comment briefly on the problem of chip waveform selection for the matched-filter. Clearly, the metric to be used is the expression for $\mathrm{SIR}_{\mathrm{MF}}^{\mathrm{cor}}$ in (25), since it represents the performance with sufficient statistics (and the limiting performance of the chip-MF). And to maximize the quantity $\mathrm{SIR}_{\mathrm{MF}}^{\mathrm{cor}}$ for given K and user powers, we must then choose $\Psi(f)$, T_c (and N) to minimize $(\sigma_\psi/N) \propto B\sigma_\psi$ under the constraints (20) and (21). Based on this, we can broadly identify a tradeoff: CL waveforms have greater normalized bandwidth B (and hence lower N), but have better correlation properties, i.e., lower σ_ψ.

When $\epsilon = 0$, the sinc waveform with $B = 1/2$ is optimum [15], and the tradeoff discussed above favors the infinite duration sinc waveform. This motivates the study of the truncated MC sinc waveform $\psi_{\mathrm{sinc}}(t)$ given in (19). It is of interest to compare the performance achieved with $\psi_{\mathrm{sinc}}(t)$ in (19) to that obtained with the TDRC pulse [13], and the rectangular pulse commonly used for analysis [16]–[18]. The SIR variation with the number of users is shown in Fig. 2. The bit SNR ($\mathcal{E}_b/N_0$) is taken to be 10 dB, and ϵ is set to 0.01, which corresponds to a 99% essential bandwidth. The processing gain for $\psi_{\mathrm{sinc}}(t)$ with $m = 9$ is taken to be $N = 31$. Now, based on the equal bandwidth and equal rate constraints, the processing gains for the TDRC and rectangular waveforms can be computed to be $N = 11$ and 2, respectively.[4] It should be noted that, with SIR averaged over the delay of the desired user, the chip-MF with chip rate sampling shows a significant loss in performance compared to that with Nyquist sampling, for the rectangular and TDRC waveforms. Moreover, there was no noticeable difference between the chip-MF with Nyquist sampling and correlation for all three waveforms.

Also, for chip waveform selection, we need to compare the curves corresponding to Nyquist sampling for each waveform. It is clear that the MC sinc waveform has the best performance, and the TDRC CL waveform is only slightly inferior. But the rectangular CL waveform can result in a significant loss in performance. Thus, the relevance of performance analyses specific to rectangular chip pulses is called into question.

B. MMSE Detection

We are now interested in studying the performance trends with the above chip waveforms for linear multiuser detection, specifically the linear MMSE detector [3], [19]. The estimate for the bit of interest is $\hat{b}_{1,p+1} = (\mathbf{Ly})_{p+1} = \mathbf{f}^\top \mathbf{y}$, where $\mathbf{y} = \mathbf{CAb} + \mathbf{n}$ as in (9). If we generate statistics by projecting onto $\{g_n(t)\}$, the linear MMSE detector is given by

$$\mathbf{f} = \left(\mathbf{CA}^2\mathbf{C}^\top + \sigma^2 \mathbf{R}_g\right)^{-1} \mathbf{c}_{p+1}$$

and the corresponding SIR is given by

$$\mathrm{SIR}_{\mathrm{MMSE}} = \frac{A_1^2 \beta_{p+1}^2}{\sum_{k \neq p+1} A_k^2 \beta_k^2 + \sigma^2 \mathbf{f}^\top \mathbf{R}_g \mathbf{f}} \tag{26}$$

where $\boldsymbol{\beta} = \mathbf{f}^\top \mathbf{C}$, and we have assumed that the powers and delays of all the users are known. The behavior of $\mathrm{SIR}_{\mathrm{MMSE}}$ as a function of K is studied by numerically averaging (26) over the sequences and delays. However, unlike in the MF study, we set the delay of the desired user to zero ($\tau_1 = 0$), since the chip-MF statistics are not equivalent to those with correlation even under this assumption. As before, we assume equal powers for all the users and fix $\mathcal{E}_b/N_0$ at 10 dB.

We begin by verifying again that chip-matched filtering followed by Nyquist sampling (i.e, setting $\nu = 1/\lceil 2B \rceil$) results in performance close to that obtained with correlation (note that $\nu = 1$ for the sinc waveform). The results are shown in Fig. 3. This justifies our claim of approximate sufficiency of the statistics produced by chip-matched filtering followed by Nyquist sampling. As in the MF case, N is set to 31 for the truncated sinc waveform, and equal rate and 99% bandwidth constraints at the transmitter yield $N = 11$ and 2 for the TDRC and rectangular waveforms, respectively. As mentioned previously, many papers on detection and estimation for CDMA systems, assume that the chip-MF is sampled at the chip rate for the sake of convenience in analysis. The results shown in Fig. 4 illustrate that chip rate sampling can lead to substantial loss in performance. The results thus far assumed one-shot detection, i.e., $M = 1$. A corresponding set of curves is shown in Fig. 5 for a window length of $M = 3$, and it is seen that the substantial loss incurred by chip rate sampling is not due to any windowing effects.

[4]The actual value of N for the rectangular chip waveform is between 1 and 2, and the results are hence optimistic.

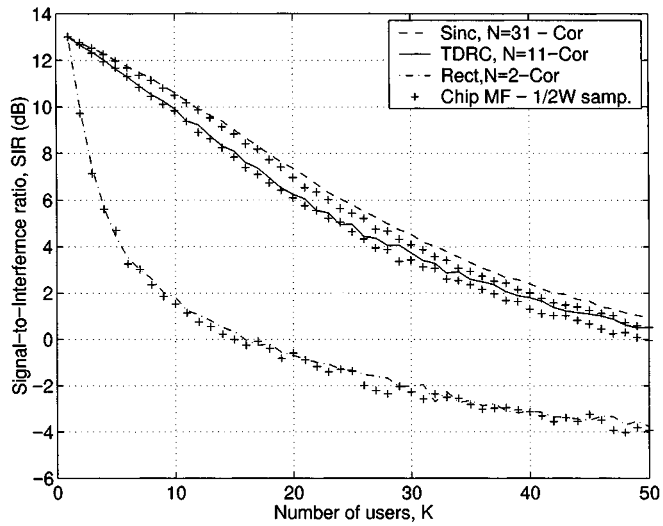


Fig. 3. One-shot MMSE detection, $\mathcal{E}_b/N_0 = 10$ dB: with Nyquist sampling of the chip-MF output, the chip-MF statistics are approximately sufficient and the performance matches that obtained via correlation for all three waveforms.

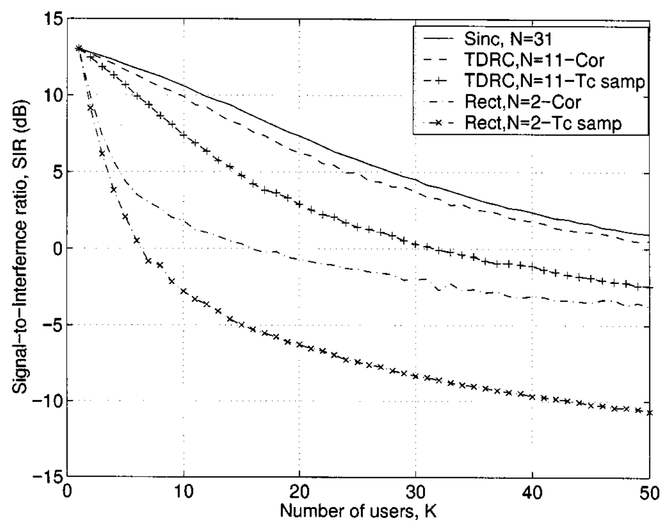


Fig. 4. One-shot MMSE detection. It is seen that T_c-sampling leads to a significant degradation for CL waveforms. The processing gains are different for the three waveforms (31, 11, and 2) as in Fig. 3.

Finally, the correlator (or Nyquist sampled chip-MF) curves in Figs. 4 and 5 again provide a fair comparison of the best performance that can be obtained with the three chip waveforms that we consider. As with the MF detector (see Fig. 2), we see that the truncated sinc MC waveform results in the best performance, with the rectangular CL waveform performing quite poorly. On the other hand, the gap between the TDRC and the sinc waveform is small. Thus, the results indicate that the better correlation properties of well-designed CL pulses can offset the decrease in processing gain, and the performance with CL pulses can approach that with MC pulses. This conclusion is function of the spillover ϵ, and as ϵ is reduced further, we expect that MC waveforms would continue to outperform CL waveforms.

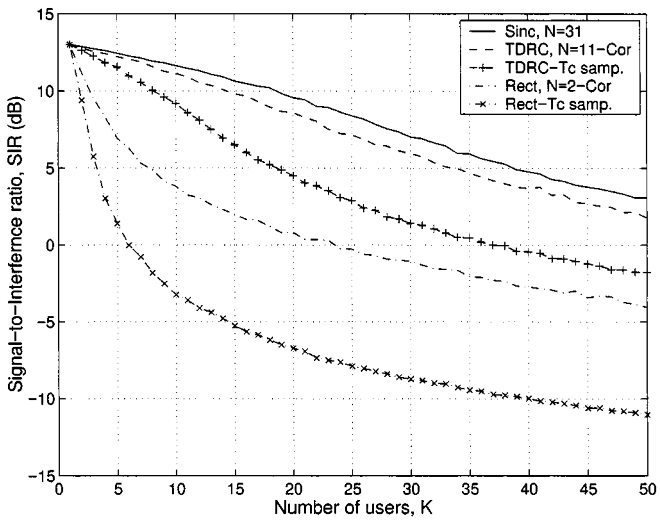


Fig. 5. MMSE detector performance—window length equal to 3.

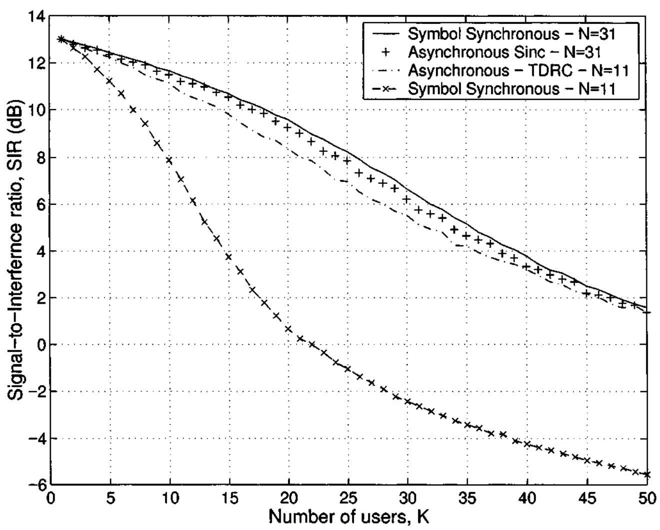


Fig. 6. MMSE detector with TDRC ($N = 11$) and sinc pulses ($N = 31$): asynchronous ($M = 3$) and synchronous performances. The performance improves in the asynchronous case since the correlation structure of the TDRC waveform allows for better signal separation. The match in performance with the sinc indicates that the synchronous case could serve as a benchmark for the asynchronous case, for given bandwidth and rate constraints.

Asynchronous Versus Synchronous Users: As an aside, it is of interest to compare performance in the completely asynchronous case to that with synchronous users ($\tau_k = 0, \forall k$). For synchronous users, it is easily seen that chip-rate sampling of the chip-MF generates N sufficient statistics per symbol, while we have N/ν approximately sufficient statistics with asynchronous users. Since ν for MC waveforms is typically greater than that for CL waveforms, the gap between the synchronous and asynchronous cases can be expected to be smaller for MC waveforms. In particular, it can be shown that the SIR for the MF with the sinc waveform is equal to the SIR in the synchronous case (see also Fig. 2). For the MMSE detector, Fig. 6 compares the two scenarios for the TDRC pulse and for the sinc pulse. The gap is seen to be much smaller with the sinc waveform. Since the sinc pulse also yields the best performance among the waveforms considered, the results indicate that synchronous performance can be a useful benchmark for asynchronous analyses.

VI. Acquisition Performance

For illustration of timing estimation, we consider the simple case of acquiring a single user under white noise (which could also be the model for the multiple-access interference). We assume that a preamble is used to allow for acquisition. The received signal in (5) becomes

$$r(t) = \sum_{m=0}^{M'-1} A_1 c_1^{(m)}(t-\tau_1) + n(t) = A_1 c_1(t-\tau_1) + n(t). \tag{27}$$

We assume that the amplitude A_1 is unknown along with the delay τ_1. Then, $s_{\{A_1,\tau_1\}}(t) = A_1 c_1(t-\tau_1)$. Since $\int c_1(t-\tau)^2 dt = MN$ and is independent of τ (ignoring edge effects), it follows from (2) that the maximum-likelihood estimator for τ_1 is given by the correlating acquisition scheme:

$$\hat{\tau}_1 = \arg\max_{\tau} \langle c_1(t-\tau), r(t)\rangle_T \tag{28}$$

which involves maximizing the integral over a continuous parameter τ. Thus, it is not possible to generate a finite set of sufficient statistics with this approach.

Alternatively, as with the MF detector, chip-matched filtering could be used to deal with the estimation problem in a more convenient discrete framework. Some recent work on this problem, especially that involving joint delay estimation (see, e.g., [7]–[9] and [20]), is based on this approach. Using (14), the ith sample at the output of the chip-MF can be seen to be

$$y_i = \sum_j A_1 \tilde{c}_{1,j} R_\psi(j + \alpha_1/T_c - \nu i) + n_i. \tag{29}$$

Here $\tilde{c}_{1,j}$ is the spreading sequence corresponding to $c_1(t)$ shifted right by $\lfloor \tau_1/T_c \rfloor$ places. Following (9), the filter outputs can be expressed in vector form as

$$\mathbf{y} = A_1 \mathbf{c}(\tau_1) + \mathbf{n} \tag{30}$$

where $\mathbf{n}$ is Gaussian with the distribution $\mathcal{N}(0, \sigma^2 \mathbf{R}_\psi)$. The ML estimator for the delay is easily seen to be

$$\hat{\tau}_1 = \arg\max_{\tau} \frac{[\mathbf{c}(\tau)^\top \mathbf{R}_\psi^{-1} \mathbf{y}]^2}{\mathbf{c}(\tau)^\top \mathbf{R}_\psi^{-1} \mathbf{c}(\tau)}. \tag{31}$$

With CL pulses and T_c sampling, it is possible to get an analytical handle on the maximization based on (15) (see, e.g., [20] and [21]). In addition, with rectangular pulses, it is possible to derive Cramer–Rao bounds since the autocorrelation function $R_\psi(u) = 1 - u$ is a simple polynomial form (see, e.g., [22]). It is hence of interest to compare systems that use CL pulses and T_c sampling with those that use CL or MC pulses and Nyquist sampling. Where analytical simplification is not known, we use a high resolution grid search to estimate τ_1. The performance of the acquisition scheme is measured in terms of the probability of acquisition error defined as

$$\mathrm{P_e} = \mathrm{Prob}\{|\hat{\tau}_1 - \tau_1| > 0.5\}. \tag{32}$$

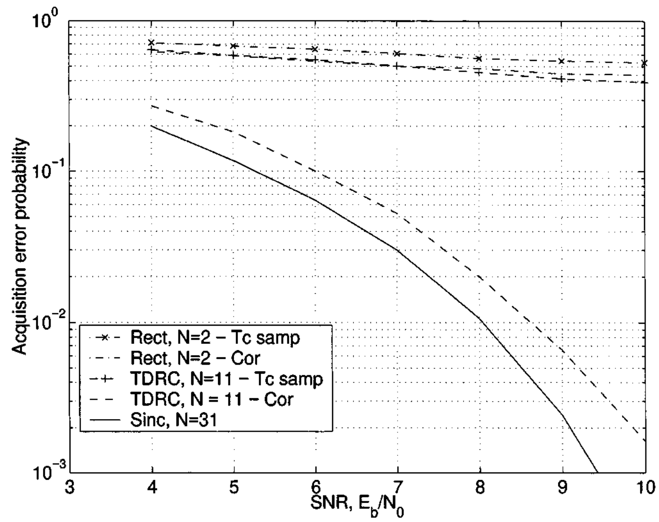


Fig. 7. Single user acquisition performance, $K = 1$.

The variation of P_e with $\mathcal{E}_b/N_0$ is shown in Fig. 7. For illustration, the observation interval is taken to be two symbols $(M = 2)$, and uncertainty in τ_1 is taken to be 30 chips in all cases. Again, the best performance with each of the three waveforms can be obtained by sampling the chip-MF at the Nyquist rate, and sampling at T_c spacing leads to a significant loss. Alternatively, the same performance can be obtained by evaluating the correlation-based statistic (28) at sufficiently fine spacing. The trends remain the same: the rectangular pulse performs poorly due to the low value of N, and the TDRC pulse provides comparable but worse performance than the MC sinc waveform. However, note that the performance comparison across chip waveforms also depends on the model for the uncertainty in τ_1. In particular, if the uncertainty in number of chips is varied across waveforms so as to correspond to a fixed time interval, the performance difference between the TDRC and sinc waveforms can be shown to be insignificant. Finally, since rectangular chip pulses with T_c sampling facilitate the derivation of the ML delay estimate, it is of interest to see how the ML estimate obtained performs with the MC sinc pulse. In other words, we use the analytical delay estimate obtained by using $\mathbf{R}_\psi$ for a rectangular waveform in (31) to a system that actually uses the MC sinc waveform at the same spreading factor. We found the performance to be extremely poor, with acquisition error probabilities of nearly 1 throughout the range of SNRs considered in Fig. 7.

VII. Conclusions and Future Directions

In this paper, we considered the problem of generating discrete statistics in an (approximately) band-limited CDMA system with an ϵ-bandwidth W. In particular, we studied the role of chip-matched filtering, a discretization technique that is commonly used in the CDMA literature. It was found that most of the signal energy is captured if the chip-MF output is sampled close to the Nyquist rate of $\lceil 2WT_c \rceil$ samples per chip-interval, and the performance using the resulting statistics is close to that obtained using statistics derived from correlation with the spreading waveforms. When chip waveforms limited to a single chip duration (CL waveforms) are used, the Nyquist rate is greater than chip rate. However, many of the papers on CDMA detection and acquisition have assumed CL waveforms and chip rate sampling of the chip-MF output for analytical convenience; we have shown that this could result in a significant performance loss.

With appropriate discretization, we considered the effect of the chip waveform on the performance of detection and acquisition schemes. Specifically, we considered three chip pulses: CL rectangular, CL TDRC, and a truncated sinc spanning multiple chips (MC pulse). Under equal rate and bandwidth constraints, we have identified a tradeoff between CL and sinc-like MC waveforms: while CL waveforms lead to lower processing gains, they have better correlation properties that lead to improved performance in the asynchronous case. However, for all three cases considered—MF, MMSE detection, and single-user acquisition—we found the tradeoff to favor the truncated sinc waveform. In particular, we showed that the rectangular waveform performs very poorly, and its widespread use in analysis of asynchronous CDMA systems needs to be questioned. On the other hand, we found that performance with the TDRC waveform is comparable to that obtained with the MC sinc waveform. Hence, appropriately designed CL waveforms could be used in practice. However, for optimum performance with CL waveforms, the chip-MF needs to be sampled at higher than the chip rate; this makes the design and analysis with CL waveforms just as cumbersome as that with MC waveforms.

Note that while studying chip-matched filtering or comparing MMSE detection and acquisition with different pulses, we did not attempt to optimize the performance measure over all possible chip waveforms—this appears to be a difficult problem. However, our study in this paper yields a framework for chip waveform design with general detection and estimation schemes, and this could be a subject for further investigation.

References

[1] J. Wozencraft and I. Jacobs, *Principle of Communication Engineering*. New York: Wiley, 1965.

[2] M. K. Varansi and B. Aazhang, "Multistage detection in asynchronous code-division multiple-access communications," *IEEE Trans. Commun.*, vol. 38, pp. 509–519, Apr. 1990.

[3] U. Madhow and M. L. Honig, "MMSE interference suppression for direct-sequence spread-spectrum CDMA," *IEEE Trans. Commun.*, vol. 42, pp. 3178–3188, Dec. 1994.

[4] Z. Xie, R. T. Short, and C. K. Rushforth, "A family of sub-optimum detectors for coherent multiuser communications," *IEEE J. Select. Areas Commun.*, vol. 8, pp. 683–690, May 1990.

[5] Y. Asano, Y. Daido, and J. M. Holtzman, "Performance evaluation for band-limited DS-CDMA communication system," in *Proc. IEEE 43th Vehicular Technology Conf.*, Secaucus, NJ, May 1993, pp. 464–468.

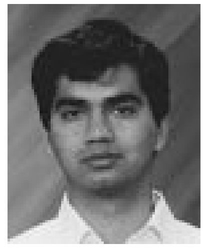
[6] T. Ostman and B. Ottersten, "Near far robust time delay estimation for asynchronous DS-CDMA systems with bandlimited pulse shapes," in *Proc. IEEE 48th Vehicular Technology Conf.*, Ottawa, ON, Canada, May 1998, pp. 1650–1654.

[7] S. Bensley and B. Aazhang, "Subspace-based channel estimation for CDMA system," *IEEE Trans. Commun.*, vol. 44, pp. 1009–1020, Aug. 1996.

[8] E. Strom *et al.*, "Propagation delay estimation in asynchronous DS-CDMA systems," *IEEE Trans. Commun.*, vol. 44, pp. 84–93, Jan. 1996.

[9] U. Madhow, "Blind adaptive interference suppression for near–far resistant acquisition and demodulation of DS-CDMA signals," *IEEE Trans. Signal Processing*, vol. 45, pp. 124–36, Jan. 1997.

[10] T. Ostman and B. Ottersten, “Low complexity asynchronous DS-CDMA detectors,” in *Proc. IEEE 46th Vehicular Technology Conf.*, Atlanta, GA, May 1996, pp. 559–563.
[11] H. J. Landau and H. O. Pollack, “PSWFs-III: The dimension of the space of essentially time- and band-limted signals,” *Bell Syst. Tech. J.*, pp. 1295–1320, July 1962.
[12] J. G. Proakis, *Digital Communications*. New York: McGraw-Hill, 1995.
[13] M. A. Landolsi and W. E. Stark, “DS-CDMA chip waveform design for minimal interference under bandwidth, phase and envelope constraints,” *IEEE Trans. Commun.*, vol. 47, pp. 1737–1746, Nov. 1999.
[14] H. V. Poor, *An Introduction to Signal Detection and Estimation*. New York: Springer-Verlag, 1994.
[15] A. Viterbi, *CDMA: Principles of Spread Spectrum Communication*. New York: Addison-Wesley, 1995.
[16] J. S. Lehnert and M. B. Pursley, “Error probabilities for binary direct sequence spread-spectrum communications with random signature sequences,” *IEEE Trans. Commun.*, vol. COM-35, pp. 87–98, Jan. 1987.
[17] J. M. Holtzman, “A simple, accurate method to calculate SSMA error probabilities,” *IEEE Trans. Commun.*, vol. 49, pp. 461–464, Mar. 1992.
[18] R. K. Morrow, “Accurate CDMA BER calculations with low computational complexity,” *IEEE Trans. Commun.*, vol. 46, pp. 1413–1417, Nov. 1998.
[19] M. Honig, U. Madhow, and S. Verdu, “Blind adaptive multiuser detection,” *IEEE Trans. Inform. Theory*, vol. 41, pp. 944–960, July 1995.
[20] S. Bensley and B. Aazhang, “Maximum-likelihood synchronization of a single user for CDMA systems,” *IEEE Trans. Commun.*, vol. 46, pp. 392–399, Mar. 1998.
[21] A. Mantravadi and V. Veeravalli, “Multi-access interference resistant acquisition for CDMA systems with long spreading sequences,” in *Proc. 1998 CISS*, Princeton, NJ, Mar. 1998, pp. 141–146.
[22] D. Zheng *et al.*, “An efficient code-timing estimator for DS-CDMA signals,” *IEEE Trans. Signal Processing*, vol. 45, pp. 82–89, Jan. 1997.



Ashok Mantravadi (S’97) received the B.Tech. degree in electronics and communications engineering from the Indian Institute of Technology, Madras, India, in 1996, and the M.S. degree from the School of Electrical Engineering at Cornell University, Ithaca, NY, in 1999. He is currently working toward the Ph.D. degree at Cornell University, where he has been a recipient of the John McMullen and Schlumberger Fellowships.

Between June and August 1997, he was an engineering intern with the CDMA system design group at Qualcomm Inc., San Diego, CA. Since August 2000, he has also been a Visiting Scholar at the University of Illinois, Urbana-Champaign. His research interests include spread-spectrum systems, wireless multiuser communications, and space–time signal processing.

Venugopal V. Veeravalli (S’86–M’92–SM’98) received the received the B. Tech. degree from the Indian Institute of Technology, Bombay, (Silver Medal Honors), in 1985, the M.S. degree from Carnegie Mellon University, Pittsburgh, PA, in 1987, and the Ph.D. degree from the University of Illinois at Urbana-Champaign in 1992, all in electrical engineering.

He joined the University of Illinois at Urbana-Champaign in 2000, where he is currently an Associate Professor in the Department of Electrical and Computer Engineering and a Research Associate Professor in the Coordinated Science Laboratory. He was an Assistant Professor at Cornell University, Ithaca, NY, from 1996 to 2000. His research interests include mobile and wireless communications, detection and estimation theory, and information theory.

Dr. Veeravalli is currently an Associate Editor for IEEE TRANSACTIONS ON INFORMATION THEORY, and an Editor for *Communications in Information and Systems* (CIS). Among the awards he has received for research and teaching are the IEEE Browder J. Thompson Best Paper Award in 1996, the National Science Foundation CAREER Award in 1998, the Presidential Early Career Award for Scientists and Engineers (PECASE) in 1999, and the Michael Tien Excellence in Teaching Award from the College of Engineering, Cornell University, in 1999.