Spectral Efficiency of MIMO Multiaccess Systems With Single-User Decoding

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***Abstract*—The use of multiple antennas at the transmitter and the receiver is considered for the uplink of cellular communication systems. The achievable spectral efficiency in bits/s/Hz is used as the criterion for comparing various design choices. The focus is on wideband code-division multiple-access (CDMA) systems when the receiver uses the matched-filter or the minimum mean-squared error detector, followed by single-user decoders. The spreading sequences of the CDMA system are assumed to be random across the users, but could be dependent across the transmit antennas of each user. Using analytical results in the large system asymptote, guidelines are provided for the sequence design across the transmit antennas and for choosing the number of antennas. In addition, comparisons are made between (random) CDMA and orthogonal multiaccess with multiple antennas. It is shown that CDMA, even with single-user decoding, can outperform orthogonal multiaccess when the number of receive antennas is sufficiently large.**

***Index Terms*—Asymptotic performance analysis, code-division multiple-access (CDMA), matched filter, minimum mean-squared error (MMSE) detection, multiple antennas, orthogonal multiaccess, random spreading sequences.**

I. Introduction

THE design of communication systems that employ multiple transmit antennas (in conjunction with multiple receive antennas) for diversity and multiplexing gains has been a topic of great interest since the initial studies for narrowband systems in [1]–[3]. More recently, transmitter diversity techniques for wideband systems such as W-CDMA and CDMA-2000 [5] have been considered for the downlink, i.e., from the base station to the mobile (e.g., [6], [7]). These studies were motivated by the greater complexity allowable at the base station and the need for higher data rates compared to the uplink. For future generation wireless systems operating at higher carrier frequencies, having multiple antennas at the mobile may be practical. In this paper, we study potential gains from using multiple antennas for *uplink* cellular systems, with a focus on wideband code-division multiple-access (CDMA) systems.

In studying such multiantenna systems over fading channels, an important aspect is the availability of channel state information (CSI) at both ends of the link. We assume perfect channel state information at the receiver (base station). While availability of CSI at the transmitter allows for better performance, the mobile is limited by allowable complexity as well as availability of channel feedback. Hence, we assume throughout that CSI is not available at the transmitter.

For the CDMA system, we assume a user-separating receiver [8], where the signal is processed by a front-end detector that feeds a suitable output to autonomous single-user (space-time) decoders. We consider two front-end detectors—the matched-filter (MF) detector and the minimum mean-squared error (MMSE) detector. The importance of these two detectors for single-antenna systems is well documented in the literature [9] and we study straightforward extensions of the detectors to the scenario with multiple transmit antennas. Clearly, the user-separating structure is suboptimal compared to an approach where the front-end is allowed to exchange information with the decoders (e.g., [10]). However, the user-separating approach is less complex and could be more relevant from a practical standpoint. In addition, we are interested in comparing CDMA with orthogonal multiaccess on the uplink; since orthogonal multiaccess schemes do not need to employ joint decoding, the single-user decoding restriction on CDMA allows for a reasonably fair comparison in terms of complexity.

We assume throughout that the spreading sequences are random and independent across the users. However, for analytical convenience, we adopt the canonical model where the received signal is symbol synchronous across all the users as well as the receive antennas. At the same time, sequences can indeed be assumed to be synchronous and controllable across the *transmit* antennas of each user, which introduces the problem of sequence selection. In this paper, we compare two extreme cases for this selection—using the same sequence and using independent random sequences across the transmit antennas. Such a comparison gives insight into the effect of sequence correlation on performance. In addition, we show that the same sequence system is a natural extension of a single-user multi-input multi-output (MIMO) system to the CDMA scenario, especially when the MF detector is used.

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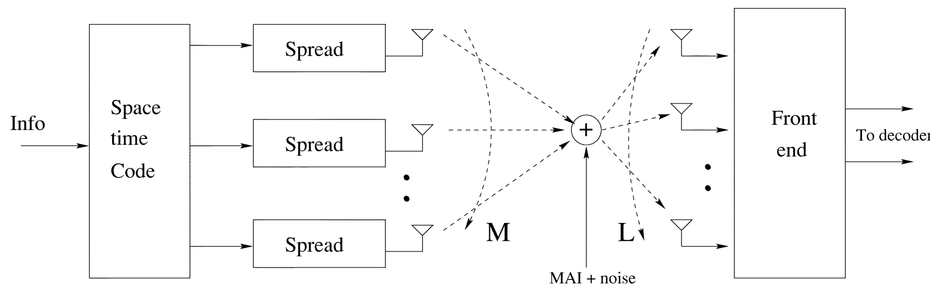

Fig. 1. CDMA schematic with multiple antennas.

Our criterion for performance is the total spectral efficiency achieved over all the users. We consider the spectral efficiency under an *ergodic* scenario, i.e., when the coherence time of the channel for each user is much smaller than the codeword length and in an *outage* framework, i.e., when the channel realizations are fixed across the codeword. To analyze the spectral efficiencies, we use recent techniques for the asymptotic analysis of such systems [11], [12]. The large system asymptote consists of letting the number of users (K) and the spreading factor (N) go to infinity with K/N kept constant. However, the number of transmit and receive antennas are kept finite. Similar analysis with multiple antennas at only the receiver is considered in [13] and [14]. Our analysis considers the more complicated situation with multiple transmit antennas as well. Furthermore, since we are considering wideband CDMA systems, we allow for frequency-selective fading and study the interplay between frequency diversity and spatial multiplexing across multiple transmit antennas.

As mentioned earlier, we are interested in comparing the spectral efficiencies achievable with CDMA to those achievable with orthogonal multiaccess. For the Gaussian scalar multiaccess channel, it is known that orthogonal multiaccess does not entail a loss in capacity for equal-power equal-rate users [15], while CDMA with random spreading and linear detection leads to a capacity penalty [12]. Comparison of CDMA and orthogonal multiaccess for scalar fading channels is considered in [16], where it is shown that CDMA with joint decoding (and no spreading) is advantageous in a single-cell scenario. The effect of fading and multiple receive antennas on CDMA with random spreading and MF/MMSE detection is studied in [14]. The results can be immediately compared to those obtained in [2] for orthogonal multiaccess with multiple receive antennas, and we do so in Section V of this paper. In addition, our results on CDMA spectral efficiency allow us to extend the comparison to the case where there are multiple antennas at the transmitter as well.

The paper is organized as follows. The CDMA system model is described in Section II, while the detectors and performance measures are described in Section III. The results from the analysis are provided in Section IV and the effect of various factors are studied within the CDMA framework. The comparison with orthogonal multiaccess is addressed in Section V and conclusions are provided in Section V.

The notation used throughout the paper is as follows: vectors are denoted by boldface lowercase letters; matrices are denoted by boldface uppercase letters; the variables k, m, ℓ, p are used to index the K users, M transmit antennas (of each user), L receive antennas and L_p multiple paths (of each user), respectively; the index p is also used as a superscript for the relevant quantities while k, m and ℓ are used as subscripts.

II. CDMA System Model and Assumptions

A schematic for a single user in a multiantenna DS/CDMA system that we consider is shown in Fig. 1, where the user is assumed to have M transmit antennas and L receive antennas. The schematic is easily extended to incorporate multiple users. The input data stream for each user is, in general, encoded using a space-time code onto the M antennas. As mentioned in Section I, we assume that the received signals of all the users are symbol synchronous. The general discrete-time model for the received signal at antenna ℓ over one symbol interval can then be written as

$$\boldsymbol{y}_\ell = \sum_{k=1}^{K} \boldsymbol{X}_{k,\ell} \boldsymbol{b}_k + \boldsymbol{w}_\ell, \qquad \ell = 1, \dots, L \tag{1}$$

where K is the number of users and $\boldsymbol{b}_k$ is the vector of M space-time code-symbols. The vector $\boldsymbol{w}_\ell$ is complex circularly Gaussian with covariance matrix $\sigma^2 \boldsymbol{I}_N$ and $\{\boldsymbol{w}_\ell\}$ are independent across ℓ. The $N \times M$ matrix $\boldsymbol{X}_{k,\ell}$ includes the effect of the spreading operation as well as the channel between user k and the receive antenna ℓ. We consider a system where each of the M coded symbols is spread by a sequence of length N. Furthermore, since we are considering a wideband CDMA system, we allow for frequency selective fading, so that each symbol is received over multiple resolvable paths. In practice, these paths would occur at different delays, but we assume that all the paths are synchronous for the sake of simplicity (see also [17]). Consequently, the matrix is

$$\boldsymbol{X}_{k,\ell} = \sum_{p=1}^{L_p} \boldsymbol{C}_{k\ell}^{(p)} \boldsymbol{\Lambda}_{k\ell}^{(p)} \boldsymbol{D}_k$$

where L_p is the number of resolvable paths and $\boldsymbol{C}_{k\ell}^{(p)}$ is the $N \times M$ matrix whose columns are the spreading sequences corresponding to $\boldsymbol{b}_k$ along the path p. The matrix $\boldsymbol{\Lambda}_{k\ell}^{(p)}$ is $M \times M$ diagonal and consists of the channel gains from the M transmit antennas to receive antenna ℓ along the path p. Similarly, the matrix $\boldsymbol{D}_k$ is $M \times M$ diagonal and consists of the amplitudes assigned to each of the M transmit antennas of user k (the phases can be absorbed into $\boldsymbol{\Lambda}_{k\ell}^{(p)}$).

We assume that the channel gains across the different paths are independent. Similarly, the receive antennas, as well as the transmit antennas for each user, are assumed to be well-separated in relation to the richness of the scattering environment. Hence, we model the channel gains as being independent across the antennas. Furthermore, the channel gains can be assumed to be independent across the users since we are considering the uplink. The channel gains for user k are normalized so that the sum of the variances across the paths is equal to 1 for each transmit-receive antenna pair. Thus we assume that frequency diversity does not result in a power gain, while multiple antennas at the receiver do give a power gain (or array gain), as may be the case in a physical scenario when CSI is available at the receiver. Finally, we assume that the channel gains are identically distributed, for the sake of brevity.

The spreading sequence from each transmit antenna is assumed to consist of N independent and identically distributed (i.i.d.) zero-mean, variance $1/N$ entries with finite eighth moments. In practice, the sequences across the different paths are related through shifts. However, we assume that the sequence matrices $\boldsymbol{C}_{k\ell}^{(p)}$ are independent for $p = 1, \ldots, L_p$. This independence assumption across paths is made for analytical simplicity (as in [17]) and can be verified to be reasonable through numerical experiments. Across the receive antennas, a good model, when the multiple antennas are co-located at the same base station, is to let the received sequence matrices be identical, i.e., $\boldsymbol{C}_{k\ell}^{(p)} = \boldsymbol{C}_{k}^{(p)}$, for $\ell = 1, 2, \ldots, L$.

III. Receiver Structures and Performance Measures

Define $\boldsymbol{y} = [\boldsymbol{y}_1^\top, \ldots, \boldsymbol{y}_L^\top]^\top$ as the overall observation vector of length LN over one symbol interval. Since the symbol vector $\boldsymbol{b}_k$ of each user is the output of a codebook, the optimum receiver for joint decoding of the information bits of all the users must process the observations $\boldsymbol{y}$ across the whole blocklength of the code. In general, such an operation is prohibitively complex. For this reason and to allows for a reasonably fair comparison between CDMA and orthogonal multiaccess, we consider below a suboptimum receiver that consists of a linear front-end detector followed by autonomous single-user decoders, as shown in Fig. 1. We have considered the limitations of such a framework for CDMA in [18], where the performance of the suboptimal receivers is compared to that of the optimum receiver.

Without loss of generality, we focus on the front-end for user 1. By (1), we have

$$\boldsymbol{y} = \boldsymbol{X}_1\boldsymbol{b}_1 + \sum_{k=2}^{K} \boldsymbol{X}_k\boldsymbol{b}_k + \boldsymbol{w} \tag{2}$$

where $\boldsymbol{X}_k$ is an $LN \times M$ matrix formed by stacking the matrices $\boldsymbol{X}_{k,\ell}, \ell = 1, \ldots, L$ and we have separated out the contribution of user 1. The output of a general linear front-end $\boldsymbol{F}_1$ is

$$\boldsymbol{z}_1 = \boldsymbol{F}_1^\dagger \boldsymbol{y} = \boldsymbol{F}_1^\dagger \boldsymbol{X}_1 \boldsymbol{b}_1 + \boldsymbol{w}_I$$

where $\boldsymbol{w}_I = \sum_{k=2}^{K} \boldsymbol{F}_1^\dagger \boldsymbol{X}_k \boldsymbol{b}_k + \boldsymbol{F}_1^\dagger \boldsymbol{w}$.

The use of the linear front-end thus results in an effective single-user (ESU) space–time channel for each user. The system performance with this front-end can be characterized in terms of the capacities of the ESU channels of all the users. However, finding these capacities involves a joint maximization of the mutual informations $I(\boldsymbol{b}_k; \boldsymbol{z}_k), k = 1, \ldots, K$, over the distributions of $\boldsymbol{b}_k, k = 1, \ldots, K$, which appears to be an intractable problem in general. We consider instead the mutual informations $I(\boldsymbol{b}_k; \boldsymbol{z}_k)$ under the assumption that the $\boldsymbol{b}_k$'s are i.i.d Gaussian across users as well as transmit antennas, which serve as lower bounds to the capacities of the ESU channels. Note that the assumption that $\boldsymbol{b}_k$ is a Gaussian vector with i.i.d. entries can be justified in orthogonal multiaccess systems[1] [1], [2]. We apply these assumptions in the CDMA scenario, with the understanding that coding and decoding are done on a single-user basis and to allow for a comparison with orthogonal multiaccess systems in Section V. In Section IV, we give a further justification for this assumption for the specific scenarios considered.

Assuming that $\boldsymbol{b}_k$'s are i.i.d. Gaussian and fixing $\{\boldsymbol{X}_k\}$, the mutual information of the effective single user (ESU) channel is given by[2]

$$I(\boldsymbol{b}_1; \boldsymbol{z}_1) = \log \frac{\left|\boldsymbol{F}_1^\dagger (\boldsymbol{P} + \boldsymbol{Q}) \boldsymbol{F}_1\right|}{\left|\boldsymbol{F}_1^\dagger \boldsymbol{Q} \boldsymbol{F}_1\right|} \tag{3}$$

where $|\cdot|$ denotes the determinant and

$$\boldsymbol{P} = \boldsymbol{X}_1\boldsymbol{X}_1^\dagger \quad \text{and} \quad \boldsymbol{Q} = \sum_{k=2}^{K} \boldsymbol{X}_k \boldsymbol{X}_k^\dagger + \sigma^2 \boldsymbol{I}_{LN}. \tag{4}$$

Since we assume that perfect CSI is available at the receiver, the matrices $\{\boldsymbol{X}_k\}_{k=1}^K$ are known and $\boldsymbol{F}_1$ can be a function of these matrices. In our analysis, we focus on two front-ends that are of significant interest. First, we consider the conventional MF detector (for the vector symbol $\boldsymbol{b}_1$) that does not need any information regarding the interferers

$$\boldsymbol{F}_1^{\text{mf}} = \boldsymbol{X}_1. \tag{5}$$

Note that, with receive diversity, i.e., $L > 1$, MF detection corresponds to despreading followed by maximal ratio combining. The mutual information of the ESU is obtained from (3) to be

$$I_{\text{mf}} = \log \left| \boldsymbol{I}_M + \left(\boldsymbol{X}_1^\dagger \boldsymbol{Q} \boldsymbol{X}_1\right)^{-1} \boldsymbol{X}_1^\dagger \boldsymbol{P} \boldsymbol{X}_1 \right| \tag{6}$$

where $\boldsymbol{P}$ and $\boldsymbol{Q}$ are as given in (4).

We compare the MF detector with a straightforward generalization of the multiuser minimum mean-squared (MMSE) detector for single antenna systems

$$\boldsymbol{F}_1^{\text{mmse}} = (\boldsymbol{P} + \boldsymbol{Q})^{-1} \boldsymbol{X}_1. \tag{7}$$

The corresponding mutual information is given by

$$I_{\text{mmse}} = \log \frac{|\boldsymbol{P} + \boldsymbol{Q}|}{|\boldsymbol{Q}|} = \log \left| \boldsymbol{I}_M + \boldsymbol{X}_1^\dagger \boldsymbol{Q}^{-1} \boldsymbol{X}_1 \right|. \tag{8}$$

Note that this mutual information also equals $I(\boldsymbol{b}_1; \boldsymbol{y})$, the mutual information prior to any front-end detection. Thus, with Gaussian code-symbols, the MMSE front-end is the "optimum" user-separating front-end in that it maximizes the mutual information between the input and output of the ESU channels [19].

[1] In particular, such independence across antennas does not, in general, imply optimality of independent scalar coding across antennas for finite blocklengths.

[2] Unless otherwise mentioned, all logarithms in this paper can be taken to have base 2.

Thus far, we have considered the mutual information for fixed $\{\boldsymbol{X}_k\}_{k=1}^K$. However, since $\boldsymbol{X}_k$ is a function of the random spreading sequences and channel gains, it is of greater interest to study the behavior of $I(\boldsymbol{b}_1;\boldsymbol{z}_1)$ as a random variable. We consider two performance measures based on the mutual information in (3): the average of the mutual information and its cumulative distribution function (cdf). The average mutual information for user k is simply

$$C_k = E\left[I(\boldsymbol{b}_k;\boldsymbol{z}_k)\right] \tag{9}$$

where the expectation is over the random matrices $\{\boldsymbol{X}_k\}_{k=1}^K$. The rate C_k can be achieved when the blocklength of the code is large compared to the coherence time of the channel. The cdf of the distribution of $I(\boldsymbol{b}_k;\boldsymbol{z}_k)$ is closely related to the notion of information outage and is a suitable measure when the matrices $\{\boldsymbol{X}_k\}$ are fixed over the time-scale of interest [1]. The achievable rate at an outage level p_0 of the ESU channel is then defined as the maximum value of R_k such that

$$P\left\{I\left(\boldsymbol{b}_k;\boldsymbol{z}_k\right) < R_k\right\} < p_0. \tag{10}$$

Finally, we can characterize the system-wide performance through the sum of the above information rates over all the users normalized to the processing gain N. This gives the spectral efficiency of the system for the ergodic (η_e) and outage cases ($\eta_o(p_0)$) as

$$\eta_e = \frac{1}{N}\sum_{k=1}^K C_k \text{ and } \eta_o\left(p_0\right) = \frac{1}{N}\sum_{k=1}^K R_k \quad \text{bits/chip}$$

respectively. We study the asymptotic limits of the above measures when K and N both go to infinity with K/N tending to a constant α and denote this limit as simply "$N \to \infty$."

IV. CDMA ANALYSIS

With a single transmit antenna and random spreading sequences, the convergence of the mutual information for each user (for MF and MMSE detectors) follows from the results in [11] and [12]. When there are multiple transmit antennas, it is reasonable to assume that we can control the powers and sequences across the antennas. Naturally, the power allocation and the correlation of the sequences would influence performance. Since there is no CSI at the transmitter, we assume that the power is equally divided across the transmit antennas of each user, so that $\boldsymbol{D}_k = \sqrt{(P_k/M)}\boldsymbol{I}_M$, where P_k is the total power of user k. For the sequences, we consider two extreme cases: using different sequences across the antennas and using the same sequence across all the antennas. Specifically, the sequences are chosen random but independent across the antennas in the former case, while a single random sequence is repeated in the latter case. The assumption of *random* sequences across the antennas is clearly restrictive, but would yield asymptotic closed-form results and serve as a benchmark for other sequence selections. For example, the results for the independent sequence system may be expected to be representative of having orthogonal sequences and a random cover (to randomize across the users), since the M random sequences of each user become orthogonal in the asymptote.

With the above random sequence models and in the large system asymptote, Central Limit Theorem arguments (that generalize the results in [20] for single antenna systems) may be used to conclude the following. For a large class of front-end detectors that includes both the MMSE (see also [21]) and MF detectors, the interference at the output of the front-end for user k converges weakly to a Gaussian distribution as $N \to \infty$, irrespective of the individual distributions of the code symbol vectors of the interferers. It follows that $I(\boldsymbol{b}_k;\boldsymbol{z}_k)$ would be maximum when $\boldsymbol{b}_k$ is a (possibly correlated) Gaussian vector. In addition, the two sequence selections that we consider above are symmetric across the transmit antennas, which gives further justification to our assumption in Section III that $\boldsymbol{b}_k$ consists of i.i.d. Gaussian entries. Keeping the above observations in mind, we refer to the asymptotes of the rates C_k in (9) and R_k in (10), as the ergodic capacity and the outage capacity of the ESU channels, respectively. Finally, we assume that all the users have equal powers, i.e., $P_k = P, \forall k$. The results are easily extended to the case with unequal user powers, but we make this assumption for simplicity in the exposition.

A. MF Detector

Since there is no coupling of the contributions from each interferer at the output of the MF detector, the asymptotic analysis of the mutual information in (6) is relatively straightforward. Define $\boldsymbol{H}_1$ as the $M \times LL_p$ matrix containing all channel gains for user 1, along the L_p paths for each of the ML transmit and receive antenna pairs. We then have the following result for the two sequence selections being considered.

Proposition 1: As $N \to \infty$, the mutual information $I(\boldsymbol{b}_1;\boldsymbol{z}_1)$ converges, for almost every realization of the sequences and channel gains, to a limit that is independent of the sequence realization.

When the sequences are independent across the transmit antennas, the limit is

$$I^\star_{\text{mf,ind}}(P,\alpha) = \log\left|1 + \frac{\gamma}{M}\sum_{\ell=1}^{L}\sum_{p=1}^{L_p}\boldsymbol{\Lambda}_{1\ell}^{(p)}\boldsymbol{\Lambda}_{1\ell}^{(p)^\dagger}\right|. \tag{11}$$

When the sequences are the same across the transmit antennas, the limit is

$$I^\star_{\text{mf,same}}(P,\alpha) = \log\left|1 + \frac{\gamma}{M}\boldsymbol{H}_1\boldsymbol{H}_1^\dagger\right|. \tag{12}$$

The quantity γ in the above equations is given by

$$\gamma = \frac{P}{\sigma^2 + \alpha P}.$$

Proof: [Outline] The mutual information is given by (6). Since $\boldsymbol{X}_1^\dagger\boldsymbol{P}\boldsymbol{X}_1 = (\boldsymbol{X}_1^\dagger\boldsymbol{X}_1)^2$, we first investigate $\boldsymbol{X}_1^\dagger\boldsymbol{X}_1$. By the strong law of large numbers (SLLN)

$$\begin{aligned}\boldsymbol{X}_1^\dagger\boldsymbol{X}_1 =& \frac{P}{M}\sum_{\ell=1}^{L}\sum_{p=1}^{L_p}\sum_{p'=1}^{L_p}\boldsymbol{\Lambda}_{k\ell}^{(p)^\dagger}\boldsymbol{C}_k^{(p)^\dagger}\boldsymbol{C}_k^{(p')}\boldsymbol{\Lambda}_{k\ell}^{(p')}\\ &\to_{a.s}\frac{P}{M}\sum_{\ell=1}^{L}\sum_{p=1}^{L_p}\boldsymbol{\Lambda}_{k\ell}^{(p)^\dagger}\boldsymbol{R}\boldsymbol{\Lambda}_{k\ell}^{(p)}\end{aligned} \tag{13}$$

where $\boldsymbol{R} = \boldsymbol{I}_M$ for the independent sequence system and $\boldsymbol{1}\boldsymbol{1}^\dagger$ for the same sequence system, while $\to_{a.s}$ denotes almost sure convergence element-wise. Consider now the term $\boldsymbol{X}_1^\dagger \boldsymbol{Q} \boldsymbol{X}_1$. We have

$$\begin{aligned}\lim_{N\to\infty} \boldsymbol{X}_1^\dagger \boldsymbol{Q} \boldsymbol{X}_1 &= \lim_{N\to\infty} \sigma^2 \boldsymbol{X}_1^\dagger \boldsymbol{X}_1 + \frac{P}{M}\sum_{k=1}^{K} \boldsymbol{X}_1^\dagger \boldsymbol{X}_k \boldsymbol{X}_k^\dagger \boldsymbol{X}_1 \\ &= \lim_{N\to\infty} \sigma^2 \boldsymbol{X}_1^\dagger \boldsymbol{X}_1 + \frac{P}{M}\sum_{k=1}^{K} \boldsymbol{X}_1^\dagger \left(E\boldsymbol{X}_k \boldsymbol{X}_k^\dagger\right) \boldsymbol{X}_1\end{aligned}$$

where we have fixed $\boldsymbol{X}_1$ and applied the SLLN element-wise. It is easy to show that

$$E\boldsymbol{X}_{k,\ell}\boldsymbol{X}_{k,\ell'}^\dagger = \frac{P}{N}\boldsymbol{I}_N \delta_{\ell\ell'}$$

where $\delta_{\ell\ell'} = 1$ if $\ell = \ell'$ and 0 otherwise. Consequently

$$\boldsymbol{X}_1^\dagger \boldsymbol{Q} \boldsymbol{X}_1 \to_{a.s} \left(\alpha P + \sigma^2\right) \boldsymbol{X}_1^\dagger \boldsymbol{X}_1.$$

Combining the above limit with (13), we get the mutual informations in Proposition 1. □

We make the following remarks on the above result:

1) The randomness in $I^\star(P,\alpha)$ is only due to the channel gains of user 1. All other parameters are captured in the deterministic limit γ/M which is the effective signal-to-interference ratio (SIR) at the output corresponding to a symbol from each transmit antenna of user 1. Note that γ is equal for both sequence selections and is independent of M and L.
2) It follows that the asymptotic ergodic capacity of the ESU channel for user 1 is given by $C(P,\alpha) = E[I^\star(P,\alpha)]$, where the expectation is over the channel gains of user 1. The asymptotic outage capacity of the ESU channel at a level p_0 is given by the maximum $R(P,\alpha)$ such that $P\{I^\star(P,\alpha) < R\} < p_0$. The ergodic and outage spectral efficiencies of the system are given by
$$\eta_e = \alpha C(P,\alpha) \text{ and } \eta_o(p_0) = \alpha R(P,\alpha) \tag{14}$$
respectively.
3) The matrices $\boldsymbol{\Lambda}_{1\ell}^{(p)}$ are diagonal. Hence, the mutual information for the independent sequence system can also be written as
$$I^\star_{\text{mf,ind}}(P,\alpha) = \sum_{m=1}^{M} \log\left(1 + \frac{\gamma}{M}G_m\right) \tag{15}$$
where G_m is the sum of LL_p independent variables, each of which is the squared magnitude of a channel gain. (Hence, each of the LL_p random variables has a mean $1/L_p$.) Thus, the use of independent sequences reduces the ESU channel to a set of M parallel channels. Since these parallel channels are independent and identically distributed, it follows that ergodic capacity is simply $ME[\log(1 + (\gamma/M)G)]$. Note that this ergodic capacity can be achieved with only independent scalar coding for each of the transmit antennas. On the other hand, achieving the outage capacity does require coding across the antennas. However, since we have a set of parallel channels, decoding would be much simpler, without involving cancellation of the interference between the data streams from different transmit antennas.
4) For the same sequence system with *flat* fading, i.e., $p = 1$, $I^\star(P,\alpha)$ reduces to
$$I^\star_{\text{mf,same}}(P,\alpha) = \log\left|1 + \frac{\gamma}{M}\boldsymbol{H}_1\boldsymbol{H}_1^\dagger\right|$$
where $\boldsymbol{H}_1$ is an $M \times L$ matrix of channel gains. It is important to note the similarity of this mutual information with that studied in the single-user case (e.g., [1], [2]). Thus, the use of the same sequence reduces the ESU channel to a single-user MIMO channel whose capacity can be achieved only through joint decoding of the data streams from different transmit antennas, see, e.g., [22].

Because of the final remark above, our interest in comparing same and independent sequence selections is further justified. Keeping all other parameters fixed, such a comparison can be summarized in the following results.

Corollary 1: For every realization of the channel gains

$$\begin{aligned} I^\star_{\text{mf,same}}(P,\alpha) &< I^\star_{\text{mf,ind}}(P,\alpha) \\ \lim_{L_p\to\infty} I^\star_{\text{mf,same}}(P,\alpha) &= \lim_{L_p\to\infty} I^\star_{\text{mf,ind}}(P,\alpha). \end{aligned} \tag{16}$$

The first inequality in (16) above is a direct consequence of Hadamard's inequality while the second result is a consequence of the strong law of large numbers applied to the summation across paths.

Corollary 1 implies that both the ergodic and outage capacities of each user are larger for the independent sequence system. It follows that the overall spectral efficiencies η_e and $\eta_o(p_0)$ in (14) are larger as well. While the above comparison was made at a fixed code symbol SNR P/σ^2 for each user, it is easy to show that the comparison is unaffected when the energy per information bit (normalized to the noise level) $\gamma_b = \mathcal{E}_b/N_0$ is fixed, so that P/σ^2 varies with the rate achieved by the user as

$$\frac{P}{\sigma^2} = \begin{cases} C\gamma_b & \text{(ergodic)} \\ R\gamma_b & \text{(outage).}\end{cases}$$

The above discussion clearly establishes that using independent sequences, which also results in simpler coding and decoding, is preferable to spreading with the same sequence. The conclusion may appear somewhat uninteresting, since there seems to be no scenario under which using the same sequence offers any advantage and a natural question is whether there exists some other sequence selection that performs better than using independent sequences. While we do not address the problem of optimal sequence selection, we note the following. The proof of Proposition 1 holds for any selection of $\boldsymbol{C}_k$ so long as the sequence at each transmit antenna consists of i.i.d. entries, with an appropriate choice of the matrix $\boldsymbol{R}$. And the independent sequence system would outperform any such selection.

The above arguments strongly motivate the use of independent sequences as a benchmark for the MF detector. With this sequence selection, we illustrate the outage spectral efficiencies as a function of α in Fig. 2 for different values of M and L and an outage level of $p_0 = 0.05$ and $\gamma_b = 10$ dB. All the numerical results presented in this paper are for Rayleigh fading, i.e., for

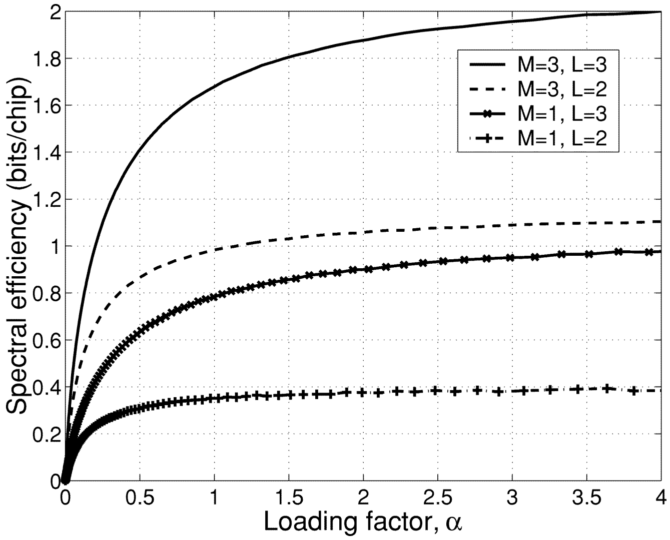

Fig. 2. Spectral efficiency for MF detector—outage, $\gamma_b = 10$ dB, $L_p = 1$.

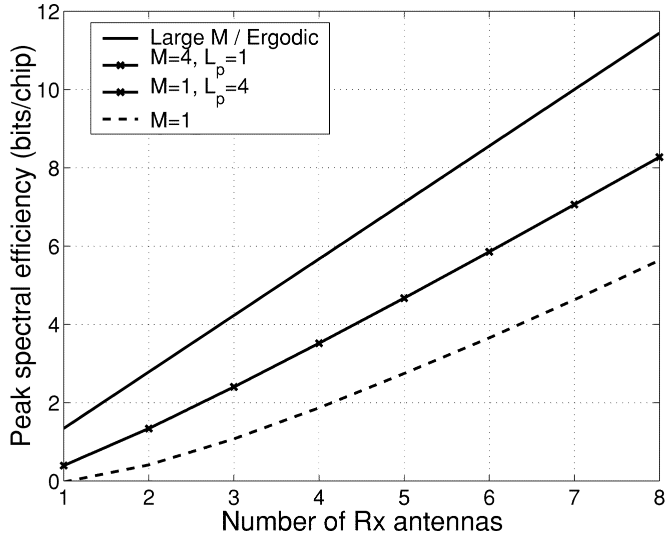

Fig. 3. Peak spectral efficiency—outage and ergodic $\gamma_b = 10$ dB. Note that M and L_p are interchangeable, so that the middle curve corresponds to both $M = 4, L_p = 1$ and $M = 1, L_p = 4$.

channel gains that are zero-mean complex, circularly Gaussian random variables.

We turn next to the effect of the number of antennas on the spectral efficiency of the independent sequence system. As seen in Fig. 2, the spectral efficiency increase with M and L for a fixed loading factor α. However, it is reasonable to assume that α is a design choice and it is then clearly of interest to operate the system at the peak spectral efficiency and study the choice of M (and L) needed to achieve it. Let us denote the peak ergodic and outage spectral efficiencies by $\eta_e^{\max}$ and $\eta_o(p_0)^{\max}$, respectively. The peak spectral efficiency exhibits slightly different behavior for ergodic and outage cases and it is easy to show the following (see also [14]). The proof is skipped for the sake of brevity.

Proposition 2: (Independent sequences) The spectral efficiencies η_e and $\eta_o(p_0)$ achieve their supremum as $\alpha \to \infty$ and

$$\eta_e^{\max} = L \log_2 e - \frac{1}{\gamma_b} \text{ bits/chip} \tag{17}$$

is independent of M, while

$$\eta_o(p_0)^{\max} = F^{-1}(p_0) \log_2 e - \frac{1}{\gamma_b} \text{ bits/chip.} \tag{18}$$

Here, $F(x)$ is the cdf of $X = (1/M)\sum_{m=1}^{M} G_m$ and G_m is as in (15).

Since the peak spectral efficiencies are obtained as $\alpha \to \infty$, N must be made much smaller than K in a finite system. Thus, spreading is not beneficial for MF detection and all bandwidth redundancy should be allocated to coding, an observation which is consistent with the single antenna case [19]. Also, $\eta_e^{\max}$ and $\eta_o(p_0)^{\max}$ for the same sequence system can be shown to be equal to the respective values for the independent sequence system given by Proposition 2. Thus, peak spectral efficiencies may be insensitive to sequence selections and conclusions drawn from the independent sequence analysis may hold more generally.

Now, Proposition 2 shows that $\eta_e^{\max}$ increases linearly in L. Thus, receive antenna diversity has a pronounced benefit for the system. On the other hand, the value of $\eta_e^{\max}$ in a system with K (equal power) users and M transmit antennas is equal to that in a system with KM users and one antenna. Thus, $\eta_e^{\max}$ is independent of M and having multiple transmit antennas may not be beneficial in the ergodic scenario.

However, the same conclusion cannot be made with the outage criterion, where the peak spectral efficiency does depend on M. In fact, for low values of the outage level p_0, $\eta_o(p_0)^{\max}$ increases with M till it approaches $\eta_e^{\max}$. This follows because the random variable X in Proposition 2 converges to L as $M \to \infty$. The variation of $\eta_o(p_0)^{\max}$ and $\eta_e^{\max}$ are shown in Fig. 3 for an outage level of $p_0 = 0.05$. As the outage level p_0 decreases, $\eta_o(p_0)$ decreases as well. But since the large M limit is deterministic and independent of p_0, it follows that larger M will have a more pronounced effect at lower outage levels.

On the other hand, note that X is in fact a sum of MLL_p random variables, normalized by ML_p. Thus M and L_p can be interchanged without affecting the peak spectral efficiency. In other words, the stability in the channel achieved by frequency diversity can replace similar effects due to multiple transmit antennas. If L_p is sufficiently large, additional transmit antennas may not improve performance significantly. On the other hand, increasing the number of receive antennas L has a decisive benefit for outage spectral efficiency as well.

B. MMSE Detector

For the MMSE detector, the mutual information of the corresponding ESU channel is given by (8)

$$I_{\text{mmse}} = \log \left| \boldsymbol{I}_M + \boldsymbol{X}_1^\dagger \boldsymbol{Q}^{-1} \boldsymbol{X}_1 \right|$$

where $\boldsymbol{Q}$ is defined in (4). The coupling between the interferers makes the analysis of the MMSE detector much more complicated. Asymptotic analysis using results from random matrix theory [23] on the eigenvalue distribution of the matrix $\boldsymbol{Q}$ was performed in [11]–[14] and [17]. The new aspect of the problem here is the introduction of multiple transmit antennas. However, for the case when we have independent sequences across the transmit antennas, the analysis becomes a straightforward application of results in [13] and [17], since the system is equivalent to that with KM users, a processing gain of LN and a modified power distribution. The asymptotic mutual information converges to

$$I^{\star}_{\text{mmse,ind}}(P,\alpha)=\sum_{m=1}^{M}\log\left(1+\frac{\gamma}{M}G_m\right) \tag{19}$$

where G_m is as in (15). The asymptotic SIR is given by an implicit equation

$$\gamma=\frac{P}{\sigma^2+\frac{\alpha PM}{L}E_G\left[\frac{G}{M+G\gamma}\right]} \tag{20}$$

where G has the same distribution as G_m in (15). In applying the results from [13], we note two technical differences from the rest of the results in this paper. First, the sequences would have to be explicitly assumed to be independent across the receive antennas when $L_p > 1$. However, the rigorous derivation in [13] for $L_p = 1$ justifies the assumption for $L_p > 1$ as well. Secondly, the convergence to (19) and (20) is in probability, while we have considered convergence in the almost sure sense elsewhere in this paper.

When we have the same sequence across the transmit antennas, the analysis is further complicated by the presence of dependent entries in the matrix $\boldsymbol{Q}$ because of the sequence repetition at the transmitter. Across the multiple paths or receive antennas, independence of sequences is a good assumption, yielding the same performance as in the dependent case [13], [17]. However, repetition across the transmit antennas changes the performance from that of the independent sequence system and it appears that analysis can only be done under some restrictive assumptions. Here, we provide results in two scenarios, which are obtained by similar applications of the results on the limiting eigenvalue distributions of random matrices.

Proposition 3:

1) With one receive antenna ($L = 1$), the mutual information of the same sequence system converges, almost surely, to the random variable

$$I^{\star}_{\text{mmse,same}}(P,\alpha)=\log\left|\boldsymbol{I}_M+\frac{\gamma}{M}\boldsymbol{H}_1\boldsymbol{H}_1^{\dagger}\right| \tag{21}$$

where

$$\begin{aligned}\gamma=&\frac{P}{\sigma^2+\frac{\alpha P}{\gamma}(L_p-f(\gamma))}\\ f(\gamma)=&E_{\boldsymbol{H}}\text{Tr}\left\{(\boldsymbol{I}_{L_p}+\frac{\gamma}{M}\boldsymbol{H}^{\dagger}\boldsymbol{H})^{-1}\right\}\end{aligned} \tag{22}$$

and $\boldsymbol{H}_1$ is an $M \times L_p$ channel gain matrix for user 1 and $\boldsymbol{H}$ has the same distribution as $\boldsymbol{H}_1$.

2) For any value of L

$$\begin{aligned}\lim_{N\to\infty}\lim_{M\to\infty}I_{\text{mmse,same}}=&LL_p\log\left(1+\frac{\gamma}{L_p}\right)\\ =&\lim_{M\to\infty}\log\left|\boldsymbol{I}_M+\frac{\gamma}{M}\boldsymbol{H}_1\boldsymbol{H}_1^{\dagger}\right|.\end{aligned} \tag{23}$$

The SIR γ is equal to that in a system with $M = L = 1$, KL_p users at power P/L_p and no fading

$$\gamma=\frac{P}{\sigma^2+\alpha L_p\frac{P}{L_p+\gamma}}. \tag{24}$$

Proof: See Appendix A. □

We make the following remarks regarding the above results.

1) When $\boldsymbol{H}$ has i.i.d. complex Gaussian entries, the function $f(\gamma)$ can be expressed in terms of the eigenvalue distribution of the Wishart matrix $\boldsymbol{H}^{\dagger}\boldsymbol{H}$ [2].
2) The quantity in the last equality in (23) also represents the asymptotic mutual information achievable, for all M and L, with an MMSE detector that knows the sequences and powers of the interferers but does not know their channel gains. It thus serves as a lower bound on the mutual information with the MMSE detector considered here and may be of some interest in its own right.

While it is not completely general, Proposition 3 gives much of the insight into the comparison between the same sequence and independent sequence systems for the MMSE detector. We first have the following result for $L = 1$, analogous to the second result in Corollary 1

$$\lim_{L_p\to\infty}I^{\star}_{\text{mmse,same}}(P,\alpha)=\lim_{L_p\to\infty}I^{\star}_{\text{mmse,ind}}(P,\alpha). \tag{25}$$

This follows because the matrix $\boldsymbol{H}\boldsymbol{H}^{\dagger}$ in (22) converges to $\boldsymbol{I}_M$ as $L_p \to \infty$. Thus, the same and independent sequence systems approach each other when there is enough frequency diversity. More generally, we may expect that channel randomization may make the performance less sensitive to sequence selections. Also, from (19), the ergodic capacity with independent sequences again corresponds to having M parallel channels for which coding and decoding can be performed separately for each antennas. For the same sequence system, space-time coding and joint decoding of the M transmitted streams would be needed. The performance variation with α for the two sequence selections is shown for $L_p = 1$ and $L_p = 3$ in Fig. 4. The values of M and L are set at three and one, respectively, while the value of γ_b is 10 dB.

We note that there is a crossover between the performance of the same and independent sequence systems with the MMSE detector, which is in contrast with the sequence comparison for the MF detector. This crossover can be explained as follows. At low loads, it is possible to separate the signals from the different transmit antennas of the desired user by using independent spreading sequences, while the same sequence system suffers from cross interference of these signals. However, at high loads, using independent sequences increases the apparent number of users and degrades the performance of the MMSE detector. Consequently, the sequence selection depends on the loading factor at which the system is operated. At the same time, note that the performance gap between the two systems is smaller for $L_p = 3$ than $L_p = 1$, so that frequency diversity may make performance less sensitive to sequence selections. Finally, Fig. 4 also shows performance obtained with finite system simulations using $N = 32$ and the match is reasonably good. In particular, when $L_p = 3$, the sequences across the paths are generated through cyclic shifts of a single sequence and the results validate the independence assumption across paths.

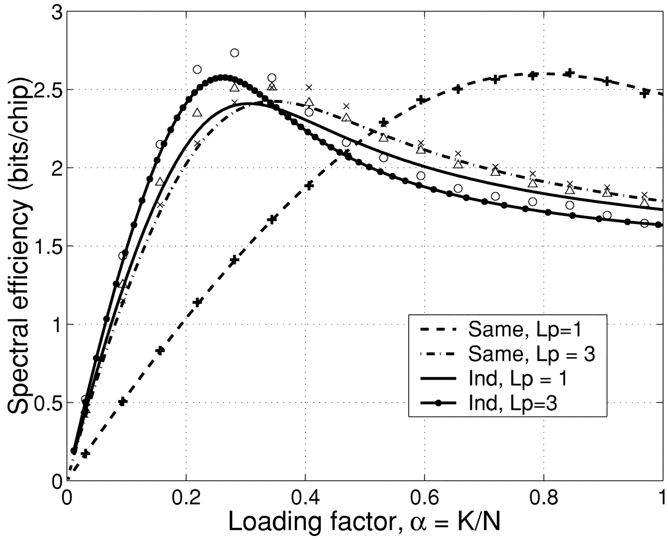

Fig. 4. Spectral efficiencies for MMSE detector—Ergodic, $\gamma_b = 10$ dB, $M = 3, L = 1$. The symbols "+," "×," "△," and "o" denote the corresponding simulation values with $N = 32$.

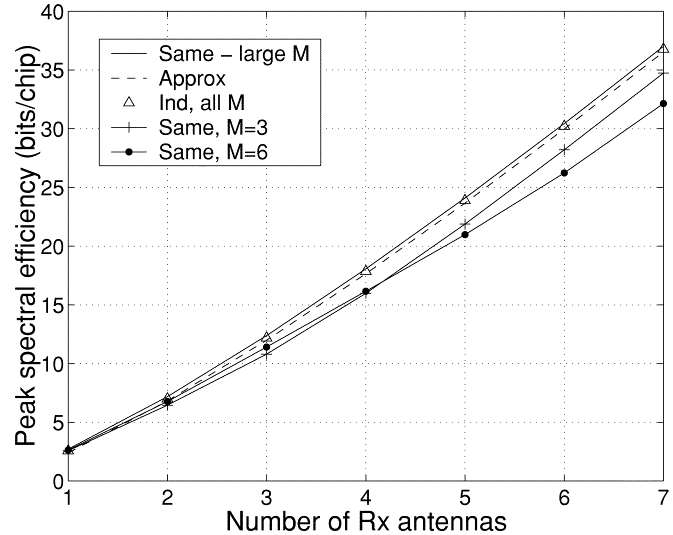

Fig. 5. Peak spectral efficiencies for MMSE detector—ergodic, $\gamma_b = 10$ dB, $L_p = 1$.

As with the MF detector, we focus on the peak spectral efficiency, assuming that α can be chosen to operate at the maximum in each case. Unlike for the MF detector, $\eta_e^{\max}$ occurs at a finite value of α for the MMSE detector, which advocates some bandwidth allocation to spreading in a finite system [19]. For $L = 1$ and $L_p = 1$, Fig. 4 shows that $\eta_e^{\max}$ for the same sequence system is slightly higher than that for the independent sequence system, suggesting a potential benefit from using the same sequence and space-time coding. We can characterize the peak spectral efficiency for the two sequence selections in greater detail as follows.

Proposition 4:

1) For the independent sequence system operating at fixed γ_b, the value of $\eta_e^{\max}$ does not depend on M.
2) For the same sequence system, the limiting (in M) $\eta_e^{\max}$ derived from (23) satisfies

$$\lim_{L\gamma_b \to \infty} \frac{\eta_e^{\max}}{L \log_2 e[\log(1 + L\gamma_b \log_2 e) - 1]} = 1. \tag{26}$$

3) As $L_p \to \infty$, $\eta_e^{\max}$ for the independent sequence system approaches the limiting (in M) $\eta_e^{\max}$ for the same sequence system derived from (23).

Proof: See Appendix B □

The value of $\eta_e^{\max}$ for the independent sequence system being equal for all M is analogous to the corresponding observation for the MF detector in Proposition 2. However, for the MMSE detector, this equality holds only with fixed $\mathcal{E}_b/N_0$ and not with fixed P/σ^2. The limit in (26) suggests the following approximation for $\eta_e^{\max}$:

$$\eta_e^{\max} \approx L \log_2 e[\log(1 + L\gamma_b \log_2 e) - 1]. \tag{27}$$

The approximation is good at large M and large $L\gamma_b$ for the same sequence system and at large L_p with independent se-quences. As discussed in Appendix B, the approximation may be expected to hold for the independent sequence system at large L as well. Note that the approximation does not depend on the fading distribution and that the variation for large L is $O(L \log L)$. The factor of $\log L$ arises due to our assumption that receive antennas result in an array gain.[3]

Fig. 5 shows the variation of $\eta_e^{\max}$ with L for $M = 1$ (and hence for all M) for the independent sequence system. For the same sequence system, $\eta_e^{\max}$ is shown for different values of M along with the large M limit obtained using (23). For $L > 1$, $\eta_e^{\max}$ for the same sequence system is obtained through numerical simulations. The approximation given in (27) is also shown. The key observations from Fig. 5 can be listed as follows.

- For a fixed value of L, the large-M limit with the same sequence is slightly better than that with independent sequences, at the value of γ_b considered.
- However, for sufficiently large L/M, the value of $\eta_e^{\max}$ for the same sequence system is smaller than that for the independent sequence system. In other words, as M increases with L fixed, the same sequence system overtakes the independent sequence system from below. However, since the limiting value is only slightly larger, we can conclude that improvements in $\eta_e^{\max}$ by using the same sequence are at best marginal.
- For the independent sequence system, as L_p increases, the ESU channel capacity increases, but the SIR falls due to the loss in multiuser diversity (see also [14]). The overall effect on $\eta_e^{\max}$ is a slight increase, since it approaches $\eta_e^{\max}$ derived from (23) as $L_p \to \infty$, by Proposition 4.
- Increasing the number of receive antennas always improves performance. The expression in (27) provides a good approximation for the increase with L, at the value of γ_b considered.

The variation of the outage spectral efficiency, $\eta_o(p_0)$, with α is similar to that in the ergodic case and we hence focus on the peak spectral efficiency, $\eta_o(p_0)^{\max}$. Fig. 6 shows the variation of $\eta_o(p_0)^{\max}$ with L. The sequence comparison and the effect of receive diversity show similar trends as with the ergodic case. The key difference is that increasing M can now increase

[3]This normalization must be contrasted with the one in [14], where the total energy across the receive antennas is kept fixed.

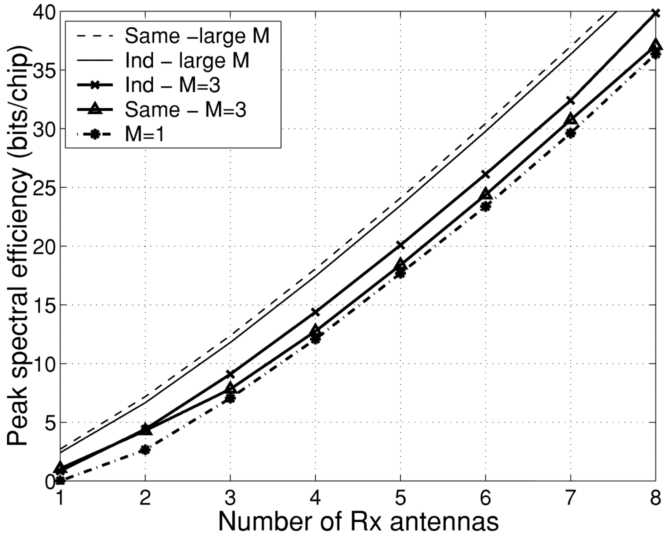

Fig. 6. Peak spectral efficiencies for MMSE detector—outage, $\gamma_b = 10$ dB, $L_p = 1$.

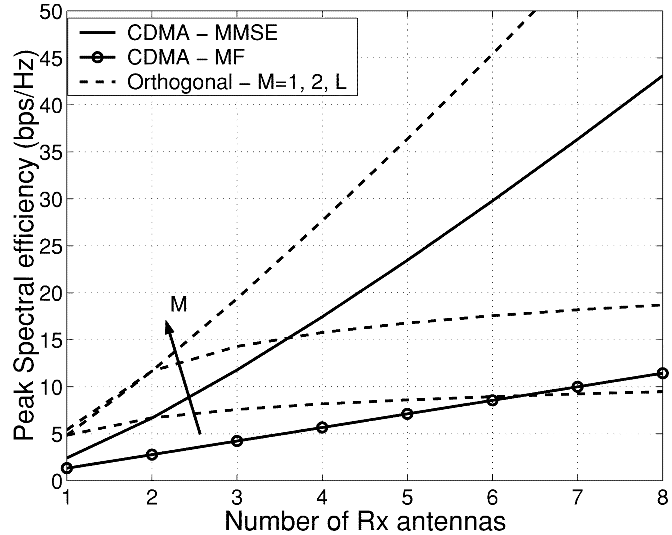

Fig. 7. Comparison with orthogonal systems—ergodic, $\gamma_b = 10$ dB.

$\eta_o(p_0)^{\max}$ for the independent sequence system. As $M \to \infty$, the value of $\eta_o(p_0)^{\max}$ for the independent sequence system converges to the corresponding ergodic curve, while $\eta_o(p_0)^{\max}$ for the same sequence system converges to $\eta_o(p_0)^{\max}$ derived from (23). These limiting curves are also shown in Fig. 6. While $\eta_o(p_0)^{\max}$ does improve with the M, we can again show that, with sufficiently large L_p, the gains achieved by increasing M may not be significant. Thus, benefits from multiple transmit antennas can alternately be achieved through increased frequency diversity.

V. Comparison With Orthogonal Systems

In this section, we compare CDMA with systems where the users are orthogonal in time (TDMA), frequency (FDMA), or some combination thereof. In terms of user coordination in the time-frequency plane, CDMA and orthogonal multiaccess systems can be considered to be extreme cases. For a single-antenna system and a single cell of users, the spectral efficiency of a random sequence CDMA system with $N > 1$ is known to be necessarily worse than orthogonal multiaccess. With one transmit antenna and L receive antennas, the results from [2] indicate that the spectral efficiency of an orthogonal system is logarithmic in L. For the CDMA system, using independent or orthogonal sequences across the transmit antennas would be a good choice, as discussed in Section IV. The corresponding peak spectral efficiencies are not sensitive to M, while they exhibit linear (or better) increases with L, as seen in (17) and (27). Thus, we note a clear contrast between orthogonal multiaccess and CDMA and it is of some interest to compare the relative performance with multiple antennas.

For orthogonal multiaccess, the relation between the ergodic spectral efficiency η and $\gamma_b = \mathcal{E}_b/N_0$ is given by the implicit equation

$$\eta = E \log \left| \boldsymbol{I}_M + \frac{1}{M} \eta \gamma_b \boldsymbol{H}\boldsymbol{H}^\dagger \right|$$

where $\boldsymbol{H}$ is an $M \times L$ matrix of channel gains between the transmit and receive antennas. The spectral efficiency for an outage scenario can be defined analogously. Here, we have assumed that the orthogonal system undergoes flat fading. This would be the case with a technique like FDMA, where the bandwidth occupied by each user is not large enough to cause frequency selective fading. If the underlying data rate is large enough to cause frequency selective fading, orthogonal techniques like orthogonal frequency-division multiplexing (OFDM) would be used to avoid complicated equalization at the receiver. Correspondingly, the CDMA system could use multiple carriers as well and the essence of the comparison can be captured by assuming a single carrier. In addition, we assume that the CDMA signal (on each carrier) undergoes flat fading, which is typically not true due to the increase in bandwidth by spreading. However, as seen in Section IV, assuming $L_p = 1$ is a slightly pessimistic assumption for MF or MMSE detection, since peak spectral efficiencies would benefit from frequency diversity.

The comparison of single carrier systems with flat fading is shown in Fig. 7 for a single cell ergodic scenario, for $\mathcal{E}_b/N_0 = 10$ dB and a varying number of transmit antennas. As L increases, we see that the value of $\eta_e^{\max}$, with even the MF, becomes larger than the spectral efficiency of the orthogonal system. This crossover is to be expected given the linear and logarithmic variations discussed above. In fact, the crossover can be deduced from the results for a single transmit antenna derived in [14]. An intuitive explanation for this crossover is that the CDMA system offers the possibility of multiplexing across the users whereas the orthogonal system does not. Furthermore, it can be shown that the crossover for the MF or MMSE detector is not significantly affected even when the users are scheduled so as to exploit multiuser diversity in the system, with the base station assigning the channel to the user whose ESU channel has the maximum rate. As M increases, $\eta_e^{\max}$ is unaffected for the MF and MMSE detectors, while the orthogonal system shows significant improvements.

For the outage criterion, the trends (not shown here) are similar to that in the ergodic scenario, except for a notable increase in the crossover point for the MF detector and the benefit derived by the CDMA system from increasing M. However, the

CDMA system is still much less sensitive to M than the orthogonal system, at the outage level of 0.05 and $\mathcal{E}_b/N_0$ of 10 dB considered, since the presence of interfering users serves to offset potential gains from multiple transmit antennas.

Note that the above CDMA analysis has been under the large system asymptote. However, the conclusions can be verified in a finite system as well, as done in Fig. 4. In particular, while peak spectral efficiencies for the MF detector occur as $\alpha \to \infty$, (which is not practicable in a finite system), the improvement over orthogonal multiaccess can be achieved with one transmit antenna and moderate values of α. Finally, while the performance of the MMSE detector is markedly better than that of the MF in Fig. 7 (and the outage results), the difference between the two detectors may be expected to be significantly reduced in the multicell scenario [19].

VI. Conclusion

We can summarize our observations regarding sequence selection across the transmit antennas in the CDMA system, the effect of the number of antennas and the comparison of CDMA to orthogonal multiaccess as follows.

- For CDMA, the independent sequence system has larger spectral efficiency than the same sequence system for the MF detector. The sequence comparison depends on the loading factor for the MMSE detector, with the same sequence system performing better at larger values of the loading factor K/N.
- Using independent sequences across transmit antennas decomposes the effective single user channel into parallel scalar channels. Ergodic capacities can be obtained without space-time coding and joint detection/decoding of the streams from different transmit antennas, while achieving outage capacities is significantly simplified. Essentially, the additional operation of spreading thats available in CDMA systems can be utilized to simplify space-time processing.
- Sequence correlation across transmit antennas may not be important for achieving the *peak* spectral efficiency in the CDMA system and performance with independent sequences can be representative, especially with enough frequency diversity.
- For the independent sequence system, the peak spectral efficiency in the ergodic sense is independent of the number of transmit antennas (M). This independence holds for both the MF and MMSE detectors.
- Under an outage criterion, the peak spectral efficiency of both detectors increases with M and eventually approaches the corresponding peak ergodic spectral efficiency. The benefit from increasing M is greater at lower outage levels. But the increase can also be achieved through frequency diversity.
- The number of receive antennas (L) has a pronounced effect for CDMA, with increases that are linear (or better) in L for fixed M. This increase holds for both ergodic and outage spectral efficiencies and for both detectors.
- At large values of L/M, CDMA with just single-user matched filtering eventually improves over orthogonal multiaccess, even for a single cell of users.

Thus, the results indicate that performance gains in CDMA are more sensitive to the number of receive antennas than the number of transmit antennas, if the parameters are chosen appropriately. In other words, the value of L should be made large and the value of M can be kept small, so that we have large L/M. But large values of L/M are precisely what would be practicable on the uplink, since it is more cost-effective to add multiple antennas at the base station than at the mobile. Combining this observation with the fact that CDMA (with single-user decoding) outperforms orthogonal multiaccess at large L/M, we are led to conclude that CDMA may be better suited to the uplink when there are multiple antennas. Note that this conclusion is made by considering only a single cell of users. In a cellular system with interference from other cells, the traditional advantages, such as greater reuse efficiency, may be additional factors favoring CDMA over orthogonal multiaccess techniques [24]. Also, as an alternative to direct sequence spreading, other nonorthogonal schemes, such as OFDM with tone sharing, could be expected to lead to similar conclusions. The underlying intuition is that allowing the users to collide in the time-frequency plane leads to a more effective utilization of the spatial dimensions at the receiver. Based on this intuition, a more effective strategy may be to allow for some form of hybrid multiaccess, where the number of interfering users increases with L/M.

At the same time, it is important to note that the above conclusions hold under the key assumption of perfect CSI at the base station. The initial channel estimation problem could be expected to be harder for the CDMA system due to the interference between the users and the presence of frequency selective fading. On the other hand, tracking the channel may be easier for the CDMA system due to the continuous transmission from each user. Thus, the overall effect of the channel estimation problem on the comparison between CDMA and orthogonal multiacess needs to be studied further. On the other hand, it is possible that these channel estimates can be fed back to the transmitters which can then do power allocation and/or beam forming to improve performance. This could be another interesting avenue for further study. Other key assumptions are that of uncorrelated fading across the antennas and equal rate requirements for the users. It is clearly of interest to rework the analysis of the paper when these assumptions are relaxed as well.

Appendix A
Proof of Proposition 3

For the MMSE detector, the Gaussian mutual information is given by

$$I_{\rm mmse} = \log\left|\boldsymbol{I}_M + \boldsymbol{X}_1^\dagger \boldsymbol{Q}^{-1}\boldsymbol{X}_1\right|$$

where $\boldsymbol{Q} = \sum_{k=2}^{K} \boldsymbol{X}_k\boldsymbol{X}_k^\dagger + \sigma^2\boldsymbol{I}_{LN}$. To analyze the mutual information, we make use of the following results, rephrased slightly for our purpose.

Lemma 1. [17, Lemma 1], [25, Lemma 3.1]: Let $\boldsymbol{A}$ be a deterministic $N \times N$ complex matrix with uniformly bounded spectral radius for all n. Let $\boldsymbol{q}$ be a length N vector with zero mean, variance $1/N$ entries, and finite eighth moment. Let $\boldsymbol{r}$ be a similar vector independent of $\boldsymbol{q}$. Then

$$\boldsymbol{q}^\dagger \boldsymbol{A}\boldsymbol{q} - \frac{1}{N}Tr\{\boldsymbol{A}\} \rightarrow_{a.s} 0 \quad \text{and} \quad \boldsymbol{q}^\dagger \boldsymbol{A}\boldsymbol{r} \rightarrow_{a.s} 0.$$

Lemma 2. [25, Th. 1]: Consider the general sample covariance matrix

$$\boldsymbol{A} = \frac{1}{n}\boldsymbol{X}^\dagger \boldsymbol{T}\boldsymbol{X}$$

where $\boldsymbol{X}$ is a $p \times n$ matrix of i.i.d. zero mean and unit variance entries and $\boldsymbol{T}$ is a positive definite matrix whose empirical eigenvalue distribution converges weakly to $H(\tau)$. Assume that $\boldsymbol{X}$, $\boldsymbol{T}$ are independent. Then, as $p, N \rightarrow \infty, p/n \rightarrow \alpha$, the Stieltjes transform of $\boldsymbol{A}$ converges, almost surely, to $m_{\boldsymbol{A}}(z)$ which satisfies the implicit equation

$$m_{\boldsymbol{A}}(z) = \left(-z + \alpha \int \frac{\tau}{1+\tau m_{\boldsymbol{A}}(z)} H(d\tau)\right)^{-1}.$$

Proof of (22): For the same sequence system with $L = 1$, we have

$$\boldsymbol{X}_k = \boldsymbol{X}_{k,1} = \sqrt{\frac{P}{M}} \sum_{p=1}^{L_p} \boldsymbol{c}_k^{(p)} \boldsymbol{1}^\dagger \boldsymbol{\Lambda}_{k1}^{(p)} = \sqrt{\frac{P}{M}} \sum_{p=1}^{L_p} \boldsymbol{c}_k^{(p)} \boldsymbol{h}_k^{(p)^\dagger}$$

where $\boldsymbol{c}_k^{(p)}$ and $\boldsymbol{h}_k^{(p)}$ are the $N \times 1$ and $M \times 1$ sequence and channel gain vectors along path p, respectively. Hence, using Lemma 1

$$\boldsymbol{X}_1^\dagger \boldsymbol{Q}^{-1} \boldsymbol{X}_1 = \frac{P}{M} \sum_{p=1}^{L_p} \boldsymbol{h}_k^{(p)} \boldsymbol{c}_k^{(p)^\dagger} \boldsymbol{Q} \boldsymbol{c}_k^{(p)} \boldsymbol{h}_k^{(p)^\dagger} \rightarrow_{a.s} \frac{1}{N}\mathrm{Tr}\{\boldsymbol{Q}\} \boldsymbol{H}_1 \boldsymbol{H}_1^\dagger$$

where $\boldsymbol{H}_1$ is an $M \times L_p$ matrix of channel gains. Furthermore,

$$\boldsymbol{Q} = \boldsymbol{C}\boldsymbol{U}\boldsymbol{C}^\dagger + \sigma^2 I$$

where $\boldsymbol{C}$ is a $N \times KL_p$ matrix of i.i.d. sequence entries and $\boldsymbol{U}$ is a $KL_p \times KL_p$ block diagonal matrices, where the kth Block of Size $L_p \times L_p$ is $(P/M)\boldsymbol{H}_k^\dagger \boldsymbol{H}_k$. The normalized trace of $\boldsymbol{Q}$ can then be derived from the eigenvalue distribution of $\boldsymbol{C}\boldsymbol{U}\boldsymbol{C}^\dagger$. This matrix is in a form to which Lemma 2 can be applied. In particular, note that the empirical spectral distribution of $\boldsymbol{U}$ can be directly related to the eigenvalue distribution of the (wishart) matrix $\boldsymbol{H}^\dagger \boldsymbol{H}$, Where $\boldsymbol{H}$ is a $M \times L_p$ matrix with the same distribution as the matrix $\boldsymbol{H}_k$, $k = 1, \ldots, K$.

Proof of (23): When $L > 1$, the $LN \times LN$ matrix $\boldsymbol{Q}$ can be written as

$$\boldsymbol{Q} = \begin{pmatrix} \boldsymbol{C}\boldsymbol{U}_{11}\boldsymbol{C}^\dagger + \sigma^2 \boldsymbol{I}_N & \ldots & \boldsymbol{C}\boldsymbol{U}_{1L}\boldsymbol{C}^\dagger \\ \vdots & & \vdots \\ \boldsymbol{C}\boldsymbol{U}_{1L}^\dagger \boldsymbol{C}^\dagger & \ldots & \boldsymbol{C}\boldsymbol{U}_{LL}\boldsymbol{C}^\dagger + \sigma^2 \boldsymbol{I}_N \end{pmatrix}$$

where $\boldsymbol{U}_{\ell\ell'}$ is a block diagonal matrix consisting of the $L_p \times L_p$ blocks $(P/M)\boldsymbol{H}_{k\ell}^\dagger \boldsymbol{H}_{k\ell'}$. Note that the matrices $\boldsymbol{C}$ are equal across the receive antennas. The analysis in [13] may justify replacing the matrices $\boldsymbol{C}$ with independent realizations $\boldsymbol{C}_\ell$, $\ell = 1, \ldots, L$ across the receive antennas and it may seem that this could facilitate the analysis. However, the real difficulty is that, even after such a replacement, the matrix has dependent entries due to the repetition across the transmit antennas. Hence, it appears that $\boldsymbol{Q}$ cannot be written in a form that allows for application of standard random matrix results on the limiting eigenvalue distribution.

Now, for fixed L and N, as we let $M \rightarrow \infty$, the independence of the channel gains and the strong law of large numbers imply that $\boldsymbol{Q}$ converges, almost surely, to a block diagonal matrix with $N \times N$ blocks, $(P/L_p)\boldsymbol{C}\boldsymbol{C}^\dagger + \sigma^2 \boldsymbol{I}_N$. Similarly, $\boldsymbol{X}_1 \boldsymbol{X}_1^\dagger$ converges to a $LN \times LN$ diagonal block matrix with $N \times N$ blocks, $(P/L_p)\boldsymbol{C}_1 \boldsymbol{C}_1^\dagger$. It follows that

$$\begin{aligned}\lim_{M\rightarrow\infty} I_{\text{mmse,same}} &= \log\left|\boldsymbol{I}_{LN} + \boldsymbol{Q}^{-1}\boldsymbol{X}_1\boldsymbol{X}_1^\dagger\right| \\ &= \sum_{\ell=1}^{L} \log\left|\boldsymbol{I}_N + \frac{P}{L_p}\boldsymbol{Q}_{\ell\ell}^{-1}\boldsymbol{C}_1\boldsymbol{C}_1^\dagger\right| \\ &= \sum_{\ell=1}^{L} \log\left|\boldsymbol{I}_{L_p} + \frac{P}{L_p}\boldsymbol{C}_1^\dagger \boldsymbol{Q}_{\ell\ell}^{-1}\boldsymbol{C}_1\right|.\end{aligned}$$

Finally, letting $N \rightarrow \infty$ and using Lemma 1 and Lemma 2, we have

$$\begin{aligned}\lim_{N\rightarrow\infty}\lim_{M\rightarrow\infty} I_{\text{mmse,same}} &= L\log\left|\boldsymbol{I}_{L_p} + \frac{\gamma}{L_p}\boldsymbol{I}_{L_p}\right| \\ &= LL_p \log\left(1 + \frac{\gamma}{L_p}\right)\end{aligned}$$

where γ is as given in (23).

Appendix B
Proof of Proposition 4

Proof of Part 1

By defining $\gamma_1 = \gamma/M$ and using $P/\sigma^2 = \gamma_b C$ in (20), the ergodic spectral efficiency with independent sequences is

$$\eta_e = \alpha C = \alpha M E \log(1 + \gamma_1 G) = (\alpha M) c(\gamma_1)$$

where $c(\gamma_1) = E \log(1 + \gamma_1 G)$ and

$$\gamma_1 = \frac{\gamma_b c(\gamma_1)}{1 + \frac{(\alpha M)\gamma_b c(\gamma_1)}{L} E_G\left[\frac{G}{1+G\gamma_1}\right]}.$$

Thus, γ_1 and η_e are functions only of the product αM and $\eta_e^{\max}$ is unaffected by M.

Proof ofPart 2

The form of the capacity in (23) and (24) is similar to the one with independent sequences, except that M is replaced by L_p and there is no fading. Thus, the argument for the first part above can be extended to imply that the peak spectral efficiency based on (23) is independent of L_p. Hence, we can set $L_p = 1$ and rewrite the implicit equation (24) as

$$\alpha = (1 + \gamma)\left(\frac{1}{\gamma} - \frac{1}{\gamma_s}\right).$$

The spectral efficiency is then given by

$$\eta_e = \alpha C = (1 + \gamma)\left(\frac{C}{\gamma} - \frac{C\sigma^2}{P}\right) = L\log_2 ef(\gamma, a). \quad (28)$$

where $a = 1/(L\gamma_b \log_2 e)$ and

$$f(\gamma, a) = (1+\gamma)\left(\frac{\log(1+\gamma)}{\gamma} - a\right).$$

To find the peak efficiency, we take the derivative of $f(\gamma, a)$ with respect to γ which yields

$$h(\gamma^\star) = \frac{1 - \frac{\log(1+\gamma^\star)}{\gamma^\star}}{\gamma^\star} = a$$

for the optimum value $\gamma^\star$. The function $h(\gamma^\star)$ is monotone decreasing and goes to zero as $\gamma^\star \to \infty$. Hence, as $a \to 0$ (or $L\gamma_b \to \infty$), $\gamma^\star \to \infty$ and $\log(1+\gamma^\star)/\gamma^\star \to 0$. It follows that $a\gamma^\star \to 1$. Hence

$$\begin{aligned}\lim_{a\to 0} f(\gamma^\star, a) &= \lim_{a\to 0}\left(1 + \frac{1}{\gamma^\star}\right)\log(1+\gamma^\star) - a\gamma^\star - a \\ &= \lim_{L\gamma_b\to\infty} \log(1 + L\gamma_b) - 1\end{aligned}$$

and the limit in (26) follows.

Proof of Part 3

For the calculation of the peak spectral efficiency with independent sequences, we can assume $M = 1$ without loss of generality. Following the same steps as for part 2 above, the spectral efficiency is then given by

$$\eta_e = \left\{\frac{1}{L}E\left[\frac{G}{1+G\gamma}\right]\right\}^{-1}\left(\frac{E\log_2(1+\gamma G)}{\gamma} - \frac{1}{\gamma_b}\right).$$

The peak can be obtained by taking the derivative with respect to γ. Now, as $L_p \to \infty$, $G \to 1$ almost surely and η_e converges to $L \log_2 e f(\gamma, a)$ for each γ. This is the same function as in (28) and hence the peak spectral efficiency approaches that for the same sequence system as $L_p \to \infty$.

A similar argument could be given for an asymptote in L, by rewriting

$$\eta_e = L\log_2 e\left\{E\left[\frac{G'}{1+G'\gamma'}\right]\right\}^{-1}\left(\frac{E\log(1+\gamma' G')}{\gamma'} - a\right)$$

where $G' = G/L$ and $\gamma' = L\gamma$ and noting that $G' \to 1$ almost surely, as $L \to \infty$.

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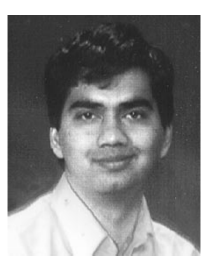

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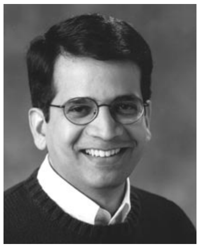

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