

# Algorithms for Dynamic Spectrum Access With Learning for Cognitive Radio

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**Abstract**—We study the problem of dynamic spectrum sensing and access in cognitive radio systems as a partially observed Markov decision process (POMDP). A group of cognitive users cooperatively tries to exploit vacancies in primary (licensed) channels whose occupancies follow a Markovian evolution. We first consider the scenario where the cognitive users have perfect knowledge of the distribution of the signals they receive from the primary users. For this problem, we obtain a greedy channel selection and access policy that maximizes the instantaneous reward, while satisfying a constraint on the probability of interfering with licensed transmissions. We also derive an analytical universal upper bound on the performance of the optimal policy. Through simulation, we show that our scheme achieves good performance relative to the upper bound and improved performance relative to an existing scheme. We then consider the more practical scenario where the exact distribution of the signal from the primary is unknown. We assume a parametric model for the distribution and develop an algorithm that can learn the true distribution, still guaranteeing the constraint on the interference probability. We show that this algorithm outperforms the naive design that assumes a worst case value for the parameter. We also provide a proof for the convergence of the learning algorithm.

**Index Terms**—Channel selection, cognitive radio, dynamic spectrum access, learning, partially observed Markov decision process (POMDP).

## I. INTRODUCTION

**C**OGNITIVE radios that exploit vacancies in the licensed spectrum have been proposed as a solution to the ever-increasing demand for radio spectrum. The idea is to sense times when a specific licensed band is not used at a particular place and use this band for unlicensed transmissions without causing interference to the licensed user (referred to as the “primary”). An important part of designing such systems is to develop an efficient channel selection policy. The cognitive radio (also called the “secondary user”) needs to adopt the best strategy for selecting channels for sensing and access. The sensing and access policies should jointly ensure that the probability of interfering with the primary’s transmission meets a given constraint.

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In the first part of this paper, we consider the design of such a joint sensing and access policy, assuming a Markovian model for the primary spectrum usage on the channels being monitored. The secondary users use the observations made in each slot to track the probability of occupancy of the different channels. We obtain a suboptimal solution to the resultant POMDP problem.

In the second part of the paper, we propose and study a more practical problem that arises when the secondary users are not aware of the exact distribution of the signals that they receive from the primary transmitters. We develop an algorithm that learns these unknown statistics and show that this scheme gives improved performance over the naive scheme that assumes a worst-case value for the unknown distribution.

## A. Contribution

When the statistics of the signals from the primary are known, we show that, under our formulation, the dynamic spectrum access problem with a group of cooperating secondary users is equivalent in structure to a single user problem. We also obtain a new analytical upper bound on the expected reward under the optimal scheme. Our suboptimal solution to the POMDP is shown via simulations to yield a performance that is close to the upper bound and better than that under an existing scheme.

The main contribution of this paper is the formulation and solution of the problem studied in the second part involving unknown observation statistics. We show that unknown statistics of the primary signals can be learned and provide an algorithm that learns these statistics online and maximizes the expected reward still satisfying a constraint on interference probability.

## B. Related Work

In most of the existing schemes [1], [2] in the literature on dynamic spectrum access for cognitive radios, the authors assume that every time a secondary user senses a primary channel, it can determine whether or not the channel is occupied by the primary. A different scheme was proposed in [3] and [4] where the authors assume that the secondary transmitter receives error-free ACK signals from the secondary’s receivers whenever their transmission is successful. The secondary users use these ACK signals to track the channel states of the primary channels. We adopt a different strategy in this paper. We assume that every time the secondary users sense a channel they see a random observation whose distribution depends on the state of the channel. Our approach is distinctly different from and more realistic than that in [1], [2] since we do not assume that the secondary users know the primary channel states perfectly through sensing. We provide a detailed comparison of our approach with that of [3] and [4] after presenting our solution. In particular, we point out

that while using the scheme of [4] there are some practical difficulties in maintaining synchronization between the secondary transmitter and receiver. Our scheme provides a way around this difficulty, albeit we require a dedicated control channel between the secondary transmitter and receiver.

The problem studied in the second part of this paper that involves learning of unknown observation statistics is new. However, the idea of combining learning and dynamic spectrum access was also used in [5] where the authors propose a reinforcement-learning scheme for learning channel idling probabilities and interference probabilities.

We introduce the basic spectrum sensing and access problem in Section II and describe our proposed solution in Section III. In Section IV, we elaborate on the problem where the distributions of the observations are unknown. We present simulation results and comparisons with some existing schemes in Section V, and our conclusions in Section VI.

## II. PROBLEM STATEMENT

We consider a slotted system where a group of secondary users monitor a set  $\mathcal{C}$  of primary channels. The state of each primary channel switches between 'occupied' and 'unoccupied' according to the evolution of a Markov chain. The secondary users can cooperatively sense any one out of the channels in  $\mathcal{C}$  in each slot, and can access any one of the  $L = |\mathcal{C}|$  channels in the same slot. In each slot, the secondary users must satisfy a strict constraint on the probability of interfering with potential primary transmissions on any channel. When the secondary users access a channel that is free during a given time slot, they receive a reward proportional to the bandwidth of the channel that they access. The objective of the secondary users is to select the channels for sensing and access in each slot in such a way that their total expected reward accrued over all slots is maximized subject to the constraint on interfering with potential primary transmissions every time they access a channel<sup>1</sup>. Since the secondary users do not have explicit knowledge of the states of the channels, the resultant problem is a constrained partially observable Markov decision process (POMDP) problem.

We assume that all channels in  $\mathcal{C}$  have equal bandwidth  $B$ , and are statistically identical and independent in terms of primary usage. The occupancy of each channel follows a stationary Markov chain. The state of channel  $a$  in any time slot  $k$  is represented by variable  $S_a(k)$  and could be either 1 or 0, where state 0 corresponds to the channel being free for secondary access and 1 corresponds to the channel being occupied by some primary user.

The secondary system includes a decision center that has access to all the observations made by the cooperating secondary users<sup>2</sup>. The observations are transmitted to the decision center over a dedicated control channel. The same dedicated channel can also be used to maintain synchronization between the secondary transmitter and secondary receiver so that the receiver

<sup>1</sup>We do not consider scheduling policies in this paper and assume that the secondary users have some predetermined scheduling policy to decide which user accesses the primary channel every time they determine that a channel is free for access.

<sup>2</sup>The scheme proposed in this paper and the analyses presented in this paper are valid even if the cooperating secondary users transmit quantized versions of their observations to the fusion center. Minor changes are required to account for the discrete nature of the observations.

can tune to the correct channel to receive transmissions from the transmitter. The sensing and access decisions in each slot are made at this decision center. When channel  $a$  is sensed in slot  $k$ , we use  $\underline{X}_a(k)$  to denote the vector of observations made by the different cooperating users on channel  $a$  in slot  $k$ . These observations represent the sampled outputs of the wireless receivers tuned to channel  $a$  that are employed by the cognitive users. The statistics of these observations are assumed to be time-invariant and distinct for different channel states. The observations on channel  $a$  in slot  $k$  have distinct joint probability density functions  $f_0$  and  $f_1$  when  $S_a(k) = 0$  and  $S_a(k) = 1$  respectively. The collection of all observations up to slot  $k$  is denoted by  $\underline{X}^k$ , and the collection of observations on channel  $a$  up to slot  $k$  is denoted by  $\underline{X}_a^k$ . The channel sensed in slot  $k$  is denoted by  $u_k$ , the sequence of channels sensed up to slot  $k$  is denoted by  $u^k$ , and the set of time slots up to slot  $k$  when channel  $a$  was sensed is denoted by  $K_a^k$ . The decision to access channel  $a$  in slot  $k$  is denoted by a binary variable  $\delta_a(k)$ , which takes value 1 when channel  $a$  is accessed in slot  $k$ , and 0 otherwise.

Whenever the secondary users access a free channel in some time slot  $k$ , they get a reward  $B$  equal to the bandwidth of each channel in  $\mathcal{C}$ . The secondary users should satisfy the following constraint on the probability of interfering with the primary transmissions in each slot

$$P(\{\delta_a(k) = 1\} | \{S_a(k) = 1\}) \leq \zeta.$$

In order to simplify the structure of the access policy, we also assume that in each slot the decision to access a channel is made using only the observations made in that slot. Hence it follows that in each slot  $k$ , the secondary users can access only the channel they sense in slot  $k$ , say channel  $a$ . Furthermore, the access decision must be based on a binary hypothesis test [6] between the two possible states of channel  $a$ , performed on the observation  $\underline{X}_a(k)$ . This leads to an access policy with a structure similar to that established in [4]. The optimal test [6] is to compare the joint log-likelihood ratio (LLR)  $\mathcal{L}(\underline{X}_a(k))$  given by

$$\mathcal{L}(\underline{X}_a(k)) = \log \left( \frac{f_1(\underline{X}_a(k))}{f_0(\underline{X}_a(k))} \right)$$

to some threshold  $\Delta$  that is chosen to satisfy

$$P(\{\mathcal{L}(\underline{X}_a(k)) < \Delta\} | \{S_a(k) = 1\}) = \zeta \quad (1)$$

and the optimal access decision would be to access the sensed channel whenever the threshold exceeds the joint LLR. Hence

$$\delta_a(k) = \mathcal{I}_{\{\mathcal{L}(\underline{X}_a(k)) < \Delta\}} \mathcal{I}_{\{u_k=a\}} \quad (2)$$

and the reward obtained in slot  $k$  can be expressed as

$$\hat{r}_k = B \mathcal{I}_{\{S_{u_k}(k)=0\}} \mathcal{I}_{\{\mathcal{L}(\underline{X}_{u_k}(k)) < \Delta\}} \quad (3)$$

where  $\mathcal{I}_E$  represents the indicator function of event  $E$ . The main advantage of the structure of the access policy given in (2) is that we can obtain a simple sufficient statistic for the resultant POMDP without having to keep track of all the past observations, as discussed later. It also has the added advantage [4] that the secondary users can set the thresholds  $\Delta$  to meet the constraint on the probability of interfering with the primary trans-

missions without relying on their knowledge of the Markov statistics.

Our objective is to generate a policy that makes optimal use of primary spectrum subject to the interference constraint. We introduce a discount factor  $\alpha \in (0, 1)$  and aim to solve the infinite horizon dynamic program with discounted rewards [7]. That is, we seek the sequence of channels  $\{u_0, u_1, \dots\}$ , such that the  $\sum_{k=0}^{\infty} \alpha^k \mathbb{E}[r_k]$  is maximized, where the expectation is performed over the random observations and channel state realizations. We can show the following relation based on the assumption of identical channels:

$$\begin{aligned} \mathbb{E}[\hat{r}_k] &= \mathbb{E} \left[ B \mathcal{I}_{\{S_{u_k}(k)=0\}} \mathcal{I}_{\{\mathcal{L}(\underline{X}_{u_k}(k)) < \Delta\}} \right] \\ &= \mathbb{E} \left[ \mathbb{E} \left[ B \mathcal{I}_{\{S_{u_k}(k)=0\}} \right. \right. \\ &\quad \left. \left. \times \mathcal{I}_{\{\mathcal{L}(\underline{X}_{u_k}(k)) < \Delta\}} \middle| S_{u_k}(k) \right] \right] \\ &= \mathbb{E} \left[ B(1 - \hat{\epsilon}) \mathcal{I}_{\{S_{u_k}(k)=0\}} \right] \end{aligned} \quad (4)$$

where

$$\hat{\epsilon} = \mathbb{P}(\{\mathcal{L}(\underline{X}_a(k)) > \Delta\} | \{S_a(k) = 0\}). \quad (5)$$

Since all the channels are assumed to be identical and the statistics of the observations are assumed to be constant over time,  $\hat{\epsilon}$  given by (5) is a constant independent of  $k$ . From the structure of the expected reward in (4) it follows that we can redefine our problem such that the reward in slot  $k$  is now given by

$$r_k = B(1 - \hat{\epsilon}) \mathcal{I}_{\{S_{u_k}(k)=0\}} \quad (6)$$

and the optimization problem is equivalent to maximizing  $\sum_{k=0}^{\infty} \alpha^k \mathbb{E}[r_k]$ . Since we know the structure of the optimal access decisions from (2), the problem of spectrum sensing and access boils down to choosing the optimal channel to sense in each slot. Whenever the secondary users sense some channel and make observations with LLR lower than the threshold, they are free to access that channel. Thus we have converted the constrained POMDP problem into an unconstrained POMDP problem as was done in [4].

### III. DYNAMIC PROGRAMMING

The state of the system in slot  $k$  denoted by

$$\underline{S}(k) = (S_1(k), S_2(k), \dots, S_L(k))^T$$

is the vector of states of the channels in  $\mathcal{C}$  that have independent and identical Markovian evolutions. The channel to be sensed in slot  $k$  is decided in slot  $k-1$  and is given by

$$u_k = \mu_k(I_{k-1})$$

where  $\mu_k$  is a deterministic function and  $I_k \triangleq (\underline{X}^k, u^k)$  represents the net information about past observations and decisions up to slot  $k$ . The reward obtained in slot  $k$  is a function of the state in slot  $k$  and  $u_k$  as given by (6). We seek the sequence of channels  $\{u_0, u_1, \dots\}$ , such that  $\sum_{k=0}^{\infty} \alpha^k \mathbb{E}[r_k]$  is maximized. It is easily verified that this problem is a standard dynamic programming problem with imperfect observations. It is known [7] that for such a POMDP problem, a sufficient statistic at the end

of any time slot  $k$ , is the probability distribution of the system state  $\underline{S}(k)$ , conditioned on all the past observations and decisions, given by  $\mathbb{P}(\{\underline{S}(k) = \underline{s}\} | I_k)$ . Furthermore, since the Markovian evolution of the different channels are independent of each other, this conditional probability distribution is equivalently represented by the set of *beliefs* about the occupancy states of each channel, i.e., the probability of occupancy of each channel in slot  $k$ , conditioned on all the past observations on channel  $a$  and times when channel  $a$  was sensed. We use  $p_a(k)$  to represent the belief about channel  $a$  at the end of slot  $k$ , i.e.,  $p_a(k)$  is the probability that the state  $S_a(k)$  of channel  $a$  in slot  $k$  is 1, conditioned on all observations and decisions up to time slot  $k$ , which is given by

$$p_a(k) = \mathbb{P}(\{S_a(k) = 1\} | \underline{X}_a^k, K_a^k).$$

We use  $\underline{p}(k)$  to denote the  $L \times 1$  vector representing the beliefs about the channels in  $\mathcal{C}$ . The initial values of the belief parameters for all channels are set using the stationary distribution of the Markov chain. We use  $P$  to represent the transition probability matrix for the state transitions of each channel, with  $P(i, j)$  representing the probability that a channel that is in state  $i$  in slot  $k$  switches to state  $j$  in slot  $k+1$ . We define

$$q_a(k) = P(1, 1)p_a(k-1) + P(0, 1)(1 - p_a(k-1)). \quad (7)$$

This  $q_a(k)$  represents the probability of occupancy of channel  $a$  in slot  $k$ , conditioned on the observations up to slot  $k-1$ . Using Bayes' rule, the belief values are updated as follows after the observation in time slot  $k$ :

$$p_a(k) = \frac{q_a(k) f_1(\underline{X}_a(k))}{q_a(k) f_1(\underline{X}_a(k)) + (1 - q_a(k)) f_0(\underline{X}_a(k))} \quad (8)$$

when channel  $a$  was selected in slot  $k$  (i.e.,  $u_k = a$ ), and  $p_a(k) = q_a(k)$  otherwise. Thus from (8) we see that updates for the sufficient statistic can be performed using only the joint LLR of the observations,  $\mathcal{L}(\underline{X}_a(k))$ , instead of the entire vector of observations. Furthermore, from (2) we also see that the access decisions also depend only on the LLRs. Hence, we conclude that this problem with vector observations is equivalent to one with scalar observations where the scalars represent the joint LLR of the observations of all the cooperating secondary users. Therefore, in the rest of this paper, we use a scalar observation model with the observation made on channel  $a$  in slot  $k$  represented by  $Y_a(k)$ . We use  $Y^k$  to denote the set of all observations up to time slot  $k$  and  $Y_a^k$  to denote the set of all observations on channel  $a$  up to slot  $k$ .

Hence the new access decisions are given by

$$\delta_a(k) = \mathcal{I}_{\{\mathcal{L}'(Y_a(k)) < \Delta'\}} \mathcal{I}_{\{u_k=a\}} \quad (9)$$

where  $\mathcal{L}'(Y_a(k))$  represents the LLR of  $Y_a(k)$  and the access threshold  $\Delta'$  is chosen to satisfy

$$\mathbb{P}(\{\mathcal{L}'(Y_a(k)) < \Delta'\} | \{S_a(k) = 1\}) = \zeta. \quad (10)$$

Similarly the belief updates are performed as in (8) with the evaluations of density functions of  $\underline{X}_a(k)$  replaced with the evaluations of the density functions  $f'_0$  and  $f'_1$  of  $Y_a(k)$

$$p_a(k) = \frac{q_a(k) f'_1(Y_a(k))}{q_a(k) f'_1(Y_a(k)) + (1 - q_a(k)) f'_0(Y_a(k))} \quad (11)$$

when channel  $a$  is accessed in slot  $k$  (i.e.,  $u_k = a$ ), and  $p_a(k) = q_a(k)$  otherwise. We use  $G(\underline{p}(k-1), u_k, Y_{u_k}(k))$  to denote the function that returns the value of  $\underline{p}(k)$  given that channel  $u_k$  was sensed in slot  $k$ . This function can be calculated using (7) and (11). The reward obtained in slot  $k$  can now be expressed as

$$r_k = B(1 - \epsilon)\mathcal{I}_{\{S_{u_k}(k)=0\}} \quad (12)$$

where  $\epsilon$  is given by

$$\epsilon = P(\{\mathcal{L}'(Y_a(k)) > \Delta'\} | \{S_a(k) = 0\}). \quad (13)$$

From the structure of the dynamic program, it can be shown that the optimal solution to this dynamic program can be obtained by solving the following Bellman equation [7] for the optimal reward-to-go function:

$$J(\underline{p}) = \max_{u \in \mathcal{C}} [B(1 - \epsilon)(1 - q_u) + \alpha E(J(G(\underline{p}, u, Y_u)))] \quad (14)$$

where  $\underline{p}$  represents the initial value of the belief vector, i.e., the prior probability of channel occupancies in slot  $-1$ , and  $\underline{q}$  is calculated from  $\underline{p}$  as in (7) by

$$q_a = P(1, 1)p_a + P(0, 1)(1 - p_a), \quad a \in \mathcal{C}. \quad (15)$$

The expectation in (14) is performed over the random observation  $Y_u$ . Since it is not easy to find the optimal solution to this Bellman equation, we adopt a suboptimal strategy to obtain a channel selection policy that performs well.

In the rest of the paper we assume that the transition probability matrix  $P$  satisfies the following regularity conditions:

$$\text{Assumption 1 : } 0 < P(j, j) < 1, \quad j \in \{0, 1\} \quad (16)$$

$$\text{Assumption 2 : } P(0, 0) > P(1, 0). \quad (17)$$

The first assumption ensures that the resultant Markov chain is irreducible and positive recurrent, while the second assumption ensures that it is more likely for a channel that is free in the current slot to remain free in the next slot than for a channel that is occupied in the current slot to switch states and become free in the next slot. While the first assumption is important the second one is used only in the derivation of the upper bound on the optimal performance and can easily be relaxed by separately considering the case where (17) does not hold.

#### A. Greedy Policy

A straightforward suboptimal solution to the channel selection problem is the greedy policy, i.e., the policy of maximizing the expected instantaneous reward in the current time slot. The expected instantaneous reward obtained by accessing some channel  $a$  in a given slot  $k$  is given by  $B(1 - \epsilon)(1 - q_a(k))$  where  $\epsilon$  is given by (13). Hence, the greedy policy is to choose the channel  $a$  such that  $1 - q_a(k)$  is the maximum.

$$u_k^{\text{gr}} = \arg \max_{u \in \mathcal{C}} \{1 - q_u(k)\}. \quad (18)$$

In other words, in every slot the greedy policy chooses the channel that is most likely to be free, conditioned on the past observations. The greedy policy for this problem is in fact equivalent to the  $Q_{\text{MDP}}$  policy, which is a standard suboptimal solution to the POMDP problem (see, e.g., [8]). It is shown in [1] and [2] that under some conditions on  $P$  and  $L$ , the greedy

policy is optimal if the observation in each slot reveals the underlying state of the channel. Hence, it can be argued that under the same conditions, the greedy policy would also be optimal for our problem at high SNR.

#### B. An Upper Bound

An upper bound on the optimal reward for the POMDP of (14) can be obtained by assuming more information than the maximum that can be obtained in reality. One such assumption that can give us a simple upper bound is the  $Q_{\text{MDP}}$  assumption [8], which is to assume that in all future slots, the state of all channels become known exactly after making the observation in that slot. The optimal reward under the  $Q_{\text{MDP}}$  assumption is a function of the initial belief vector, i.e., the prior probabilities of occupancy of the channels in slot  $-1$ . We represent this function by  $J^Q$ . In practice, a reasonable choice of initial value of the belief vector is given by the stationary distribution of the Markov chains. Hence for any solution to the POMDP that uses this initialization, an upper bound for the optimal reward under the  $Q_{\text{MDP}}$  assumption is given by  $J^U = J^Q(p^* \underline{1})$  where  $p^*$  represents the probability that a channel is occupied under the stationary distribution of the transition probability matrix  $P$ , and  $\underline{1}$  represents an  $L \times 1$  vector of all 1's.

The first step involved in evaluating this upper bound is to determine the optimal reward function under the assumption that all the channel states become known exactly after making the observation in each slot including the current slot. We call this function  $\check{J}$ . That is, we want to evaluate  $\check{J}(\underline{x})$  for all binary strings  $\underline{x}$  of length  $L$  that represent the  $2^L$  possible values of the vector representing the states of all channels in slot  $-1$ . The  $Q_{\text{MDP}}$  assumption implies that the functions  $J^Q$  and  $\check{J}$  satisfy the following equation:

$$J^Q(\underline{z}) = \max_{u \in \mathcal{C}} \{ [B(1 - \epsilon)(1 - q_u) + \sum_{\underline{x} \in \{0, 1\}^L} \alpha P(\{\underline{S}(0) = \underline{x}\}) \check{J}(\underline{x})] \} \text{ s.t. } \underline{p}(-1) = \underline{z} \quad (19)$$

where  $\underline{p}(-1)$  denotes the *a priori* belief vector about the channel states in slot  $-1$  and  $q_u$  is obtained from  $p_u(-1)$  just as in (15). Hence the upper bound  $J^U = J^Q(p^* \underline{1})$  can be easily evaluated using  $P$  once the function  $\check{J}$  is determined.

Now we describe how one can solve for the function  $\check{J}$ . Under the assumption that the states of all the channels become known exactly at the time of observation, the optimal channel selected in any slot  $k$  would be a function of the states of the channels in slot  $k - 1$ . Moreover, the sensing action in the current slot would not affect the rewards in the future slots. Hence the optimal policy would be to maximize the expected instantaneous reward, which is achieved by accessing the channel that is most likely to be free in the current slot. Now under the added assumption stated in (17) earlier<sup>3</sup>, the optimal policy would always select some channel that was free in the previous time slot, if there is any. If no channel is free in the previous time slot, then the optimal policy would be to select any one of the channels in  $\mathcal{C}$ , since all of them are equally likely to be free in the current slot. Hence the derivation of the optimal total reward for this problem is straightforward as illustrated below. The total

<sup>3</sup>It is easy to see that a minor modification of the derivation of the upper bound works when assumption (17) does not hold.

reward for this policy is a function of the state of the system in the slot preceding the initial slot, i.e.,  $\underline{S}(-1)$ .

$$\begin{aligned}\tilde{J}(\underline{x}) &= \max_{u \in \mathcal{C}} \mathbb{E}[\mathcal{I}_{\{S_u(0)=0\}} \\ &\quad + \alpha \tilde{J}(\underline{S}(0)) | \{\underline{S}(-1) = \underline{x}\}] \\ &= \begin{cases} \kappa P(0,0) + \alpha V(\underline{x}) & \text{if } \underline{x} \neq \underline{1} \\ \kappa P(1,0) + \alpha V(\underline{x}) & \text{if } \underline{x} = \underline{1} \end{cases}\end{aligned}$$

where  $V(\underline{x}) = \mathbb{E}[\tilde{J}(\underline{S}(0)) | \{\underline{S}(-1) = \underline{x}\}]$ ,  $\kappa = B(1 - \epsilon)$ , and  $\underline{1}$  is an  $L \times 1$  string of all 1's. This means that we can write

$$\begin{aligned}\tilde{J}(\underline{x}) &= \kappa \left\{ P(0,0) \sum_{k=0}^{\infty} \alpha^k \right. \\ &\quad \left. - (P(0,0) - P(1,0))w(\underline{x}) \right\} \\ &= \kappa \left\{ \frac{P(0,0)}{1 - \alpha} - (P(0,0) - P(1,0))w(\underline{x}) \right\} \quad (20)\end{aligned}$$

where

$$w(\underline{x}) \triangleq \mathbb{E} \left[ \sum_{M \geq -1: \underline{S}(M) = \underline{1}} \alpha^{M+1} \middle| \{\underline{S}(-1) = \underline{x}\} \right]$$

is a scalar function of the vector state  $\underline{x}$ . Here the expectation is over the random slots when the system reaches state  $\underline{1}$ . Now by stationarity we have

$$w(\underline{x}) = \mathbb{E} \left[ \sum_{M \geq 0: \underline{S}(M) = \underline{1}} \alpha^M \middle| \{\underline{S}(0) = \underline{x}\} \right]. \quad (21)$$

We use  $\mathbb{P}$  to denote the matrix of size  $2^L \times 2^L$  representing the transition probability matrix of the joint Markov process that describes the transitions of the vector of channel states  $\underline{S}(k)$ . The  $(i, j)$ th element of  $\mathbb{P}$  represents the probability that the state of the system switches to  $\underline{y}$  in slot  $k+1$  given that the state of the system is  $\underline{x}$  in slot  $k$ , where  $\underline{x}$  is the  $L$ -bit binary representation of  $i-1$  and  $\underline{y}$  is the  $L$ -bit binary representation of  $j-1$ . Using a slight abuse of notation we represent the  $(i, j)$ th element of  $\mathbb{P}$  as  $\mathbb{P}(\underline{x}, \underline{y})$  itself. Now (21) can be solved to obtain

$$w(\underline{x}) = \sum_{\underline{y}} \alpha \mathbb{P}(\underline{x}, \underline{y}) w(\underline{y}) + \mathcal{I}_{\{\underline{x} = \underline{1}\}}. \quad (22)$$

This fixed point equation which can be solved to obtain

$$\underline{w} = (\mathbb{I} - \alpha \mathbb{P})^{-1} \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}_{2^L \times 1} \quad (23)$$

where  $\underline{w}$  is a  $2^L \times 1$  vector whose elements are the values of the function  $w(\underline{x})$  evaluated at the  $2^L$  different possible values of the vector state  $\underline{x}$  of the system in time slot  $-1$ . Again, the  $i$ th element of vector  $\underline{w}$  is  $w(\underline{x})$  where  $\underline{x}$  is the  $L$ -bit binary representation of  $i-1$ . Thus  $\tilde{J}$  can now be evaluated by using relation (20) and the expected reward for this problem under the  $Q_{\text{MDP}}$  assumption can be calculated by evaluating  $J^U = J^Q(p^* \underline{1})$  via (19). This optimal value yields an analytical upper bound on the optimal reward of the original problem (14).

### C. Comparison With [4] for Single User Problem

Although we have studied a spectrum access scheme for a cooperative cognitive radio network, it can also be employed by a single cognitive user. Under this setting, our approach to the spectrum access problem described earlier in this section is similar to that considered in [3] and [4] in that sensing does not reveal the true channel states but only a random variable whose distribution depends on the current state of the sensed channel. As a result, the structure of our optimal access policy and the sufficient statistic are similar to those in [4]. In this section, we compare the two schemes.

The main difference between our formulation and that in [4] is that in our formulation the secondary users use the primary signal received on the channel to track the channel occupancies, while in [4] they use the ACK signals exchanged between the secondary transmitter and receiver. Under the scheme of [4], in each slot, the secondary receiver transmits an ACK signal upon successful reception of a transmission from the secondary receiver. The belief updates are then performed using the single bit of information provided by the presence or absence of the ACK signal. The approach of [4] was motivated by the fact that, under that scheme, the secondary receiver knows in advance the channel on which to expect potential transmissions from the secondary transmitter in each slot, thus obviating the need for control channels for synchronization. However, such synchronization between the transmitter and receiver is not reliable in the presence of interfering terminals that are hidden [9] from either the receiver or transmitter, because the ACK signals will no longer be error-free. In this regard, we believe that a more practical solution to this problem would be to set aside a dedicated control channel of low capacity for the purpose of reliably maintaining synchronization, and use the observations on the primary channel for tracking the channel occupancies. In addition to guaranteeing synchronization, our scheme provides some improvement in utilizing transmission opportunities over the ACK-based scheme, as we show in Section V-A.

Another difference between our formulation and that in [4] is that we assume that the statistics of channel occupancies are independent and identical while [4] considers the more general case of correlated and nonidentical channels. However, the scheme we proposed in Section III can be easily modified to handle this case, with added complexity. The sufficient statistic would now be the *posteriori* distribution of  $\underline{S}(k)$ , the vector of states of all channels, and the access thresholds on different channels would be nonidentical and depend on the statistics of the observations the respective channels. We avoid elaborating on this more general setting to keep the presentation simple.

## IV. THE CASE OF UNKNOWN DISTRIBUTIONS

In practice, the secondary users are typically unaware of the primary's signal characteristics and the channel realization from the primary [10]. Hence, cognitive radio systems have to rely on some form of noncoherent detection such as energy detection while sensing the primary signals. Furthermore, even while employing noncoherent detectors, the secondary users are also unaware of their locations relative to the primary and hence are not aware of the shadowing and path loss from the primary to the secondary. Hence, it is not reasonable to assume that the

secondary users know the exact distributions of the observations under the primary-present hypothesis, although it can be assumed that the distribution of the observations under the primary-absent hypothesis is known exactly. This scenario can be modeled by using a parametric description for the distributions of the received signal under the primary-present hypothesis. We denote the density functions of the observations under the two possible hypotheses as,

$$\begin{aligned} S_a(k) = 0 : Y_a(k) &\sim f_{\theta_0} \\ S_a(k) = 1 : Y_a(k) &\sim f_{\theta_a} \\ \text{where } \theta_a &\in \Theta, \quad \forall a \in \{1, 2, \dots, L\} \end{aligned} \quad (24)$$

where the parameters  $\{\theta_a\}$  are unknown for all channels  $a$ , and  $\theta_0$  is known. We use  $\mathcal{L}_\theta(\cdot)$  to denote the log-likelihood function under  $f_\theta$  defined by

$$\mathcal{L}_\theta(x) \triangleq \log \left( \frac{f_\theta(x)}{f_{\theta_0}(x)} \right), \quad x \in \mathbb{R}, \theta \in \Theta. \quad (25)$$

In this section, we study two possible approaches for dealing with such a scenario, while restricting to greedy policies for channel selection. For ease of illustration, in this section we consider a secondary system comprised of a single user, although the same ideas can also be applied for a system with multiple cooperating users.

#### A. Worst-Case Design for Nonrandom $\theta_a$

A close examination of Section III reveals two specific uses for the density function of the observations under the  $S_a(k) = 1$  hypothesis. The knowledge of this density was of crucial importance in setting the access threshold in (10) to meet the constraint on the probability of interference. The other place where this density was used was in updating the belief probabilities in (11). When the parameters  $\{\theta_a\}$  are nonrandom and unknown, we have to guarantee the constraint on the interference probability for all possible realizations of  $\theta_a$ . The optimal access decision would thus be given by

$$\hat{\delta}_a(k) = \mathcal{I}_{\{u_k=a\}} \prod_{\theta \in \Theta} \mathcal{I}_{\{\mathcal{L}_\theta(Y_a(k)) < \tau_\theta\}} \quad (26)$$

where  $\tau_\theta$  satisfies

$$\mathbb{P}(\{\mathcal{L}_\theta(Y_a(k)) < \tau_\theta\} | \{S_a(k) = 1, \theta_a = \theta\}) = \zeta. \quad (27)$$

The other concern that we need to address in this approach is: what distribution do we use for the observations under  $S_a(k) = 1$  in order to perform the updates in (11). An intelligent solution is possible provided the densities described in (24) satisfy the condition that there is a  $\theta^* \in \Theta$  such that for all  $\theta \in \Theta$  and for all  $\tau \in \mathbb{R}$  the following inequality holds:

$$\begin{aligned} &\mathbb{P}(\{\mathcal{L}_{\theta^*}(Y_a(k)) > \tau\} | \{S_a(k) = 1, \theta_a = \theta\}) \\ &\geq \mathbb{P}(\{\mathcal{L}_{\theta^*}(Y_a(k)) > \tau\} | \{S_a(k) = 1, \theta_a = \theta^*\}). \end{aligned} \quad (28)$$

The condition (28) is satisfied by several parameterized densities including an important practical example discussed later. Under (28), a good suboptimal solution to the channel selection problem would be to run the greedy policy for channel selection using  $f_{\theta^*}$  for the density under  $S_a(k) = 1$  while performing the

updates of the channel beliefs in (11). This is a consequence of the following lemma.

*Lemma 4.1:* Assume (28) holds. Suppose  $f_{\theta^*}$  is used in place of  $f_1$  for the distribution of the observations under  $S_a(k) = 1$  while performing belief updates in (11). Then:

- (i) For all  $\gamma \in \Theta$  and for all  $\beta, p \in [0, 1]$

$$\begin{aligned} &\mathbb{P}_\gamma(\{p_a(k) > \beta\} | \{S_a(k) = 1, p_a(k-1) = p\}) \\ &\geq \mathbb{P}_{\theta^*}(\{p_a(k) > \beta\} | \{S_a(k) = 1, p_a(k-1) = p\}) \end{aligned} \quad (29)$$

where  $\mathbb{P}_\theta$  represents the probability measure when  $\theta_a = \theta$ .

- (ii) Conditioned on  $\{S_a(k) = 0\}$ , the distribution of  $p_a(k)$  given any value for  $p_a(k-1)$  is identical for all possible values of  $\theta_a$ .

*Proof:*

- (i) Clearly, (29) holds with equality when channel  $a$  is not sensed in slot  $k$  (i.e.,  $u_k \neq a$ ). When  $u_k = a$ , it is easy to see that the new belief given by (11) is a monotonically increasing function of the log-likelihood function,  $\mathcal{L}_{\theta^*}(Y_a(k))$ . Hence, (29) follows from (28).  
(ii) This is obvious since the randomness in  $p_a(k)$  under  $\{S_a(k) = 0\}$  is solely due to the observation  $Y_a(k)$  whose distribution  $f_{\theta_0}$  does not depend on  $\theta_a$ . ■

Clearly, updating using  $f_{\theta^*}$  in (11) is optimal if  $\theta_a = \theta^*$ . When  $\theta_a \neq \theta^*$ , the tracking of beliefs are guaranteed to be at least as accurate, in the sense described in Lemma 4.1. Hence, under (28), a good suboptimal solution to the channel selection problem would be to run the greedy policy for channel selection using  $f_{\theta^*}$  for the density under  $S_a(k) = 1$  while performing the updates of the channel beliefs in (11). Furthermore, it is known that [11] under (28), the set of likelihood ratio tests in the access decision of (26) can be replaced with a single likelihood ratio test under the worst case parameter  $\theta^*$  given by

$$\hat{\delta}_a(k) = \mathcal{I}_{\{u_k=a\}} \mathcal{I}_{\{\mathcal{L}_{\theta^*}(Y_a(k)) < \tau_{\theta^*}\}}. \quad (30)$$

The structure of the access decision given in (30), and the conclusion from Lemma 4.1 suggests that  $\theta^*$  is a worst-case value of the parameter  $\theta_a$ . Hence, the strategy of designing the sensing and access policies assuming this worst possible value of the parameter is optimal in the following min-max sense: The average reward when the true value of  $\theta_a \neq \theta^*$  is expected to be no smaller than that obtained when  $\theta_a = \theta^*$  since the tracking of beliefs is worst when  $\theta_a = \theta^*$  as shown in Lemma 4.1. This intuitive reasoning is seen to hold in the simulation results in Section V-B.

#### B. Modeling $\theta_a$ as Random

In Section V-B, we show through simulations that the worst-case approach of the previous section leads to a severe decline in performance relative to the scenario where the distribution parameters in (24) are known accurately. In practice it may be possible to learn the value of these parameters online. In order to learn the parameters  $\{\theta_a\}$  we need to have a statistical model for these parameters and a reliable statistical model for the channel state process. In this section we model the parameters  $\{\theta_a\}$  as random variables, which are i.i.d. across the channels and independent of the Markov process as well as the noise process.

In order to assure the convergence of our learning algorithm, we also assume that the cardinality of set  $\Theta$  is finite<sup>4</sup> and let  $|\Theta| = N$ . Let  $\{\mu_i\}_1^N$  denote the elements of set  $\Theta$ . The prior distribution of the parameters  $\{\theta_a\}$  is known to the secondary users. The beliefs of the different channels no longer form a sufficient statistic for this problem. Instead, we keep track of the following set of *a posteriori* probabilities which we refer to as *joint beliefs*:

$$\{P(\{(\theta_a, S_a(k)) = (\mu_i, j)\} | I_k) : \forall i, j, a\}. \quad (31)$$

Since we assume that the parameters  $\{\theta_a\}$  take values in a finite set, we can keep track of these joint beliefs just as we kept track of the beliefs of the states of different channels in Section III. For the initial values of these joint beliefs we use the product distribution of the stationary distribution of the Markov chain and the prior distribution on the parameters  $\{\theta_a\}$ . We store these joint beliefs at the end of slot  $k$  in an  $L \times N \times 2$  array  $Q(k)$  with elements given by

$$Q_{a,i,j}(k) = P(\{(\theta_a, S_a(k)) = (\mu_i, j)\} | Y_a^k, K_a^k). \quad (32)$$

The entries of the array  $Q(k)$  corresponding to channel  $a$  represent the joint *a posteriori* probability distribution of the parameter  $\theta_a$  and the state of channel  $a$  in slot  $k$  conditioned on the information available up to slot  $k$  which we called  $I_k$ . Now define

$$H_{a,i,j}(k) = \sum_{\ell \in \{0,1\}} P(\ell, j) Q_{a,i,\ell}(k-1).$$

Again, the values of the array  $H(k)$  represent the *a posteriori* probability distributions about the parameters  $\{\theta_a\}$  and the channel states in slot  $k$  conditioned on  $I_{k-1}$ , the information up to slot  $k-1$ . The update equations for the joint beliefs can now be written as follows:

$$Q_{a,i,j}(k) = \begin{cases} \lambda H_{a,i,0}(k) f_{\theta_0}(Y_a(k)) & \text{if } j = 0 \\ \lambda H_{a,i,1}(k) f_{\mu_i}(Y_a(k)) & \text{if } j = 1 \end{cases}$$

when channel  $a$  was accessed in slot  $k$ , and  $Q_{a,i,j}(k) = H_{a,i,j}(k)$  otherwise. Here  $\lambda$  is just a normalizing factor.

It is shown in Appendix that, for each channel  $a$ , the *a posteriori* probability mass function of parameter  $\theta_a$  conditioned on the information up to slot  $k$ , converges to a delta-function at the true value of parameter  $\theta_a$  as  $k \rightarrow \infty$ , provided we sense channel  $a$  frequently enough. This essentially means that we can learn the value of the actual realization of  $\theta_a$  by just updating the joint beliefs. This observation suggests that we could use this knowledge learned about the parameters in order to obtain better performance than that obtained under the policy of Section IV-A. We could, for instance, use the knowledge of the true value of  $\theta_a$  to be more liberal in our access policy than the satisfy-all-constraints approach that we used in Section IV-A when we did a worst-case design. With this in mind, we propose the following algorithm for choosing the threshold to be used in each slot for determining whether or not to access the spectrum.

<sup>4</sup>We do discuss the scenario when  $\Theta$  is a compact set in the example considered in Section V-B.

Assume channel  $a$  was sensed in slot  $k$ . We first arrange the elements of set  $\Theta$  in increasing order of the *a posteriori* probabilities of parameter  $\theta_a$ . We partition  $\Theta$  into two groups, a “lower” partition and an “upper” partition, such that all elements in the lower partition have lower *a posteriori* probability values than all elements in the upper partition. The partitioning is done such that the number of elements in the lower partition is maximized subject to the constraint that the *a posteriori* probabilities of the elements in the lower partition add up to a value lower than  $\zeta$ . These elements of  $\Theta$  can be ignored while designing the access policy since the sum of their *a posteriori* probabilities is below the interference constraint. We then design the access policy such that we meet the interference constraint conditioned on parameter  $\theta_a$  taking any value in the upper partition. The mathematical description of the algorithm is as follows. Define

$$b_a^i(k) \triangleq \sum_{j \in \{0,1\}} Q_{a,i,j}(k-1).$$

The vector  $(b_a^1(k), b_a^2(k), \dots, b_a^N(k))^\top$  represents the *a posteriori* probability mass function of parameter  $\theta_a$  conditioned on  $I_{k-1}$ , the information available up to slot  $k-1$ . Now let  $\pi_k(i) : \{1, 2, \dots, N\} \mapsto \{1, 2, \dots, N\}$  be a permutation of  $\{1, 2, \dots, N\}$  such that  $\{\mu_{\pi_k(i)}\}_{i=1}^N$  are arranged in increasing order of *a posteriori* probabilities, i.e.,

$$i \geq j \Leftrightarrow b_a^{\pi_k(i)}(k) \geq b_a^{\pi_k(j)}(k)$$

and let  $N_a(k) = \max\{c \leq N : \sum_{i=1}^c b_a^{\pi_k(i)}(k) < \zeta\}$ . Now define set  $\Theta_a(k) = \{\mu_{\pi_k(i)} : i \geq N_a(k)\}$ . This set is the upper partition mentioned earlier. The access decision on channel  $a$  in slot  $k$  is given by,<sup>5</sup>

$$\tilde{\delta}_a(k) = \mathcal{I}_{\{u_k=a\}} \prod_{\theta \in \Theta_a(k)} \mathcal{I}_{\{\mathcal{L}_\theta(Y_a(k)) < \tau_\theta\}} \quad (33)$$

where  $\tau_\theta$  satisfy (27). The access policy given above guarantees that

$$P(\{\tilde{\delta}_a(k) = 1\} | \{S_a(k) = 1\}, Y^{k-1}, K^{k-1}) < \zeta \quad (34)$$

when the same holds without conditioning on  $Y^{k-1}$  and  $K^{k-1}$ . Hence, the interference constraint is met on an average, averaged over the *a posteriori* distributions of  $\theta_a$ . Now it is shown in Appendix that the *a posteriori* probability mass function of parameter  $\theta_a$  converges to a delta function at the true value of parameter  $\theta_a$  almost surely. Hence, the constraint is asymptotically met even conditioned on  $\theta_a$  taking the correct value. This follows from the fact that, if  $\mu_{i^*}$  is the actual realization of the random variable  $\theta_a$ , and  $b_a^{i^*}(k)$  converges to 1 almost surely, then, for sufficiently large  $k$ , (33) becomes:  $\tilde{\delta}_a(k) = \mathcal{I}_{\{u_k=a\}} \mathcal{I}_{\{\mathcal{L}_{\mu_{i^*}}(Y_a(k)) < \tau_{\mu_{i^*}}\}}$  with probability one and hence the claim is satisfied.

It is important to note that the access policy given in (33) need not be the optimal access policy for this problem. Unlike in Section II, here we are allowing the access decision in slot  $k$

<sup>5</sup>The access policy obtained via the partitioning scheme is simple to implement but is not the optimal policy in general. The optimal access decision on channel  $a$  in slot  $k$  would be given by a likelihood-ratio test between  $f_{\theta_0}$  and the mixture density  $\sum_{\theta \in \Theta} r_\theta(k-1) f_\theta$  where  $r_\theta(k-1)$  represents the value of the posterior distribution of  $\theta_a$  after slot  $k-1$ , evaluated at  $\theta$ . However, setting thresholds for such a test is prohibitively complex.

to depend on the observations in all slots up to  $k$  via the joint beliefs. Hence, it is no longer obvious that the optimal test should be a threshold test on the LLR of the observations in the current slot even if parameter  $\theta_a$  is known. However, this structure for the access policy can be justified from the observation that it is simpler to implement in practice than some other policy that requires us to keep track of all the past observations. The simulation results that we present in Section V-B also suggest that this scheme achieves substantial improvement in performance over the worst-case approach, thus further justifying this structure for the access policy.

Under this scheme the new greedy policy for channel selection is to sense the channel which promises the highest expected instantaneous reward which is now given by

$$\tilde{u}_k^{\text{gr}} = \arg \max_{a \in \mathcal{C}} \left\{ \sum_{i=1}^N h_{a,i,0}(k)(1 - \epsilon_a(k)) \right\} \quad (35)$$

where

$$\epsilon_a(k) = \mathbb{P} \left( \bigcup_{\theta \in \Theta_a(k)} \{ \mathcal{L}_\theta(Y_a(k)) > \tau_\theta \} \mid \{ S_a(k) = 0 \} \right).$$

However, in order to prove the convergence of the *a posteriori* probabilities of the parameters  $\{\theta_a\}$ , we need to make a slight modification to this channel selection policy. In our proof, we require that each channel is accessed frequently. To enforce that this condition is satisfied, we modify the channel selection policy so that the new channel selection scheme is as follows:

$$\tilde{u}_k^{\text{mod}} = \begin{cases} \mathcal{C}_j & \text{if } k \equiv j \pmod{CL}, \quad j \in \mathcal{C} \\ \tilde{u}_k^{\text{gr}} & \text{else} \end{cases} \quad (36)$$

where  $C > 1$  is some constant and  $\{\mathcal{C}_j : 1 \leq j \leq L\}$  is some ordering of the channels in  $\mathcal{C}$ .

## V. SIMULATION RESULTS AND COMPARISONS

### A. Known Distributions

We consider a simple model for the distributions of the observations and illustrate the advantage of our proposed scheme over that in [4] by simulating the performances obtained by employing the greedy algorithm on both these schemes. We also consider a combined scheme that uses both the channel observations and the ACK signals for updating beliefs.

We simulated the greedy policy under three different schemes. Our scheme, which we call  $G_1$ , uses only the observations made on the channels to update the belief vectors. The second one,  $G_2$ , uses only the ACK signals transmitted by the secondary receiver, while the third one,  $G_3$ , uses both observations as well as the error-free ACK signals. We have performed the simulations for two different values of the interference constraint  $\zeta$ . The number of channels was kept at  $L = 2$  in both cases and the transition probability matrix used was

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$$

where the first index represents state 0 and the second represents state 1. Both channels were assumed to have unit bandwidth,

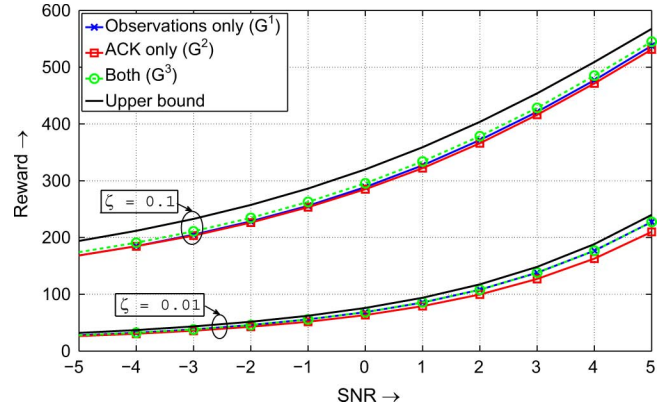


Fig. 1. Comparison of performances obtained with greedy policy that uses observations and greedy policy that uses ACKs. Performance obtained with greedy policy that uses both ACKs as well as observations and the upper bound are also shown.

$B = 1$  and the discount factor was set to  $\alpha = 0.999$ . Such a high value of  $\alpha$  was chosen to approximate the problem with no discounts which would be the problem of practical interest. As we saw in Section III, the spectrum access problem with a group of cooperating secondary users is equivalent to that with a single user. Hence, in our simulations we use a scalar observation model with the following simple distributions for  $Y_a(k)$  under the two hypotheses:

$$\begin{aligned} S_a(k) = 0 \text{ (primary OFF)} : Y_a(k) &\sim \mathcal{N}(0, \sigma^2) \\ S_a(k) = 1 \text{ (primary ON)} : Y_a(k) &\sim \mathcal{N}(\mu, \sigma^2). \end{aligned} \quad (37)$$

It is easy to verify that the LLR for these observations is an increasing linear function of  $Y_a(k)$ . Hence the new access decisions are made by comparing  $Y_a(k)$  to a threshold  $\tau$  chosen such that

$$\mathbb{P}(\{Y_a(k) < \tau\} \mid \{S_a(k) = 1\}) = \zeta \quad (38)$$

and access decisions are given by

$$\delta_a(k) = \mathcal{I}_{\{Y_a(k) < \tau\}} \mathcal{I}_{\{u_k = a\}}. \quad (39)$$

The belief updates in (8) are now given by

$$p_a(k) = \frac{q_a(k)f(\mu, \sigma^2, Y_a(k))}{q_a(k)f(\mu, \sigma^2, Y_a(k)) + (1 - q_a(k))f(0, \sigma^2, Y_a(k))}$$

when channel  $a$  was selected in slot  $k$  (i.e.,  $u_k = a$ ), and  $p_a(k) = q_a(k)$  otherwise. Here  $q_a(k)$  is given by (7) and  $f(x, y, z)$  represents the value of the Gaussian density function with mean  $x$  and variance  $y$  evaluated at  $z$ . For the mean and variance parameters in (37) we use  $\sigma = 1$  and choose  $\mu$  so that  $\text{SNR} = 20 \log_{10}(\mu/\sigma)$  takes values from  $-5$  dB to  $5$  dB. In the case of cooperative sensing, this SNR represents the effective signal-to-noise ratio (SNR) in the joint LLR statistic at the decision center,  $\mathcal{L}(\underline{X}_a(k))$ . We perform simulations for two values of the interference constraint,  $\zeta = 0.1$  and  $\zeta = 0.01$ .

As seen in Fig. 1, the strategy of using only ACK signals ( $G_2$ ) performs worse than the one that uses all the observations ( $G_1$ ), especially for  $\zeta = 0.01$ , thus demonstrating that relying only on ACK signals compromises on the amount of information that



can be learned. We also observe that the greedy policy attains a performance that is within 10% of the upper bound. It is also seen in the figure that the reward values obtained under  $G_1$  and  $G_3$  are almost equal. For  $\zeta = 0.01$ , it is seen that the two curves are overlapping. This observation suggests that the extra advantage obtained by incorporating the ACK signals is insignificant especially when the interference constraint is low. The explanation for this observation is that the ACK signals are received only when the signal transmitted by the secondary transmitter successfully gets across to its receiver. For this to happen the state of the primary channel should be “0” and the secondary must decide to access the channel. When the value of the interference constraint  $\zeta$  is low, the secondary accesses the channel only when the value of the  $Y_a(k)$  is low. Hence the observations in this case carry a significant amount of information about the states themselves and the additional information that can be obtained from the ACK signals is not significant. Thus learning using only observations is almost as good as learning using both observations as well as ACK signals in this case.

### B. Unknown Distributions

We compare the performances of the two different approaches to the spectrum access problem with unknown distributions that we discussed in Section IV. We use a parameterized version of the observation model we used in the example in Section V-A. We assume that the primary and secondary users are stationary and assume that the secondary user is unaware of its location relative to the primary transmitter. We assume that the secondary user employs some form of energy detection, which means that the lack of knowledge about the location of the primary manifests itself in the form of an unknown mean power of the signal from the primary. Using Gaussian distributions as in Section V-A, we model the lack of knowledge of the received primary power by assuming that the mean of the observation under  $H_1$  on channel  $a$  is an unknown parameter  $\theta_a$  taking values in a finite set of positive values  $\Theta$ . The new hypotheses are

$$\begin{aligned} S_a(k) = 0 : Y_a(k) &= N_a(k) \\ S_a(k) = 1 : Y_a(k) &= \theta_a + N_a(k) \end{aligned}$$

$$\text{where } N_a(k) \sim \mathcal{N}(0, \sigma^2), \quad \theta_a \in \Theta, \min(\Theta) > 0. \quad (40)$$

For the set of parameterized distributions in (40), the log-likelihood ratio function  $\mathcal{L}_\theta(x)$  defined in (25) is linear in  $x$  for all  $\theta \in \Theta$ . Hence comparing  $\mathcal{L}_\theta(Y_a(k))$  to a threshold is equivalent to comparing  $Y_a(k)$  to some other threshold. Furthermore, for this set of parameterized distributions, it is easy to see that the conditional cumulative distribution function (cdf) of the observations  $Y_a(k)$  under  $H_1$ , conditioned on  $\theta_a$  taking value  $\theta$ , is monotonically decreasing in  $\theta$ . Furthermore, under the assumption that  $\min \Theta > 0$ , it follows that choosing  $\theta^* = \min \Theta$  satisfies the conditions of (28). Hence the optimal access decision under the worst-case approach given in (30) can be written as

$$\hat{\delta}_a(k) = \mathcal{I}_{\{u_k=a\}} \mathcal{I}_{\{Y_a(k) < \tau_w\}} \quad (41)$$

where  $\tau_w$  satisfies

$$\text{P}(\{Y_a(k) < \tau_w\} | \{S_a(k) = 1, \theta_a = \theta^*\}) = \zeta \quad (42)$$

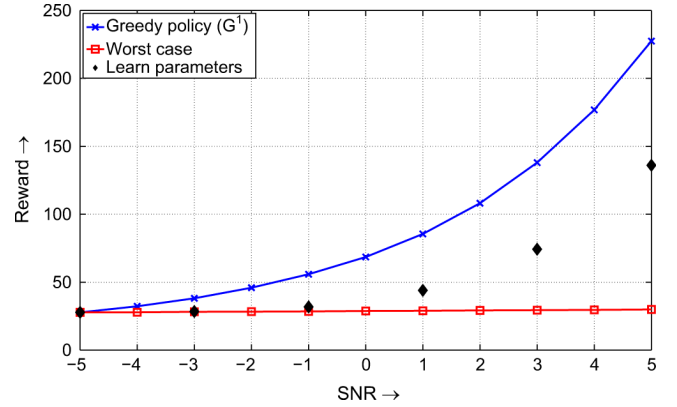


Fig. 2. Comparison of performances obtained with worst-case approach and learning approach. Performance obtained with greedy policy is also shown. The value of  $\zeta = 0.01$ .

where  $\theta^* = \min \Theta$ . Thus the worst-case solution for this set of parameterized distributions is identical to that obtained for the problem with known distributions described in (37) with  $\mu$  replaced by  $\theta^*$ . Thus the structures of the access policy, the channel selection policy, and the belief update equations are identical to those derived in the example shown in Section V-A with  $\mu$  replaced by  $\theta^*$ .

Similarly, the access policy for the case of random  $\theta_a$  parameters given in (33) can now be written as

$$\tilde{\delta}_a(k) = \mathcal{I}_{\{u_k=a\}} \mathcal{I}_{\{Y_a(k) < \tau_r(k)\}} \quad (43)$$

where  $\tau_r(k)$  satisfies

$$\text{P}(\{Y_a(k) < \tau_r(k)\} | \{S_a(k) = 1, \theta_a = \theta^\#(k)\}) = \zeta \quad (44)$$

where  $\theta^\#(k) = \min \Theta_a(k)$ . The belief updates and greedy channel selection are performed as described in Section IV-B. The quantity  $\epsilon_a(k)$  appearing in (35) can now be written as

$$\epsilon_a(k) = \text{P}(\{Y_a(k) > \tau_r(k)\} | \{S_a(k) = 0\}).$$

We simulated the performances of both the schemes on the hypotheses described in (40). We used the same values of  $L$ ,  $P$ ,  $\alpha$  and  $\sigma$  as in Section V-A. We chose set  $\Theta$  such that the SNR values in dB given by  $20 \log(\mu_i/\sigma)$  belong to the set  $\{-5, -3, -1, 1, 3, 5\}$ . The prior probability distribution for  $\theta_a$  was chosen to be the uniform distribution on  $\Theta$ . The interference constraint  $\zeta$  was set to 0.01. Both channels were assumed to have the same values of true SNR while the simulations were performed. The reward was computed over 10 000 slots since the remaining slots do not contribute significantly to the reward. The value of  $C$  in (36) was set to a value higher than the number of slots considered so that the greedy channel selection policy always uses the second alternative in (36). Although we require (36) for our proof of convergence of the *a posteriori* probabilities in the Appendix, it was observed in simulations that this condition was not necessary for convergence.

The results of the simulations are given in Fig. 2. The net reward values obtained under the worst-case design of Section IV-A and that obtained with the algorithm for learning  $\theta_a$  given in Section IV-B are plotted. We have also included the

rewards obtained with the greedy algorithm  $G_1$  with known  $\theta_a$  values; these values denote the best rewards that can be obtained with the greedy policy when the parameters  $\theta_a$  are known exactly. Clearly, we see that the worst-case design gives us almost no improvement in performance for high values of actual SNR. This is because the threshold we choose is too conservative for high SNR scenarios leading to several missed opportunities for transmitting. The minimal improvement in performance at high SNR is due to the fact that the system now has more accurate estimates of the channel beliefs although the update equations were designed for a lower SNR level. The learning scheme, on the other hand, yields a significant performance advantage over the worst-case scheme for high SNR values as seen in the figure. It is also seen that there is a significant gap between the performance with learning and that with known  $\theta_a$  values at high SNR values. This gap is due to the fact that the *posteriori* probabilities about the  $\theta_a$  parameters take some time to converge. As a result of this delay in convergence a conservative access threshold has to be used in the initial slots thus leading to a drop in the discounted infinite horizon reward. However, if we were using an average reward formulation for the dynamic program rather than a discounted reward formulation, we would expect the two curves to overlap since the loss in the initial slots is insignificant while computing the long-term average reward.

*Remark 5.1:* So far in this paper, we have assumed that the cardinality of set  $\Theta$  is finite. The proposed learning algorithm can also be adapted for the case when  $\Theta$  is a compact set. A simple example illustrates how this may be done. Assuming parameterized distributions of the form described in (40), suppose that the value of  $\theta_a$  in dB is uniformly distributed in the interval  $[-4.5, 4.5]$  and that we compute the *posteriori* probabilities of  $\theta_a$  assuming that its value in dB is quantized to the finite set  $\Theta = \{-5, -4, \dots, 5\}$ . Now if the actual realization of  $\theta_a$  is between 1 and 2 dB, say 1.5 dB, then we expect to see low *posteriori* probabilities for all elements of  $\Theta$  except 1 and 2 dB and in this case it would be safe to set the access threshold assuming an SNR of 1 dB. Although this threshold is not the best that can be set for the actual realization of  $\theta_a$ , it is still a significant improvement over the worst-case threshold which would correspond to an SNR of  $-4.5$  dB. We expect the *a posteriori* probabilities of all elements of  $\Theta$  other than 1 and 2 dB to converge to 0, but the *a posteriori* probabilities of these two values may not converge; they may oscillate between 0 and 1 such that their sum converges to 1. A rigorous version of the above argument would require some ordering of the parameterized distributions as in (28).

## VI. CONCLUSIONS AND DISCUSSION

The results of Section V-A and the arguments we presented in Section III-C clearly show that our scheme of estimating the channel occupancies using the observations yields performance gains and may have practical advantages over the ACK-based scheme that was proposed in [4]. We believe that these advantages are significant enough to justify using our scheme even though it necessitates the use of dedicated control channels for synchronization.

For the scenario where the distributions of the received signals from the primary transmitters are unknown and belong to

a parameterized family, the simulation results in Section V-B suggest that designing for worst-case values of the unknown parameters can lead to a significant drop in performance relative to the scenario where the distributions are known. Our proposed learning-based scheme overcomes this performance drop by learning the primary signal's statistics. The caveat is that the learning procedure requires a reliable model for the state transition process if we need to give probabilistic guarantees of the form (34) and to ensure convergence of the beliefs about the  $\theta_a$  parameters.

In most of the existing literature on sensing and access policies for cognitive radios that use energy detectors, the typical practice is to consider a worst-case mean power under the primary-present hypothesis. The reasoning behind this approach is that the cognitive users have to guarantee that the probability of interfering with any primary receiver located within a protected region [9], [10] around the primary transmitter is below the interference constraint. Hence, it is natural to assume that the mean power of the primary signal is the worst-case one, i.e., the mean power that one would expect at the edge of the protected region. However, the problem with this approach is that by designing for the worst-case distribution, the secondary users are forced to set conservative thresholds while making access decisions. Hence, even when the secondary users are close to the primary transmitter and the SNR of the signal they receive from the primary transmitter is high, they cannot efficiently detect vacancies in the primary spectrum. Instead, if they were aware that they were close to the transmitter they could have detected spectral vacancies more efficiently as demonstrated by the improvement in performance at higher SNRs observed in the simulation example in Section V-A. This loss in performance is overcome by the learning scheme proposed in Section IV-B. By learning the value of  $\theta_a$  the secondary users can now set more liberal thresholds and, hence, exploit vacancies in the primary spectrum better when they are located close to the primary transmitter. Thus, using such a scheme would produce a significant performance improvement in overall throughput of the cognitive radio system.

## APPENDIX

Here we show that for each channel  $a$ , the *a posteriori* probability mass function of parameter  $\theta_a$  converges to a delta-function at the true value of the parameter almost surely under the algorithm described in Section IV-B.

*Theorem A.1:* Assume that the transition probability matrix  $P$  satisfies (16). Further assume that the conditional densities of the observations given in (24) satisfy

$$\int f_{\mu_i}(y) |\log(f_{\mu_i}(y))| dy < \infty \text{ for all } \mu_i \in \Theta \quad (45)$$

and that all densities in (24) are distinct. Then, under the channel sensing scheme that was introduced in (36) for each channel  $a$ ,

$$P(\{\theta_a = \mu_i\} | Y^n) \xrightarrow[n \rightarrow \infty]{a.s.} \mathcal{I}_{\{\theta_a = \mu_i\}}, \text{ for all } \mu_i \in \Theta.$$

*Proof:* Without loss of generality, we can restrict ourselves to the proof of the convergence of the *a posteriori* distribution of  $\theta_1$ , the parameter for the first channel. By the modified sensing scheme introduced in (36), it can be seen that channel 1 is sensed at least every  $ML$  slots. Hence, if the *a posteriori*

distribution converges for an algorithm that senses channel 1 exactly every  $ML$  slots, it should converge even for our algorithm, since our algorithm updates the *a posteriori* probabilities more frequently. Furthermore by considering an  $ML$ -times undersampled version of the Markov chain that determines the evolution of channel 1, without loss of generality, it is sufficient to show convergence for a sensing policy in which channel 1 is sensed in every slot. It is obvious that since condition (16) holds for the original Markov chain, it holds even for the undersampled version. So now we assume that an observation  $Y_k$  is made on channel 1 in every slot  $k$ . We use  $Y^k$  to represent all observations on channel 1 up to slot  $k$ .

We use  $\mu_i^* \in \Theta$  to represent the true realization of random variable  $\theta_1$  with  $i^* \in \{1, \dots, N\}$ , and  $\pi$  to denote the prior distribution of  $\theta_1$ . The *a posteriori* probability mass function of  $\theta_1$  evaluated at  $\mu_i$  after  $n$  time slots can be expressed as

$$P(\{\theta_1 = \mu_j\} | Y^n) = \frac{P_j(Y^n)\pi(\mu_j)}{\sum_i P_i(Y^n)\pi(\mu_i)} \quad (46)$$

where we use the notation  $P_i(\cdot)$  to denote the distribution of the observations conditioned on  $\theta_1$  taking the value  $\mu_i \in \Theta$ . It follows from [12, Theorem 1, Theorem 2, and Lemma 6] that conditioned on  $\{\theta_1 = \mu_{i^*}\}$  we have

$$\frac{P_{i^*}(Y^n)}{P_i(Y^n)} \xrightarrow[n \rightarrow \infty]{a.s.} \infty \text{ for all } i \neq i^*.$$

Hence, it follows from (46) that conditioned on  $\{\theta_1 = \mu_{i^*}\}$  we have

$$\frac{P_{i^*}(Y^n)\pi(\mu_{i^*})}{\sum_i P_i(Y^n)\pi(\mu_i)} \xrightarrow[n \rightarrow \infty]{a.s.} 1$$

which further implies that conditioned on  $\{\theta_1 = \mu_{i^*}\}$  we have

$$P_j(Y^n)\pi(\mu_j) \xrightarrow[n \rightarrow \infty]{a.s.} \mathcal{I}_{\{i^*=j\}}.$$

Since this holds for all possible realizations  $\mu_{i^*} \in \Theta$  of  $\theta_1$ , the result follows. ■

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