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# Hybrid Acquisition of Direct Sequence CDMA Signals[1]

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In direct sequence code division multiple access (DS-CDMA) systems, signal acquisition is necessary before communication can commence. Recent work has shown that the problem of acquisition may be even more restrictive than the problem of error control in limiting the capacity and performance of these systems. Passive matched filters and parallel search schemes have been shown to be able to acquire signals rapidly, but they do so at the cost of high or prohibitive complexity. In contrast, straight serial search schemes have lower complexity but acquire the signal much more slowly. In this paper, we study *hybrid* active correlation schemes that provide flexibility in the trade-off between acquisition speed and complexity. These techniques test several phases concurrently and either decide that a particular phase is correct, in which case the decision is verified by a binary hypothesis test, or that none of the phases are correct, in which case another group of phases is tested. Two new hybrid acquisition schemes are presented; the first is based on a fixed-sample-size weighted MAP test, and the second is based on an $M$-ary sequential hypothesis test called MSPRT. It is shown that the weighted MAP-based scheme hypothesis test called MSPRT. It is shown that the weighted MAP-based scheme outperforms a standard MAP-based scheme. It is also shown that considerable performance gain can be obtained using sequential testing. In particular, it is shown through numerical examples that MSPRT-based schemes are several times faster than corresponding fixed-sample-size schemes.

**KEY WORDS:** Hybrid acquisition; CDMA; sequential detection; MSPRT.

## 1. INTRODUCTION

*Acquistion* in direct sequence spread-spectrum systems can be defined as the process of coarsely aligning the phase, or delay, of a locally generated despreading signal with that of an incoming (spread) signal. Recent work has shown that in direct sequence code division multiple access (DS/CDMA) systems including personal communication systems and networks (PCS), the problem of acquisition may be even more restrictive on system capacity and performance than the problem of error control [1]. For this reason, the design of efficient acquisition schemes is a critical component in the overall design of CDMA systems.

In general, the phase of the incoming sequence is said to be acquired if it is determined to within some fraction (typically one-half) of a chip duration. A *tracking* stage can then lock onto the signal with greater accuracy [2]. There are two basic approaches to acquiring the phase. The first is to use a passive matched filter that is matched to a section of the incoming sequence; a peak in the output of the filter determines the phase.[4] A second approach is to use a single correlator or a set of correlators to exhaustively test the various possible phases of the incoming sequence, either serially or in parallel. Passive matched filter schemes acquire signals rapidly, since the time taken to cover all possible phases

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[4] An alternative approach is to declare the phase as matched at the first moment the output of the filter exceeds a fixed threshold.

in the phase uncertainty interval is simply the length of the matched section plus the length of the phase uncertainty interval. However, building matched filters of appropriate length for reliable acquisition can be prohibitively expensive, especially in a low signal-to-noise ratio environment (such as would tend to occur in systems subject to multiple access interference and fading). For this reason, active correlation schemes tend to be more practical.

Among active correlation schemes there are two extremes, straight serial search schemes and parallel search schemes. Straight serial search schemes use only one correlator and check only one phase at a time [2, 3], whereas parallel search schemes use a large bank of correlators to test all possible phases concurrently [4, 5]. Straight serial search schemes (also known as sliding correlators) appear to be a practical choice because of their simplicity. However, acquisition via straight serial search schemes can be quite slow. Parallel search schemes have much shorter acquisition times, but this benefit is at the expense of much greater complexity. This has led to the development of hybrid schemes that trade off the speed of parallel search schemes with the simplicity of straight serial search schemes. Naturally these hybrid schemes are of great practical interest.

The hybrid schemes simply test more than one phase at the testing stage, using $M - 1$ correlators (where $M$ is greater than 2 but much smaller than the total number of phases) each tuned to a different phase. The testing stage is thus faced with an $M$-ary hypothesis testing problem—each of $M - 1$ of these hypotheses correspond to deciding that a particular phase is correct, and an additional hypothesis corresponds to deciding that none of these $M - 1$ phases are correct.

The use of hybrid schemes has been discussed and studied previously in the literature [6–11]. In much of this work (see, for example, Section 1 of [7], Section 2 of [10]), it has been assumed that it is optimal for the testing stage to use a bank of $M - 1$ correlators that correlate over a fixed number of chip intervals[5] and declare that the largest output corresponds to the correct phase if the output exceeds a preselected threshold.

This test, which we refer to as the *maximum correlation test*, has three significant drawbacks. First, the correlator outputs can only be viewed as sufficient statistics if the hypothesis that none of the $M - 1$ candidate phases is correct is modeled as a noise-only signal, which is referred to as the "zero sequence" model [12]. But it has been shown that the zero sequence model is far from accurate and that the null hypothesis is much better modeled as a random binary sequence [12].

Second, the structure of the test (choose the largest correlation if it exceeds a threshold) does not have any known optimality with respect to minimizing acquisition time. The only exception to this statement is the degenerate case of $M = 2$ (straight serial search), because in this case the test can be expressed in terms of a likelihood ratio [13]. Even here, however, the optimality is limited to a class of schemes that use the zero sequence model for the null hypothesis.

Third, the threshold for the maximum correlation test is unspecified. In general, a determination of the threshold that results in the minimum average acquisition time can be obtained through the use of simulations only.

In this paper, we show that a *weighted maximum a posteriori probability* (weighted MAP) test is "approximately optimal" (in a manner defined precisely in Section 5) among all fixed sample size (FSS) tests for the minimization of the average acquisition time. The weighted MAP is similar in structure to the maximum correlation test, except that the likelihood ratio of the chip correlations is computed using the random sequence model for the null hypothesis. As with the maximum correlation test, a detection decision is made if the largest of these likelihood ratios exceeds a threshold, but, in contrast, the best threshold for the weighted MAP test can be calculated very easily without resorting to simulations. We study the performance of hybrid schemes based on this new weighted MAP test.

The optimality discussed above is restricted to FSS tests. For straight serial search schemes, the average acquisition time can be reduced considerably by using a *sequential* binary test (in particular, the sequential probability ratio test (SPRT)), as opposed to a FSS test, at the testing stage [12, 14]. A similar gain in hybrid testing is highly desirable. To achieve this gain, an appropriately designed $M$-ary sequential test must be utilized. The use of sequential multihypothesis decision theory with hybrid acquisition has not been investigated to date.

In this paper, we investigate the use of a test known as the MSPRT for acquisition. The MSPRT, a generalization of the SPRT, is described and analyzed in [15]. It is shown in [15] that the MSPRT is typically two to three times faster than the best FSS test, a gain similar to that obtained by using the SPRT for binary hypothesis testing. In this paper, we show that the weighted MAP test outperforms a standard MAP test, and we also show that the MSPRT significantly outperforms both the MAP and the weighted MAP schemes.

[5] If the carrier phase is unknown, the correlation over a chip interval is taken to be the sum of the squares of an in-phase and a quadrature correlation.

The paper is organized as follows. The system model is given in Section 2. In Section 3, detailed descriptions of the weighted MAP-based and MSPRT-based acquisition schemes are given. A general framework for the analysis of hybrid acquisition schemes is presented in Section 4. The design of these hybrid schemes to minimize the average acquisition time is discussed in Section 5. Numerical results comparing various acquisition schemes and directions for further research are given in Section 6.

## 2. SYSTEM MODEL

The model that is used here for the DS/SS received signal $r(t)$ is the standard one (see, e.g., [2]),

$$r(t) = \sqrt{2V}\, a(t)\, c(t + \delta T_c) \cos(\omega_0 t + \theta) + n(t) \quad (1)$$

where $V$ is the received signal power, $a(\cdot)$ is the data signal, $c(\cdot)$ is the spreading signal, $\omega_0$ is the carrier frequency, $T_c$ is the chip period of the spreading signal, $\delta T_c$ is the phase of the spreading signal, $\theta$ is the phase of the carrier, and $n(\cdot)$ is additive channel noise.

The spreading signal $c(t)$ is given by

$$c(t) = \sum_{j=-\infty}^{\infty} c^{(0)}(j)\Pi_{T_c}(t - jT_c)$$

where $\Pi_{T_c}(\cdot)$ is a unit rectangular pulse of duration $T_c$ and $\{c^{(0)}\}$ is the code sequence used for spreading. This sequence is a doubly infinite binary sequence of the form

$$(\ldots, c^{(0)}(0), c^{(0)}(1), c^{(0)}(2), \ldots) \quad (2)$$

where $c^{(0)}(j) \in \{-1, 1\}$. The $k$th phase of $\{c^{(0)}\}$ is denoted by $\{c^{(k)}\}$; i.e.,

$$c^{(k)}(j) = c^{(0)}(j + k), \qquad j = \ldots, -1, 0, 1, \ldots \quad (3)$$

The acquisition problem is to determine the unknown phase, $\delta T_c$, of the spreading signal. It is assumed that the phase uncertainty is restricted to the interval $[0, LT_c)$, where $L$ is some positive integer. Since spreading sequences are periodic, the maximum value that $L$ can take is the period of the spreading sequence.

Several assumptions are made with respect to a receiver acquiring the spread signal. First, it is assumed that the receiver is perfectly synchronized to the carrier, and that coherent demodulation of the carrier is carried out prior to acquisition. Second, it is assumed that there is no data modulation during the acquisition process. This assumption is well justified since DS/SS transmissions usually include a training period during which the carrier is modulated only by the spreading signal [2]. Third, it is assumed that the chip boundaries of the spreading signal $c(\cdot)$ are aligned with that of the locally generated signal. This assumption is justified in systems where the chip boundaries can be determined prior to acquisition. These three assumptions lead to a considerable simplification of the acquisition problem, and provide a good starting point for the study of hybrid schemes. In Section 6, we discuss how these assumptions may be relaxed.

Based on the above assumptions, we may correlate the received signal $r(t)$ with $\cos(\omega_0 t + \theta)$, set the data signal $a(t)$ to 1, and assume that the phase $\delta$ of the spreading signal is an integer (denoted by $d$). Then, with a slight abuse of notation, we have the following equivalent baseband model for acquisition:

where $d$ is an integer belonging to $\{0, \ldots, L - 1\}$, and $n(t)$ is white Gaussian noise with spectral density $N_0/2$. The acquisition problem then reduces to determining the value of $d$.

## 3. HYBRID ACQUISITION SCHEMES

The general structure of hybrid acquisition schemes is shown in Fig. 1. The receiver divides the $L$ phases exhaustively into groups of phases, each containing $M - 1$ phases. These groups are then tested serially for the phase of the incoming sequence. Let $\{d_1, \ldots, d_{M-1}\}$ be a particular group of phases. To test this group, the receiver generates the spreading signal at each of the $M - 1$ phases, and correlates these $M - 1$ signals with

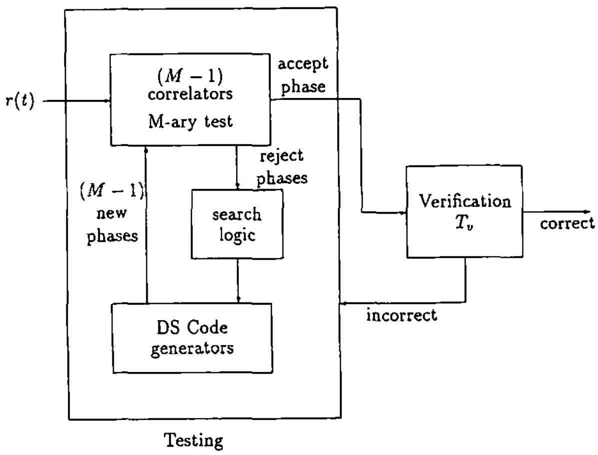


**Fig. 1.** Structure of hybrid acquisition schemes.

the received signal $r(t)$ (of Eq. (4)). The outputs of the $M - 1$ correlators are then used in an $M$-ary hypothesis test in the testing stage, the $M$ hypotheses being

$$H_0: d \neq d_1, \ldots, d_{M-1}$$

and, for $k = 1, \ldots, M - 1$,

$$H_k: d = d_k$$

If the decision made at the testing stage is in favor of $H_i$, for some $i$, then this decision is checked at the verification stage. Otherwise, the test proceeds to test the next set of $M - 1$ phases. It is assumed that the verification stage employs a very long correlation time $T_v$ and that its decisions are virtually error-free. If the phase $d_i$ is verified, then it is assumed that acquisition is complete. Otherwise, a new set of $M - 1$ phases is tested.

A more precise mathematical description of the testing stage is now given. The received signal $r(t)$ is fed into a bank of $M - 1$ correlators. The $\ell$-th correlator correlates $r(t)$ with $c(t + d_\ell T_c)$, and its output at time $(n + 1)T_c$, after appropriate normalization, is given by

$$\begin{aligned} X_\ell(n) &= \sqrt{\frac{2}{N_0 T_c}} \int_{nT_c}^{(n+1)T_c} r(t)c(t + d_\ell T_c)\, dt \\ &= pc^{(d)}(n)c^{(d_\ell)}(n) + W_\ell(n) \end{aligned}$$

where $p = \sqrt{2VT_c/N_0}$, $\{c^{(d)}\}$ and $\{c^{(d_\ell)}\}$ are as defined by (3), and for fixed $\ell$, $\{W_\ell(n), n = 1, 2, \ldots\}$ are independent and identically distributed (i.i.d.) Gaussian random variables with mean 0 and variance 1.

The vector $(X_1(n), X_2(n), \ldots, X_{M-1}(n))^T$ of correlator outputs is denoted by $\mathbf{X}(n)$. The sequence $\{\mathbf{X}(1), \mathbf{X}(2), \ldots\}$ is used in the $M$-ary test to decide which hypothesis is correct. If a sequential test is employed, this data also determines the stopping time at which the decision is made. The $M$ hypotheses may be written in terms of the correlator outputs as

$$H_0: \mathbf{X}(n) = p\mathbf{C}(n) + \mathbf{W}(n), \quad d \neq d_1, \ldots, d_{M-1} \tag{5}$$

and, for $k = 1, \ldots, M - 1$,

$$H_k: \mathbf{X}(n) = p\mathbf{C}_k(n) + \mathbf{W}(n) \tag{6}$$

where the $i$th component of $\mathbf{C}(n)$ is $c^{(d)}(n)c^{(d_i)}(n)$, the $i$th component of $\mathbf{W}(n)$ is $W_i(n)$, and the $i$th component of $\mathbf{C}_k(n)$ equals 1 if $i = k$ and $c^{(d_k)}(n)c^{(d_i)}(n)$ otherwise.

In order to proceed with the design of the hypothesis test, the joint probability distributions of $\mathbf{X}(n)$ under the $M$ hypotheses need to be determined. There are two problems that are encountered here. First, the hypothesis $H_0$ is a composite hypotheses since, under $H_0$, $d$ can take any of $(L - M + 1)$ possible values. Note that checking for each value of $d$ in (5) essentially results in the implementations of a parallel search scheme. Second, the joint distribution of $\{W_1(n), \ldots, W_{M-1}(n)\}$ depends on the cross-correlation of the sequences $\{c^{(d_1)}\}, \ldots, \{c^{(d_{M-1})}\}$, and an exact characterization of this joint distribution is cumbersome and inappropriate for implementation.

The above two problems are alleviated by using the following alternative test:

$$H_0: \mathbf{X}(n) = p\mathbf{R}(n) + \mathbf{W}(n) \tag{7}$$

and, for $k = 1, \ldots, M - 1$,

$$H_k: \mathbf{X}(n) = p\mathbf{R}_k(\mathrm{n}) + \mathbf{W}(n) \tag{8}$$

where the $i$th component of $\mathbf{R}(n)$ is $R_i(n)$, the $i$th component of $\mathbf{R}_k(n)$ equals 1 if $i = k$ and $R_i(n)$ otherwise, and where $\{R_1\}, \ldots, \{R_{M-1}\}$ are mutually independent sequences of independent random variables that take the values $+1$ and $-1$ with equal probability. We refer to such sequences as *random binary sequences*.

This test is obtained by approximating the sequences $\{c^{(d)}\}$ for all $d \neq d_k$ by mutually independent random binary sequences, if $H_k$ is the true hypothesis. (Included are the cases $d = d_j$ for $j \neq k$.) This model is referred to as the *random sequence model* [12]. In the context of straight serial search schemes, it has been shown in [12] that the random sequence model is very accurate—analytical results obtained using the random sequence model approximation closely match those obtained by simulation of the actual system. The modeling of mutual independence between sequences is motivated by the fact that spreading sequences (a typical example being PN sequences) are designed to have good autocorrelation properties [16]; i.e., the various phase-shifted versions of the code sequence have very little correlation.

With these approximations, it is easily seen that for any $j, k \in \{0, \ldots, L - 1\}$ such that $j \neq k$, $\{c^{(j)}(n)c^{(k)}(n)\}$ is a random binary sequence, and $\{W_1(n), \ldots, W_{M-1}(n)\}$ are independent random variables. The hypotheses of (5) and (6) can hence be approximated by (7) and (8).

The probability densities of the $\mathbf{X}(n)$ under the $M$ hypotheses can now be derived. Note that $\{\mathbf{X}(1), \mathbf{X}(2), \ldots\}$ are i.i.d. random vectors. Let $f_j$ denote the density of $\mathbf{X}(n)$ under $H_j$. Then

$$f_0(\mathbf{x}) = \prod_{\ell=1}^{M-1} \frac{1}{2\sqrt{2\pi}} \left[\exp\left(-\frac{(x_\ell - p)^2}{2}\right) + \exp\left(-\frac{(x_\ell + p)^2}{2}\right)\right]$$

and

$$f_k(\mathbf{x}) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x_k - p)^2}{2}\right) \prod_{\substack{\ell=1 \\ \ell \neq k}}^{M-1} \frac{1}{2\sqrt{2\pi}} \cdot \exp\left(-\frac{(x_\ell - p)^2}{2}\right) + \exp\left(-\frac{(x_\ell + p)^2}{2}\right)\Bigg]$$

where $\mathbf{x} = (x_1, \ldots, x_{M-1})^T$.

The $M$ hypotheses have prior probabilities associated with them. Let $\pi_j$ denote the prior probability of $H_j$. It is assumed that the $L$ possible values for the unknown phase are equally likely. Thus

$$\pi_0 = \frac{L - M + 1}{L}$$

$$\pi_k = \frac{1}{L}, \qquad k = 1, \ldots, M - 1$$

The $M$-ary test used in the testing stage is either a FSS test or a sequential test. The FSS tests use a fixed number (say $N$) of correlator output vectors for decision making. Two FSS tests are considered. The first of these is the *maximum a posteriori probability* (MAP) test that uses the following decision rule:

Choose hypothesis $H_m$ if

$$m = \arg\max_j \left(\pi_j \prod_{i=1}^{N} f_j(\mathbf{X}(i))\right)$$

While the MAP test minimizes the overall error probability [17], it is not the FSS test which results in the minimum average acquisition time. For this reason, the following *weighted* MAP test is also considered:

Choose hypothesis $H_m$ if

$$m = \arg\max_j \left(w_j \pi_j \prod_{i=1}^{N} f_j(\mathbf{X}(i))\right)$$

The weights are chosen so as to minimize the expected acquisition time—it is shown in Section 6 that the optimum weighted MAP test performs significantly better than the MAP test.

The sequential test that is considered here is the MSPRT introduced in [15]. The number of samples $N_A$ used by the sequential test is a random variable which is determined by a stopping rule. The stopping and decision rules for the MSPRT are given by

$N_A$ = first $n \geq 1$ such that

$$\cdot \frac{\pi_k \prod_{i=1}^{n} f_k(\mathbf{X}(i))}{\sum_{j=0}^{M-1} \pi_j \left(\prod_{i=1}^{n} f_j(\mathbf{X}(i))\right)} > \frac{1}{1 + A_k}$$

for at least one $k$

and

Choose hypothesis $H_m$ if

$$m = \arg\max_j \left(\pi_j \prod_{i=1}^{N} f_j(\mathbf{X}(i))\right)$$

where the parameters $A_j$ are positive and are chosen here to minimize the expected acquisition time.

The verification stage is now described in more detail. This stage uses a single correlator and a binary FSS test. If the phase being verified is $d_i$, then $r(t)$ is correlated with $c(t - d_i T_c)$. The output of the correlator at time $(n + 1)T_c$ after appropriate normalization is

$$X(n) = pc^{(d)}(n)c^{(d_i)}(n) + W(n)$$

where $\{W(n), n = 1, 2, \ldots\}$ are i.i.d. Gaussian random variables with mean 0 and variance 1.

Using the random sequence model, the hypotheses $H_0'$ and $H_1'$ that represent, respectively, $d \neq d_i$ and $d = d_i$, are given by

$$H_0'\colon X(n) = pR(n) + W(n)$$

$$H_1'\colon X(n) = p + W(n)$$

where $R(n)$ is a random binary sequence.

Let $f_0'$ and $f_1'$ denote the density of $X(n)$ under $H_0'$ and $H_1'$, respectively. Then

$$f_0'(x) = \frac{1}{2\sqrt{2\pi}} \left[\exp\left(-\frac{(x - p)^2}{2}\right) + \exp\left(-\frac{(x + p)^2}{2}\right)\right]$$

$$f_1'(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{(x - p)^2}{2}\right)$$

The test used in the verification stage is a FSS likelihood ratio test that uses $T_v$ samples. It has the form

$$\text{Choose } H_0' \text{ if } \prod_{i=1}^{T_v} f_0'(X(i)) > \prod_{i=1}^{T_v} f_1'(X(i)) \text{ and } H_1' \text{ otherwise}$$

where $T_v$ is chosen so that the error probabilities under each hypothesis are negligibly small (i.e., smaller than a prespecified tolerance). Note that this test is a maximum likelihood (ML) test [17].

## 4. ANALYSIS

As mentioned in the previous section, the FSS and MSPRT tests used in the testing stage are designed to minimize the expected value of the acquisition time $T_{\text{acq}}$. It is thus of interest to derive a general expression for the expected acquisition time for hybrid schemes. Toward this end, we define the following. Let $T$ represent the number of samples taken at the testing stage. Let $E_0[T]$ and $E_1[T]$ represent the expected value of $T$ under $H_0$ and $H_1$, respectively. (By symmetry, $E_k[T] = E_1[T]$, for $k = 1, \ldots, M - 1$.) Note that $T$ is considered to be a random variable in general, with the understanding that $E_0[T] = E_1[T] = T$ for FSS tests. Furthermore, let

$$P_F = \text{Prob(choose one of } H_j, j \neq 0 | H_0 \text{ is true)}$$

$$P_M = \text{Prob(choose } H_0 | H_j \text{ is true)}, j \neq 0$$

$$P_E = \text{Prob(choose one of } H_k,\ k \neq j,\ k \neq 0 | H_j \text{ is true)}, j \neq 0$$

The fact that $P_M$ and $P_E$ do not depend on $j$ is also due to symmetry. Finally, denote the total number of phase groups being tested by $n_0$. Note that

$$n_0 = \left\lceil \frac{L}{M-1} \right\rceil$$

where $\lceil x \rceil$ denotes the smallest integer $\geq x$.

With these definitions, it is straightforward to draw the block diagram of the acquisition process, under the condition that correct phase is tested in the $i$th group, as shown in Fig. 2. With the help of this diagram and the analysis method using signal flow graph techniques given in [13], the following expression for average acquisition time is derived in the Appendix:

$$E[T_{\text{acq}}] = \frac{(n_0 - 1)(1 + P_M + P_E)(E_0[T] + P_F T_v)}{2(1 - P_M - P_E)} + \frac{E_1[T] + (1 - P_M)T_v}{1 - P_M - P_E} \tag{9}$$

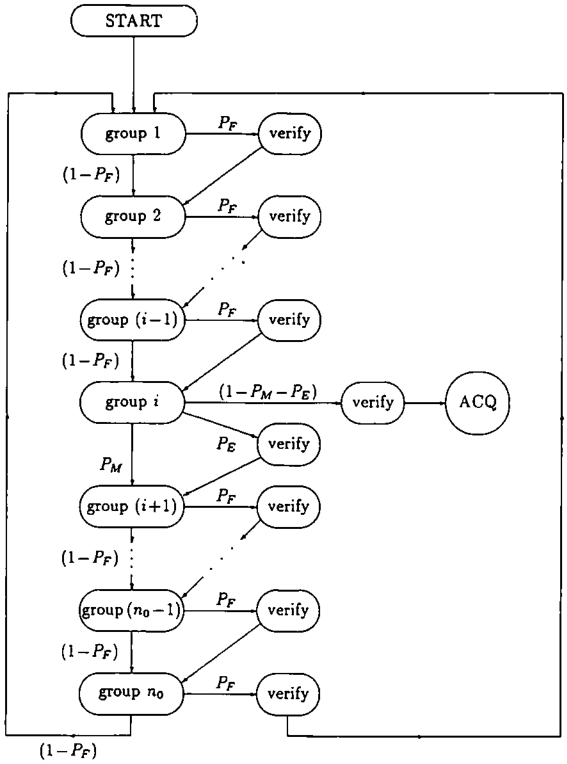


Fig. 2. Block diagram of hybrid acquisition schemes.

This expression, when simplified for the case of straight serial search schemes, is equivalent to the result in [13]. Due to the fact that the authors of [13] do not include the final verification in their definition of acquisition time, the two expressions differ by $T_v$ samples.

## 5. ACQUISITION AND VERIFICATION STAGE DESIGN

The verification stage and various choices for acquisition stage each have parameters that must be specified. The general principle in the design of these parameters is to choose them in such a way so as to minimize the expected acquisition time while not exceeding a specified (low) probability of incorrect acquisition. Symmetry and other techniques are used wherever possible to avoid multidimensional numerical searching techniques.

### 5.1. Verification Stage Design

The verification test has as a parameter the verification time $T_v$. We choose to design $T_v$ so that both types of error probabilities (declaring the wrong phase as correct and declaring the correct phase as incorrect) are less than $10^{-8}$. The Chernoff bound is used to bound these probabilities under the random sequence model. (It is shown in [12] that the random sequence model, when used as an analytical tool, provides high accuracy.) The Chernoff bounding approach gives

$$
\begin{aligned}
&P(\text{Choose } H_1' | H_0' \text{ is true}) \\
&= P\left( \prod_{i=1}^{T_v} \left( \frac{f_1'(X(i))}{f_0'(X(i))} \right)^{\theta} > 1 | H_0' \text{ is true} \right) \\
&\leq [\rho(\theta)]^{T_v}
\end{aligned}
$$

where

$$\rho(\theta) = \int_{-\infty}^{\infty} f_1(y)^{\theta} f_0(y)^{1-\theta}\, dy$$

Similarly, it can be shown that

$$P(\text{Choose } H_0' | H_1' \text{ is true}) \leq [\rho(1 - \theta)]^{T_v}$$

Let $\rho^* = \min_\theta \rho(\theta)$. Then clearly both error probabilities are bounded by $(\rho^*)^{T_v}$, which can be obtained numerically as a function of $p$. Design is then completed by choosing $T_v$ to satisfy

$$T_v \geq \frac{-8}{\log_{10} \rho^*}$$

### 5.2. MAP Test Design

The only parameter that can be designed in this test is $N$, the number of samples. Because analytical determination of $P_M$, $P_E$, and $P_F$ is difficult, simulation is employed to determine these quantities as a function of $N$. These values are used in (9) to compute $E[T_{acq}]$, and the value of $N$ that minimizes $E[T_{acq}]$ is chosen.

### 5.3. Weighted MAP Test Design

Design of this test requires determination of $N$ and $\omega_0$, $\omega_1, \ldots, \omega_{M-1}$. By symmetry considerations we have $\omega_1 = \cdots = \omega_{M-1}$. Furthermore, multiplying each $\omega_i$ by the same constant leaves the test unchanged, so that without loss of generality we can take $\omega_1 = 1$. This specifies all parameters except for $N$ and $\omega_0$.

Further design simplification is possible through the use of Bayesian decision theory. Towards this end, we assume that $P_M$, $P_E$, and $P_F$ are "small." Then

$$(1 - P_M - P_E)^{-1} \approx 1 + P_M + P_E$$

$$\frac{1 + P_M + P_E}{1 - \mathbf{P}_M - \mathbf{P}_E} \approx 1 + 2P_M + 2P_E$$

and

$$\frac{1 + P_M}{1 - P_M - P_E} \approx 1 + P_E$$

Inserting these approximations into (9) gives

$$
\begin{aligned}
E[T_{acq}] \approx\ & \frac{N(n_0 - 1)}{2} (1 + 2P_M + 2P_E) \\
& + N(1 + P_M + P_E) \\
& + T_v\left(1 + P_E + \frac{n_0 - 1}{2} P_F\right)
\end{aligned} \tag{10}
$$

Note that, for fixed $N$, (10) is a linear function of $P_M$, $P_E$, and $P_F$, and also, minimizing (10) is equivalent to minimizing

$$Nn_0P_M + (Nn_0 + T_v)P_E + \frac{T_v(n_0 - 1)}{2} P_F \tag{11}$$

We now assume that $Nn_0 >> T_v$ and $n_0 >> 1$. Then a good approximation to (11) is

$$Nn_0P_M + Nn_0P_E + \frac{T_v n_0}{2} P_F \tag{12}$$

Now, the Bayes risk function can be defined [18]

$$R(\delta) = \sum_{i=0}^{M-1} \pi_i \sum_{j=0}^{M-1} C(i, j) \int_\Gamma \delta(j | \mathbf{y}) f_i(\mathbf{y})\, d\mathbf{y}$$

where $C(i, j)$ is the cost of deciding $H_j$ when $H_i$ is true, $\mathbf{y}$ is abbreviated notation for $\{\mathbf{X}(1), \ldots, \mathbf{X}(N)\}$, and $\delta(j | \mathbf{y})$ is a decision rule (one if $H_j$ is chosen and zero otherwise).[6]

Let $\Gamma_j = \{\mathbf{y}: \delta(j | \mathbf{y}) = 1\}$. Then

$$
\begin{aligned}
R(\delta) &= \sum_{i=0}^{M-1} \sum_{j=0}^{M-1} C(i, j) \int_{\Gamma_j} f(\mathbf{y} | H_i)\, d\mathbf{y} \\
&= \sum_{i=0}^{M-1} \pi_i \sum_{j=0}^{M-1} C(i, j) \alpha_{i,j}
\end{aligned} \tag{13}
$$

where $\alpha_{i,j} = P(\text{Accept } H_j | H_i \text{ is true})$. Setting (12) equal to (13) gives

$$
C(i, j) = \begin{cases} T_v n_0 / 2\pi_0 & i = 0, j \neq 0 \\ 0 & i = j \\ Nn_0 / \pi_i(M - 1) & \text{otherwise} \end{cases}
$$

[6] A nonrandomized decision rule is sufficient in this case.

The decision rule that minimizes this risk is [18]

$$\delta(i|\mathbf{y}) = \begin{cases} 0 & \text{if } B_i(\mathbf{y}) > m(\mathbf{y}) \\ 1 & \text{if } B_i(\mathbf{y}) = m(\mathbf{y}) \text{ and} \\ & B_\ell(\mathbf{y}) > m(\mathbf{y}) \text{ for all } \ell \neq i \end{cases}$$

where

$$B_\ell(\mathbf{y}) = \sum_{i=0}^{M-1} \pi_i C(i, \ell) f_i(\mathbf{y})$$

and $m(\mathbf{y}) = \min_\ell B_\ell(\mathbf{y})$.

Substituting the appropriate expressions leads to the weighted MAP test with $\omega_1 = \cdots = \omega_{M-1} = 1$ and

$$\omega_0 = \frac{T_v}{2N} \frac{M-1}{L-(M-1)} \tag{14}$$

The Bayesian approach shows that the weighted MAP test with these weights is "approximately optimal" among FSS tests. To fully design the test, a simulation is used to determine $P_M$, $P_E$, and $P_F$ as a function of $N$, $\omega_0$ is chosen according to (14), and $N$ is chosen to minimize $E[T_{acq}]$. Additional simulations with varying values of $\omega_0$ verify that (14) is indeed optimum.

### 5.4. MSPRT Design

The MSPRT is fully specified by choosing values for $A_0, A_1, \ldots, A_{M-1}$. By symmetry we have $A_1 = \cdots = A_{M-1}$. Once again, because analysis is difficult, simulation is employed to determine the values of $A_0$ and $A_1$ that minimize $E[T_{acq}]$.

## 6. NUMERICAL RESULTS AND CONCLUSIONS

Comparisons of the performance of the MAP test, the weighted MAP test, and the MSPRT are given in Figs. 3 through 5 for acquisition of a pseudonoise (PN) sequence of period 1023. The sequence is generated by the primitive polynomial 3023 (in octal notation) of degree 10. The results in these figures are for the best possible tests in the sense that all parameters have been chosen to minimize $E[T_{acq}]$. The optimal values for these parameters, as well as the designed verification time $T_v$, are given in Tables I through IV.

The figures show that the weighted MAP test provides uniformly lower expected acquisition time than the MAP test, and the MSPRT provides significantly lower mean acquisition time than both of the other tests. Furthermore, the ratios of expected acquisition times for the various tests are very insensitive to $p$. Between the MSPRT and the weighted FSS test there is roughly a speedup factor of 4 for $M = 2$, decreasing to 1.7 at

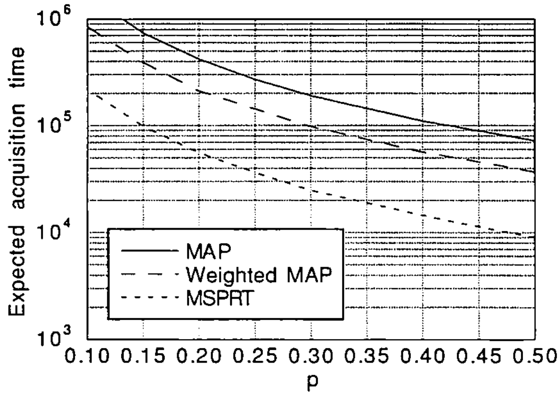


Fig. 3. Performance comparison of hybrid acquisition schemes, $M = 2$.

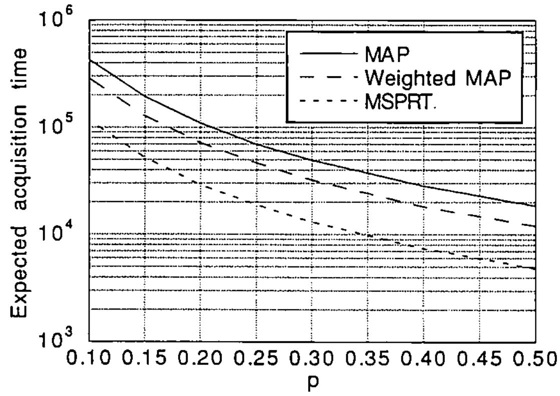


Fig. 4. Performance comparison of hybrid acquisition schemes, $M = 5$.

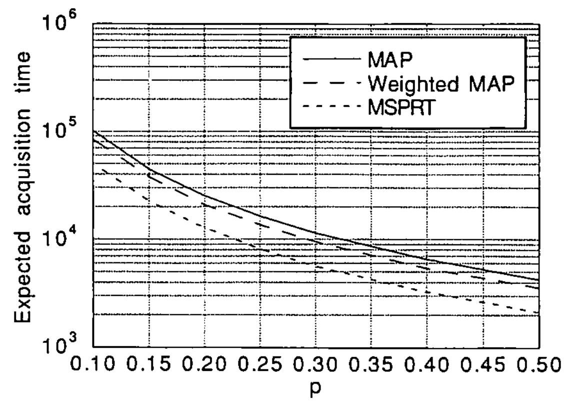


Fig. 5. Performance comparison of hybrid acquisition schemes, $M = 21$.

**Table I.** Parameters of the Verification Stage

| $p$ | $\rho^*$ | $T_v$ |
|---|---|---|
| 0.10 | 0.9988 | 14750 |
| 0.15 | 0.9972 | 6550 |
| 0.20 | 0.9950 | 3650 |
| 0.25 | 0.9922 | 2350 |
| 0.30 | 0.9888 | 1650 |
| 0.40 | 0.9803 | 950 |
| 0.50 | 0.9695 | 600 |

**Table II.** Parameters of the MAP Test

| $p$ | $M = 2$ $N$ | $M = 5$ $N$ | $M = 21$ $N$ |
|---|---|---|---|
| 0.10 | 2350 | 2345 | 2180 |
| 0.15 | 1020 | 1060 | 1030 |
| 0.20 | 575 | 590 | 595 |
| 0.25 | 380 | 370 | 380 |
| 0.30 | 265 | 280 | 255 |
| 0.40 | 155 | 160 | 150 |
| 0.50 | 95 | 100 | 100 |

**Table III.** Parameters of the Weighted MAP Test

| | $M = 2$ | | $M = 5$ | | $M = 21$ | |
|---|---|---|---|---|---|---|
| $p$ | $N$ | $\omega_0$ | $N$ | $\omega_0$ | $N$ | $\omega_0$ |
| 0.10 | 940 | 0.007 | 1280 | 0.02 | 1835 | 0.08 |
| 0.15 | 395 | 0.008 | 650 | 0.02 | 840 | 0.08 |
| 0.20 | 250 | 0.007 | 320 | 0.02 | 480 | 0.08 |
| 0.25 | 155 | 0.007 | 235 | 0.02 | 295 | 0.08 |
| 0.30 | 115 | 0.007 | 160 | 0.02 | 200 | 0.08 |
| 0.40 | 55 | 0.008 | 95 | 0.02 | 120 | 0.08 |
| 0.50 | 45 | 0.008 | 60 | 0.02 | 80 | 0.08 |

**Table IV.** Parameters of the MSPRT

| | $M = 2$ | | $M = 5$ | | $M = 21$ | |
|---|---|---|---|---|---|---|
| $p$ | $A_0$ | $A_1$ | $A_0$ | $A_1$ | $A_0$ | $A_1$ |
| 0.10 | 1.0E-3 | 6.1E-2 | 2.3E-3 | 7.0E-3 | 5.0E-3 | 3.4E-2 |
| 0.15 | 7.0E-4 | 3.1E-2 | 2.2E-3 | 7.0E-3 | 5.0E-3 | 5.0E-2 |
| 0.20 | 7.0E-4 | 3.1E-2 | 1.6E-3 | 7.0E-3 | 5.5E-3 | 5.0E-2 |
| 0.25 | 7.0E-4 | 3.1E-2 | 1.6E-3 | 7.0E-3 | 5.5E-3 | 4.2E-2 |
| 0.30 | 7.0E-4 | 1.2E-1 | 1.6E-3 | 6.0E-3 | 5.5E-3 | 6.2E-2 |
| 0.40 | 6.0E-4 | 1.2E-1 | 1.6E-3 | 8.0E-3 | 5.0E-3 | 5.0E-2 |
| 0.50 | 6.0E-4 | 6.4E-2 | 2.5E-3 | 1.6E-2 | 5.0E-3 | 3.0E-2 |

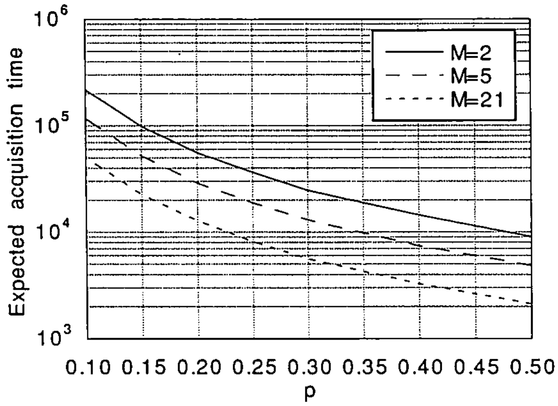


**Fig. 6.** Performance comparison of the MSPRT for hybrid acquisition.

$M = 21$. Between the MSPRT and the MAP test, the corresponding speedup factors are 8 for $M = 2$ and 2 for $M = 21$.

In Fig. 6 we compare the MSPRT tests for varying values of $M$. This figure shows that the expected acquisition time decreases with increasing $M$, as is expected. Thus, the MSPRT provides the trade-off between performance and complexity that is the main goal of hybrid schemes, and it also provides the shortest mean acquisition times among the three tests considered here.

Our simulations reveal that each of these tests is quite insensitive to the values of the parameters. (The fluctuations in the optimal parameter values in Table IV are due to this fact.) This is a desirable feature for any test for acquisition. However, we note that each of these tests also depend on $p$ explicitly (through the likelihood ratios). The total sensitivity of the tests to $p$ is beyond the scope of this paper. If in fact there is significant overall sensitivity to $p$, suboptimal statistics with little or no dependence on $p$ may be preferable in applications with strong and rapid signal-to-noise fluctuations.

In conclusion, several tests for hybrid acquisition have been presented and their performance has been characterized. It has been shown that the MSPRT has superior performance with regards to expected acquisition time. There are also several avenues for future research. Of primary interest is the relaxing of the three assumptions given in Section 2. In particular, the use of noncoherent demodulation, demodulation in the presence of data, and demodulation in the absence of chip boundary synchronization should be explored. In all three cases, it is expected that generalizations of the MSPRT approach can be utilized through suitable modifications of the hypothesis models given in Section 3.

## APPENDIX: DERIVATION OF EXPECTED ACQUISITION TIME

Let $M_j(z) := \Sigma_{n=1}^{\infty} z^n \mathrm{P}(T = n|H_j)$ be the moment generating function (MGF) of $T$ under $H_j$, $j = 0, 1$. Also let $M_{\mathrm{acq},i}(z)$ denote the MGF of $T_{\mathrm{acq}}$ given that the true hypothesis is tested in the $i$th group. Then, using signal flow graph techniques (see [13]), the following expression is easily derived:

$$M_{\mathrm{acq},i}(z) = \frac{M_0(z)[1 - P_F + P_F z^{T_v}]^{i-1} M_1(z)}{\{1 - M_0(z)[1 - P_F + P_F z^{T_v}]^{n_0 - 1} M_1(z)[P_M + P_E z^{T_v}]\}} \cdot (1 - P_M - P_E) z^{T_v}$$

Let $M_{\mathrm{acq}}(z)$ denote the unconditional MGF of $T_{\mathrm{acq}}$. Then, since all phases have been assumed to be equally likely,

$$M_{\mathrm{acq}}(z) = \frac{1}{n_0} \sum_{i=1}^{n_0} M_{\mathrm{acq},i}(z)$$

Now, it is a standard result that

$$\mathrm{E}[T_{\mathrm{acq}}] = \left[\frac{d}{dz} M_{\mathrm{acq}}(z)\right]_{z=1}$$

From this equation and using the fact that $M_j(1) = 1$ and $M_j'(z) = \mathrm{E}_j[T]$, $j = 0, 1$, the expression for the expected acquisition time given in Eq. (9) is obtained.

Note that higher-order moments of $T_{\mathrm{acq}}$ may also be derived from $M_{\mathrm{acq}}(z)$. In particular, an expression for $\mathrm{Var}(T_{\mathrm{acq}})$ (given in terms of the means and variances of $T$ under $H_0$ and $H_1$) can be obtained using

$$\mathrm{Var}(T_{\mathrm{acq}}) = \left[\frac{d^2}{dz^2} M_{\mathrm{acq}}(z)\right]_{z=1} + \mathrm{E}[T_{\mathrm{acq}}] - (\mathrm{E}[T_{\mathrm{acq}}])^2$$

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