A Convergent Version of the Max SINR Algorithm for the MIMO Interference Channel

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***Abstract*—The problem of designing linear transmit signaling strategies for the multiple input, multiple output (MIMO) interference channel is considered. For this problem, the best known iterative solution, in terms of maximizing signal to interference plus noise ratio (SINR) at the receivers, is the Max SINR algorithm. However, there is no proof that the Max SINR algorithm converges. In this paper, a modification to the Max SINR algorithm is proposed, in which a power control step is used to make a metric similar to the sum rate increase monotonically with each iteration, thus making the modified Max SINR algorithm convergent. It is further shown that with successive interference cancellation (SIC), the metric that the modified Max SINR algorithm optimizes is exactly the sum rate. Finally, simulations are used to demonstrate that the performance of the modified Max SINR algorithm, unlike other convergent alternatives, is nearly identical to that of the original Max SINR algorithm.**

***Index Terms*—Wireless networks, interference channels, iterative algorithms, throughput.**

I. Introduction

TRADITIONALLY interference has been managed in cellular networks through cell planning and treating any remaining interference as noise. Now, with many cellular networks being interference limited, there has been an increased interest in more sophisticated schemes to manage interference through base station cooperation. These questions are especially pertinent today as mobile data demand is expected to grow significantly over the next few years [1].

The natural setting to consider managing interference is the interference channel in which each transmitter communicates with a paired receiver in the presence of interference from other pairs. As a result there is considerable interest in understanding the capacity of the interference channel (IC). Unfortunately, while some achievable rate regions such as the Han-Kobayashi region are known [2], the actual capacity region of the channel is unknown. In response to this difficulty several techniques have been developed to approximate the capacity region of the IC through degrees of freedom analysis (see, e.g., [3]), and computing the capacity within a constant number of bits (see, e.g., [4]). These methods have provided some insight into how to communicate on the IC and suggest that there is significant value in more sophisticated ways to manage interference.

Based on insights gained from capacity analyses, a number of algorithms for managing interference have been proposed. In [5] an algorithm, based on the idea of interference alignment (IA) introduced in [3], was proposed in which transmitters and receivers take turns adjusting their beamforming vectors to reduce the interference leakage, under the assumption of channel reciprocity. This algorithm monotonically reduces the total interference leakage power and therefore converges. In [6] a similar algorithm without the reciprocity assumption was introduced. In [7] an extension of the algorithm in [6] was provided that gives better sum rate performance.

Another iterative algorithm, Max SINR, was proposed in [5] that gives better sum rate performance than the minimum leakage algorithm. The Max SINR algorithm starts with arbitrary transmit beamforming vectors and then designs receivers to optimize the signal-to-interference-plus-noise ratio (SINR) at each receiver. Next, the algorithm alternates the direction of communication and repeatedly optimizes the SINR at each receiver. This algorithm outperforms the other algorithms mentioned before, but there is no proof that it actually converges, and in fact no examples are known in which Max SINR fails to converge. In [8] and [9] algorithms similar to Max SINR are proposed that seek to minimize the sum mean square error (MSE) of all the receivers. These algorithms are slightly more complicated as Lagrange multipliers need to be computed to design the transmit beamformers. In this paper, we are primarily interested in Max SINR, so we do not consider the algorithms presented in [8] and [9].

In this paper, we propose a convergent version of Max SINR with nearly identical sum rate performance. First, a power control step performed in each iteration insures that the same SINRs can be achieved in both directions of communication. Using this observation, we show that a performance metric similar to sum rate converges. In addition, using successive interference cancellation (SIC) at the receiver makes the sum rate a monotonically increasing function of the SINRs, and therefore the sum rate converges. Since the sum rate for the interference channel is a non-convex function, the new algorithm, like the other proposed algorithms, converges to a local maximum (not necessarily a global maximum). Finally, we use simulations to verify that the sum rate performance of the Modified Max SINR algorithm is nearly identical to that of the original Max SINR algorithm.

For the case of a MIMO downlink channel, an algorithm similar to Max SINR was proposed and convergence demonstrated by SINR duality in [10]. We provide an algorithm

Manuscript received August 26, 2012; revised November 27 and December 23, 2012; accepted February 22, 2013. The associate editor coordinating the review of this paper and approving it for publication was H. Jafarkhani.

This work was supported by the NSF under awards CNS 0831670 and CCF 0904619, through the University of Illinois at Urbana-Champaign. This paper was presented in part at ISIT 2011, St. Petersburg, Russia.

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Digital Object Identifier 10.1109/TWC.2013.041913.121289

for the MIMO interference channel with multiple pairs of transmitters and receivers, which is fundamentally different from the centralized algorithm for the MIMO downlink with only one transmitter and several receivers given by the authors in [10].

A. Overview

We first give an overview of the interference channel model. Next, we describe interference alignment and transmit strategies for the interference channel including the Max SINR algorithm. Finally, we introduce a convergent version of the Max SINR algorithm and study its performance through simulations.

B. Notation

Notation: Scalars are lower case, vectors are lower case bold, and matrices are upper case bold. Furthermore, $\boldsymbol{A}(d)$ is the d^{th} column of the matrix $\boldsymbol{A}$. When we use this notation in general we will refer to a collection of matrices $\boldsymbol{A}_i$, and therefore in our notation $\boldsymbol{A}_i(d)$ is the d^{th} column of the i^{th} matrix $\boldsymbol{A}_i$. Also, $\boldsymbol{x}(\ell)$ is the ℓ^{th} element of the vector $\boldsymbol{x}$, $\boldsymbol{I}$ is the identity matrix, $\boldsymbol{A}^\dagger$ is the conjugate transpose of $\boldsymbol{A}$, and $\mathcal{K} = \{1, 2, \ldots, K\}$ is the set of all users. $\text{diag}(x_1, \ldots, x_N)$ is an $N \times N$ diagonal matrix with $x_1, \ldots, x_N$ on the diagonal. For iterative algorithms presented in this paper, $\boldsymbol{A}_i^{(n)}$ denotes the value of the matrix $\boldsymbol{A}_i$ at iteration n. The proper complex Gaussian vector with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$ is denoted by $\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$.

II. System Model and Problem Statement

In this section we present our system model and summarize several IA inspired communication strategies for the IC including Max SINR. Then we describe the convergence issues with Max SINR and the need to overcome them to develop a convergent version of Max SINR.

A. System Model

Consider a K-user multiple input and multiple output (MIMO) interference channel with the k^{th} user's channel having M_k inputs and N_k outputs. The MIMO interference channel model here can arise from the use of multiple antennas at transmitters and receivers, or through the use of symbol extensions in time or frequency (e.g., OFDM subcarriers) [3]. We have the following system model from the perspective of receiver k:

$$\tilde{\boldsymbol{y}}_k = \sum_{j=1}^{K} \boldsymbol{H}_{kj} \tilde{\boldsymbol{x}}_j + \tilde{\boldsymbol{z}}_k \tag{1}$$

where $\tilde{\boldsymbol{y}}_k$ is the $N_k \times 1$ receive vector, $\tilde{\boldsymbol{z}}_k$ is the $N_k \times 1$ additive white Gaussian noise (AWGN) vector normalized to have unit covariance so that $\tilde{\boldsymbol{z}}_k \sim \mathcal{CN}(0, \boldsymbol{I}_{N_k})$, $\boldsymbol{H}_{kj}$ is the $N_k \times M_j$ matrix of channel coefficients between transmitter j and receiver k, and $\tilde{\boldsymbol{x}}_j$ is the $M_j \times 1$ transmitted signal vector from user j.

We assume that user k sends a total of d_k (complex) symbols for each of use of the MIMO channel, with d_k chosen judiciously as discussed below. The vector of d_k symbols of user k is denoted by $\boldsymbol{x}_k$. Let the quantity $\boldsymbol{\rho}_{k\ell}$ be the power allocated to symbol ℓ of user k, $\boldsymbol{x}_k(\ell)$. Then we use complex Gaussian signaling, i.e.,

$$\boldsymbol{x}_k \sim \mathcal{CN}\left(\boldsymbol{0}, \text{diag}(\boldsymbol{\rho}_{k1}, \ldots, \boldsymbol{\rho}_{kd_k})\right)$$

with $\boldsymbol{\rho}_{k\ell}$ chosen to satisfy a power constraint P_k, i.e.,

$$\sum_{\ell=1}^{d_k} \boldsymbol{\rho}_{k\ell} \leq P_k \tag{2}$$

For later use we define a $\sum_{j=1}^{K} d_j \times 1$ vector of all the stream powers:

$$\boldsymbol{\rho} = [\boldsymbol{\rho}_{11}, \ldots, \boldsymbol{\rho}_{1d_1}, \ldots, \boldsymbol{\rho}_{K1}, \ldots, \boldsymbol{\rho}_{Kd_K}]^\top \tag{3}$$

The symbols $\boldsymbol{x}_k$ are then linearly transformed to form the channel input $\tilde{\boldsymbol{x}}_k$, i.e.,

$$\tilde{\boldsymbol{x}}_k = \boldsymbol{V}_k \, \boldsymbol{x}_k$$

where the matrix $\boldsymbol{V}_k$ is a matrix of unit norm transmit (beamforming) vectors of size $M_k \times d_k$. Similarly, at the receiver, the channel outputs $\tilde{\boldsymbol{y}}_k$ are linearly transformed to form the input to the decoder $\boldsymbol{y}_k$, i.e.,

$$\boldsymbol{y}_k = \boldsymbol{U}_k^\dagger \, \tilde{\boldsymbol{y}}_k$$

where $\boldsymbol{U}_k$ is the $N_k \times d_k$ matrix of unit norm receive (beamforming) vectors used by receiver k. This yields the effective system model

$$\boldsymbol{y}_k = \sum_{j=1}^{K} \boldsymbol{U}_k^\dagger \boldsymbol{H}_{kj} \boldsymbol{V}_j \boldsymbol{x}_j + \boldsymbol{z}_k \tag{4}$$

which will be used in the remainder of the paper. In this equation, $\boldsymbol{z}_k \sim \mathcal{CN}(0, \boldsymbol{U}_k^\dagger \boldsymbol{U}_k)$.

1) Channel Knowledge: Throughout this paper we will assume global channel knowledge. This means that the channel coefficients are fixed and known at all transmitters and receivers. For example, receiver k could estimate the channels $\boldsymbol{H}_{kj}$ for all j, feed the estimated channels back to transmitter k, and exchange the estimated channels with all other transmitters.

2) Choosing d_k: We now discuss how one might choose d_k, which we will refer to as the number of (symbol) *streams* for user k. One possible approach is based on the idea of interference alignment, in which the goal is to find matrices $\boldsymbol{V}_k$ and $\boldsymbol{U}_k$ for each k to completely eliminate the interference at all the receivers at high signal to noise ratio (SNR), i.e.,

$$\begin{aligned} \text{rank}\left(\boldsymbol{U}_k^\dagger \boldsymbol{H}_{kk} \boldsymbol{V}_k\right) &= d_k \; \forall k \\ \boldsymbol{U}_k^\dagger \boldsymbol{H}_{kj} \boldsymbol{V}_j &= \boldsymbol{0} \quad \forall j \neq k \end{aligned} \tag{5}$$

Using tools from algebraic geometry, there has been some progress on answering the question of when (5) can be solved. In [11], the authors derived a necessary condition for alignment given by

$$\sum_{k=1}^{K} d_k(M_k + N_k - 2d_k) \geq \sum_{j \neq k} d_j d_k \tag{6}$$

which we use as a guideline to choose d_k. There has been further work on determining feasibility of solving (5) in [12],

[13], and [14]. All of these papers are primarily concerned with answering the question of whether (5) can be solved for given system parameters, but not necessarily how to solve (5).

3) Some Useful Definitions: The next four definitions will be useful later. Define the stream-to-stream link gain as

$$G_{k\ell}^{js} = \left|\boldsymbol{U}_k(\ell)^\dagger \boldsymbol{H}_{kj} \boldsymbol{V}_j(s)\right|^2 \tag{7}$$

This is the effective gain from stream s of user j to stream ℓ of user k.

Define the noise-plus-interference covariance matrices as

$$\boldsymbol{B}_{k\ell} = \boldsymbol{I}_{N_k} + \sum_{(j,s)\neq(k,\ell)} \boldsymbol{\rho}_{js} \boldsymbol{H}_{kj} \boldsymbol{V}_j(s) \boldsymbol{V}_j(s)^\dagger \boldsymbol{H}_{kj}^\dagger \tag{8}$$

Define the sum rate using linear transmit and receive strategies as

$$R_{\text{sum}} = \sum_{k=1}^{K} \log\det\left(I_{N_r} + \boldsymbol{H}_{kk} \boldsymbol{V}_k \boldsymbol{D}_k \boldsymbol{V}_k^\dagger \boldsymbol{H}_{kk}^\dagger \boldsymbol{B}_k^{-1}\right) \tag{9}$$

with $\boldsymbol{D}_k = \text{diag}(\boldsymbol{\rho}_{k1}, \ldots, \boldsymbol{\rho}_{kd_k})$ and

$$\boldsymbol{B}_k = \boldsymbol{I}_{N_k} + \sum_{j\neq k} \boldsymbol{H}_{kj} \boldsymbol{V}_j \boldsymbol{D}_j \boldsymbol{V}_j^\dagger \boldsymbol{H}_{kj}^\dagger$$

Finally, define the signal to interference and noise ratio (SINR) for a stream as

$$\text{SINR}_{k\ell} = \frac{\boldsymbol{\rho}_{k\ell} G_{k\ell}^{k\ell}}{1 + \sum_{(j,s)\neq(k,\ell)} \boldsymbol{\rho}_{js} G_{k\ell}^{js}} \tag{10}$$

B. Communication Strategies for the Interference Channel

As discussed in section II, we know that interference alignment through linear transmit strategies achieves a good sum rate at high SNR; however, there are two major practical issues: how to choose d_k and how to find IA solutions. The first problem can be solved by using the necessary conditions in (6) to select d_k. The second problem is harder to solve, as there is no known simple way to find IA transmit and receive vectors by solving (5). It is therefore of interest to find approximate or iterative methods to find IA solutions, and there has been substantial work along these lines. This section summarizes iterative algorithms to find IA solutions. For the algorithms in this section we will consider a uniform power constraint P, so $P_k = P$ for all k.

1) Reciprocal Channel: The algorithms presented in this section all involve working with a reciprocal channel corresponding to reversing the direction of communication. In the reciprocal channel the transmitters become the receivers and the receivers become the transmitters. The roles of $\boldsymbol{V}_k$ and $\boldsymbol{U}_k$ are reversed and d_k is kept the same as in the original channel. From now on an arrow above a quantity will indicate the direction of communication. The channel model for the reciprocal channel is given by

$$\overleftarrow{\boldsymbol{y}}_k = \sum_{j=1}^{K} \overleftarrow{\boldsymbol{U}}_k^\dagger \overleftarrow{\boldsymbol{H}}_{kj} \overleftarrow{\boldsymbol{V}}_j \overleftarrow{\boldsymbol{x}}_j + \overleftarrow{\boldsymbol{z}}_k \tag{11}$$

where $\overleftarrow{\boldsymbol{H}}_{kj} = \overrightarrow{\boldsymbol{H}}_{jk}^\dagger$. The original channel will also be referred to as the forward channel and the reciprocal channel as the reverse channel when appropriate.

2) Min Leakage Algorithm: One of the first algorithms inspired by IA, Min Leakage, was presented in [5] and seeks to minimize the leaked interference power, defined below in (12), at each receiver. If the leaked interference power equals zero, then (5) is satisfied, and this algorithm yields an IA solution as desired. This algorithm works by repeatedly reversing the direction of communication designing receive vectors to minimize the leaked interference power in each direction.

To design receive vectors, we compute the interference covariance matrix $\boldsymbol{Q}_k$ at each receiver, i.e.,

$$\boldsymbol{Q}_k = \sum_{\substack{j=1\\ j\neq k}}^{K} \frac{P}{d_j} \boldsymbol{H}_{kj} \boldsymbol{V}_j \boldsymbol{V}_j^\dagger \boldsymbol{H}_{kj}^\dagger$$

The leaked interference power at receiver k is then

$$\text{Tr}\left(\boldsymbol{U}_k^\dagger \boldsymbol{Q}_k \boldsymbol{U}_k\right) \tag{12}$$

To minimize this quantity we choose the receive vectors to be the d_k least dominant eigenvectors of $\boldsymbol{Q}_k$ [5]. The Min Leakage algorithm makes a metric known as weighted leakage interference decrease monotonically, and hence the algorithm converges to a local optimum [5]. The Min Leakage algorithm is summarized in Algorithm 1. The notation $\overrightarrow{\boldsymbol{V}}_k^{(n)}$ indicates the value of the transmit vectors in the forward direction during iteration n. A similar notation is used for other quantities.

Algorithm 1 Min Leakage Algorithm

1: Choose $\{\overrightarrow{\boldsymbol{V}}_k^{(1)}\}$ such that

$$\|\overrightarrow{\boldsymbol{V}}_k^{(1)}(\ell)\| = 1$$

and set

$$\boldsymbol{\rho}_{k\ell} = \frac{P}{d_k}$$

$\forall k \in \mathcal{K}, \ell \in \{1, \ldots, d_k\}$.

2: Next, we give the steps to compute the new transmit and receive vectors. Choose the receive vectors to minimize leakage. Compute $\overrightarrow{\boldsymbol{Q}}_k$ at each receiver and take $\overrightarrow{\boldsymbol{U}}_k^{(1)}$ to be the d_k least dominant eigenvectors $\forall k \in \mathcal{K}$.

3: Reverse the direction of communication. Set

$$\overleftarrow{\boldsymbol{V}}_k^{(1)}(\ell) = \overrightarrow{\boldsymbol{U}}_k^{(1)}(\ell)$$

$\forall k \in \mathcal{K}, \ell \in \{1, \ldots, d_k\}$.

4: Compute $\overleftarrow{\boldsymbol{Q}}_k$ at each receiver and take $\overleftarrow{\boldsymbol{U}}_k^{(1)}$ to be the d_k least dominant eigenvectors $\forall k \in \mathcal{K}$.

5: Reverse the direction of communication. Set

$$\overrightarrow{\boldsymbol{V}}_k^{(2)}(\ell) = \overleftarrow{\boldsymbol{U}}_k^{(1)}(\ell)$$

$\forall k \in \mathcal{K}, \ell \in \{1, \ldots, d_k\}$.

6: Repeat steps 2 through 5 until convergence.

3) Other IA Inspired Algorithms: Several other algorithms based on the idea of trying to mimic interference alignment have been proposed. One other example is the Peters-Heath algorithm from [6], which works in a similar way to Min Leakage without the requirement of reciprocity. Other work includes a modification of the Peters-Heath algorithm to achieve a better sum rate [7], and several other similar algorithms for interference alignment including [15] and [16].

4) Max SINR Algorithm: The IA inspired algorithms all attempt to indirectly solve (5). However, there may be many (or even infinite) alignment solutions to (5) that all achieve different sum rate performance. For example, with $M_k = N_k = N$, $K < 2N - 1$, and $d_k = 1$, the authors of [11] demonstrated that there are an infinite number of solutions to (5). Therefore, it is not desirable to simply find any IA solution, but instead, to find IA solutions that have a good sum rate.

The Max SINR algorithm tackles the issue of achieving a good sum rate by copying the basic structure of the Min Leakage algorithm but with a different metric than interference leakage. In the Max SINR algorithm proposed in [5], interference leakage is replaced with the SINR of each stream defined in (10). The goal is to design receive vectors for each stream to maximize the SINR of each stream, or equivalently to minimize the mean squared error (MSE). It is known through simulation that this algorithm produces a good sum rate for all SNR (or P) compared to the Min Leakage algorithm which performs well for high SNR. Also, at high SNR it can be verified through simulations that the interference leakage goes to zero, so Max SINR produces IA solutions like Min Leakage. The Max SINR algorithm is summarized in Algorithm 2.

Algorithm 2 Max SINR Algorithm

1: Choose $\{\overrightarrow{\boldsymbol{V}}_k^{(1)}\}$ such that

$$\|\overrightarrow{\boldsymbol{V}}_k^{(1)}(\ell)\| = 1$$

and set

$$\boldsymbol{\rho}_{k\ell} = \frac{P}{d_k}$$

$\forall k \in \mathcal{K}, \ell \in \{1, \ldots, d_k\}$.

2: Next, we give the steps to compute the new transmit and receive vectors. Compute the MMSE RX vectors $\overrightarrow{\boldsymbol{U}}_k^{(1)}$ $\forall k \in \mathcal{K}$ and then $\overrightarrow{R}_{\text{sum}}^{(1)}$ defined in (9).

3: Reverse the direction of communication. Set

$$\overleftarrow{\boldsymbol{V}}_k^{(1)}(\ell) = \frac{\overrightarrow{\boldsymbol{U}}_k^{(1)}(\ell)}{\|\overrightarrow{\boldsymbol{U}}_k^{(1)}(\ell)\|}$$

$\forall k \in \mathcal{K}, \ell \in \{1, \ldots, d_k\}$. Calculate the sum rate of the reciprocal network denoted $\overleftarrow{R}_{\text{sum-switch}}^{(1)}$.

4: Compute the MMSE RX vectors $\overleftarrow{\boldsymbol{U}}_k^{(1)}$ $\forall k \in \mathcal{K}$ and then the sum rate of the reciprocal network denoted $\overleftarrow{R}_{\text{sum}}^{(1)}$.

5: Reverse the direction of communication. Set

$$\overrightarrow{\boldsymbol{V}}_k^{(2)}(\ell) = \frac{\overleftarrow{\boldsymbol{U}}_k^{(1)}(\ell)}{\|\overleftarrow{\boldsymbol{U}}_k^{(1)}(\ell)\|}$$

$\forall k \in \mathcal{K}, \ell \in \{1, \ldots, d_k\}$. Now calculate the sum rate of the forward network $\overrightarrow{R}_{\text{sum-switch}}^{(1)}$.

6: Repeat steps 2 through 5.

5) Comparison of Algorithms: Fig.1 gives a representative example of the performance of the Max SINR algorithm against the Min Leakage and Peters-Heath algorithms. As the plot shows, the Max SINR algorithm outperforms the Min Leakage and Peters-Heath algorithms at low SNRs, but the performance gap disappears at high SNRs. This makes sense since interference alignment is sum rate optimal at high SNRs and the Min Leakage and Peters-Heath algorithms aim for IA solutions. The performance for other system parameters is similar, so this plot is representative of the difference between these algorithms.

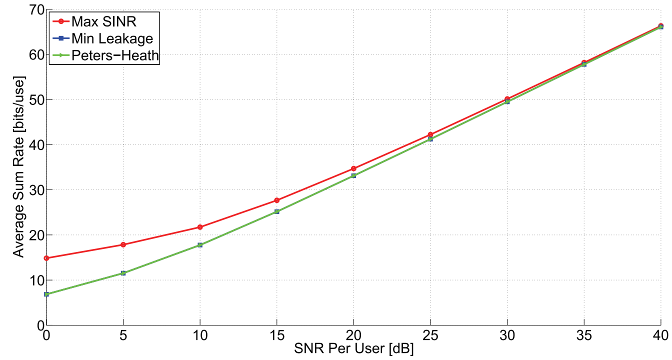


Fig. 1: Sum rate vs. P for $K = 5$, $M_k = N_k = 3$, $d_k = 1$.

C. Problem Statement

In simulations of the Max SINR algorithm, the sum rate appears to converge, but there is no proof that this actually occurs. To begin we summarize a couple of different behaviors observed in Max SINR. First, experimentally it is sometimes the case that $\overrightarrow{R}_{\text{sum}}^{(n)} > \overleftarrow{R}_{\text{sum-switch}}^{(n)}$, but after optimization $\overrightarrow{R}_{\text{sum}}^{(n)} \leq \overleftarrow{R}_{\text{sum}}^{(n)}$. In the forward channel a similar process occurs and so $\overleftarrow{R}_{\text{sum}}^{(n)} \leq \overrightarrow{R}_{\text{sum}}^{(n+1)}$. An example of this phenomenon is shown in Fig.2a. Second, the convergence need not be monotone as seen in Fig.2b. Due to the complexity of the sum rate and MMSE receiver expressions and the potentially complicated convergence behavior, it is difficult to prove that the sum rate converges. Our goal in this paper is to develop a way to overcome this difficulty. The major contribution of this paper is to develop an algorithm similar to Max SINR with the same good sum rate performance but guaranteed convergence by adding a power control step in each direction of communication.

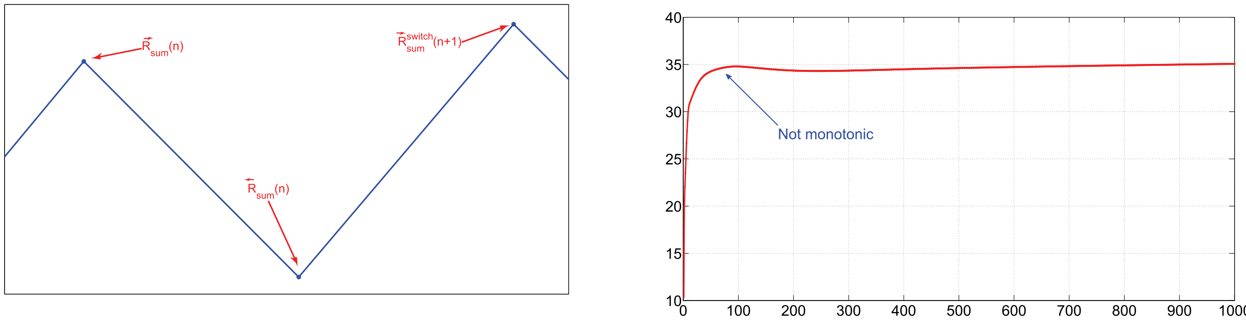
III. Convergent Version of the Max SINR Algorithm

In this section we demonstrate a way to modify the Max SINR algorithm to guarantee convergence. To do this we consider a slightly different metric than the sum rate called the sum stream rate:

$$R_{\text{sum-stream}} = \sum_{k=1}^{K} \sum_{\ell=1}^{d_k} \log(1 + \text{SINR}_{k\ell})$$

$R_{\text{sum-stream}}$ is the sum of the rates achieved by decoding each stream individually treating all other streams as noise. In contrast, the sum rate given in (9) allows joint decoding of streams $1, \ldots, d_k$ at receiver k.

Next, we show that adjusting the power allocation appropriately under a sum power constraint when switching the



(a) Max SINR convergence behavior

(b) Max SINR convergence not monotone

Fig. 2: Max SINR convergence behavior.

direction of communication will guarantee that the same SINR for each stream is achieved on the forward and reciprocal networks. By combining these two ideas we can show that the sum stream rate monotonically increases and therefore converges.

Finally, we show that using successive interference cancellation (SIC) means the sum rate is exactly the sum stream rate. Therefore, with SIC, the sum rate also converges.

A. Sum Power Constraint

In developing a convergent version of the Max SINR algorithm, we will require a power control step in which we impose a sum power constraint across all the users:

$$\sum_{k=1}^{K}\sum_{\ell=1}^{d_k}\boldsymbol{\rho}_{k\ell} \le KP \tag{13}$$

compared to the individual power constraint given in (2). The sum power constraint can also be written as

$$\mathbf{1}^\top\boldsymbol{\rho} \le KP$$

with $\boldsymbol{\rho}$ defined in (3). Imposing a sum constraint can be justified in some cases in which the main issue is to manage interference and in which each individual transmitter can transmit at high power. Instead of imposing an individual power constraint to manage interference we can impose a sum constraint and manage interference more intelligently with linear transmit and receive strategies.

For example, consider the femtocell/macrocell interference model from [17] in which several femotocells and an umbrella macrocell interfere with each other. In this case the femtocells can transmit at high power if needed, so there is a high individual power constraint on the femtocells. One approach to managing interference is to impose a much smaller individual power constraint to control the coverage of the femtocells. Another approach is to impose a sum power constraint over all the femtocells set at the maximum power at which any given femtocell can transmit at, and use beamforming to manage interference. In this section, we consider the latter approach and impose a sum power constraint.

B. SINR Duality

In this section we show that by selecting the power appropriately we can achieve the same SINRs on the forward and reciprocal channels. In our proof, we extend the ideas from [18] to allow for SIC. We note that the idea of SINR duality has been applied to the MIMO uplink/downlink in [19] and [20]. The work in [18] is applicable to the MIMO IC and is more relevant to our work, and therefore we follow its development.

Let $\gamma_{k\ell}$ be the target SINR for stream ℓ of user k. Following [18], define

$$\boldsymbol{D} = \mathrm{diag}\{\frac{\gamma_{11}}{\overrightarrow{G}_{11}^{11}},\ldots,\frac{\gamma_{Kd_K}}{\overrightarrow{G}_{Kd_K}^{Kd_K}}\} \tag{14}$$

and

$$\boldsymbol{G}\left(\sum_{m=1}^{k-1}d_m+\ell,\sum_{n=1}^{j-1}d_n+s\right)=\begin{cases}0, & \text{if } (k,\ell)=(j,s),\\ \overrightarrow{G}_{k\ell}^{js}, & \text{else}\end{cases} \tag{15}$$

with $G_{k\ell}^{js}$ defined in (7). Set

$$\boldsymbol{A} = \boldsymbol{D}^{-1} - \boldsymbol{G}$$

For the forward and reciprocal directions, respectively, we use the power allocations

$$\overrightarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-1}\mathbf{1} \tag{16}$$
$$\overleftarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-\top}\mathbf{1} \tag{17}$$

The following lemma shows that it is possible to achieve the same SINRs on the forward and reciprocal channels. The lemma also shows that, as long as we meet the power constraint initially, $\overrightarrow{\boldsymbol{\rho}}^*$ and $\overleftarrow{\boldsymbol{\rho}}^*$ will meet it too. The proof of this lemma follows from [18] and is developed in the appendix.

Lemma 1. *Suppose we choose a power allocation $\boldsymbol{\rho}$ such that $\mathbf{1}^\top\boldsymbol{\rho} \le KP$, and choose transmit and receive vectors $\{\boldsymbol{V}_k\}$ and $\{\boldsymbol{U}_k\}$, respectively. Let $\gamma_{k\ell}$ be the resulting SINRs. Then*

$$\mathbf{1}^\top\overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top\overleftarrow{\boldsymbol{\rho}}^* \le KP$$

and

$$\overrightarrow{\mathrm{SINR}}_{k\ell}(\overrightarrow{\boldsymbol{\rho}}^*) = \overleftarrow{\mathrm{SINR}}_{k\ell}(\overleftarrow{\boldsymbol{\rho}}^*) = \gamma_{k\ell}$$

Remark on SIC: For Lemma 1 to hold, $\boldsymbol{A}^\top$ must describe the reciprocal network, or equivalently $\boldsymbol{G}^\top$ must describe the interference. This means that if we use SIC we must reverse the order of cancellation in the reciprocal network.

1) Computing Power Required for Duality: Equations (16) and (17) give a centralized way to compute the required powers to achieve equal SINRs in both the forward and reciprocal directions. We now discuss how to use the framework from [21] to compute the required powers in a distributed way. For either direction of communication, define the vector valued function $\boldsymbol{I}(\boldsymbol{\rho})$ by

$$\boldsymbol{I}_{k\ell}(\boldsymbol{\rho}) = \frac{\gamma_{k\ell}\boldsymbol{\rho}_{k\ell}}{\mathrm{SINR}_{k\ell}(\boldsymbol{\rho})}$$

Then this is a standard interference function [21], and therefore we can use the well-known totally asynchronous power control algorithm from [21] in Algorithm 3, which is guaranteed to converge to $\boldsymbol{\rho}^*$ by Theorem 4 of [21].

Algorithm 3 Distributed Power Control Algorithm

1: Choose an initial power vector $\boldsymbol{\rho}^{(1)}$ such that $\mathbf{1}^\top\boldsymbol{\rho}^{(1)} \leq KP$.
2: To compute the next power allocation compute

$$\boldsymbol{\rho}^{(2)} = \boldsymbol{I}(\boldsymbol{\rho}^{(1)})$$

3: Repeat step 2 until convergence.

C. Modified Max SINR

In this section we describe a modified version of the Max SINR algorithm in which the sum rate converges. The Modified Max SINR algorithm follows the same basic principle as the Max SINR algorithm with two key differences. First, the Modified Max SINR algorithm uses the sum stream metric as a convergence criterion. Second, the Modified Max SINR algorithm changes the powers for each stream when it reverses the direction of communication to achieve SINR duality. The Modified Max SINR algorithm is summarized in Algorithm 4.

Computing Power Allocation to Achieve SINR Duality: In order to achieve SINR duality we need to compute the appropriate power allocation every time we change the direction of communication, using either (16) and (17) or a distributed power control algorithm.

For the approach in (16) and (17), assuming the users estimate their own row of $\boldsymbol{A}$ and exchange their estimates, each user can solve $\boldsymbol{A}\overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}$ to find its power allocation. In this case the Modified Max SINR algorithm converges but is no longer fully decentralized.

The other option is to use the distributed algorithm from [21]. Then the Modified Max SINR algorithm is convergent and distributed, but the iterative power control algorithm must be run at every step, which greatly increases the required number of transmissions from each user. In many cases, it may better to solve the centralized problem instead of running the iterative power control algorithm.

Theorem 1. *With Modified Max SINR, $\overrightarrow{R}^{(n)}_{\mathrm{sum-stream}}$ converges. Also, the Modified Max SINR algorithm meets the power constraint at every step.*

Algorithm 4 Modified Max SINR Algorithm

1: Choose $\{\overrightarrow{\boldsymbol{V}}^{(1)}_k\}$ and $\overrightarrow{\boldsymbol{\rho}}^{(1)}$ that satisfy the power constraint.
2: Next, we give the steps to compute the new transmit and receive vectors. Compute the MMSE RX vectors $\overrightarrow{\boldsymbol{U}}^{(1)}_k$ $\forall k \in \mathcal{K}$ and then $\overrightarrow{R}^{(1)}_{\mathrm{sum-stream}}$.
3: Reverse the direction of communication. Calculate the power allocation $\overleftarrow{\boldsymbol{\rho}}^{(1)}$ to achieve SINR duality. Set

$$\begin{aligned}\overleftarrow{\boldsymbol{V}}^{(1)}_k(\ell) &= \frac{\overrightarrow{\boldsymbol{U}}^{(1)}_k(\ell)}{\|\overrightarrow{\boldsymbol{U}}^{(1)}_k(\ell)\|}\\ \overleftarrow{\boldsymbol{U}}^{(1)}_k(\ell) &= \overrightarrow{\boldsymbol{V}}^{(1)}_k(\ell)\end{aligned}$$

$\forall k \in \mathcal{K}, \ell \in \{1,\ldots,d_k\}$. Now calculate the sum stream rate of the reciprocal network denoted $\overleftarrow{R}^{(1)}_{\mathrm{sum-stream-switch}}$.
4: Compute the MMSE RX vectors $\overleftarrow{\boldsymbol{U}}^{(1)}_k$ $\forall k \in \mathcal{K}$ and then the sum stream rate of the reciprocal network denoted $\overleftarrow{R}^{(1)}_{\mathrm{sum-stream}}$.
5: Reverse the direction of communication. Calculate the power allocation $\overrightarrow{\boldsymbol{\rho}}^{(2)}$ to achieve SINR duality. Set

$$\begin{aligned}\overrightarrow{\boldsymbol{V}}^{(2)}_k(\ell) &= \frac{\overleftarrow{\boldsymbol{U}}^{(1)}_k(\ell)}{\|\overleftarrow{\boldsymbol{U}}^{(1)}_k(\ell)\|}\\ \overrightarrow{\boldsymbol{U}}^{(2)}_k(\ell) &= \overleftarrow{\boldsymbol{V}}^{(1)}_k(\ell)\end{aligned}$$

$\forall k \in \mathcal{K}, \ell \in \{1,\ldots,d_k\}$. Now calculate the sum stream rate on the forward network denoted $\overrightarrow{R}^{(1)}_{\mathrm{sum-stream-switch}}$.
6: Repeat steps 2 through 5 until convergence of $\overrightarrow{R}^{(n)}_{\mathrm{sum-stream}}$.

Proof: The initial power allocation $\overrightarrow{\boldsymbol{\rho}}^{(1)}$ meets the power constraint, so the constraint will continue to be met due to Lemma 1. Now

$$\begin{aligned}\overrightarrow{R}^{(n)}_{\mathrm{sum-stream}} &\overset{\text{(a)}}{=} \overleftarrow{R}^{(n)}_{\mathrm{sum-stream-switch}}\\ &\overset{\text{(b)}}{\leq} \overleftarrow{R}^{(n)}_{\mathrm{sum-stream}}\\ &\overset{\text{(c)}}{=} \overrightarrow{R}^{(n)}_{\mathrm{sum-stream-switch}}\\ &\overset{\text{(d)}}{\leq} \overrightarrow{R}^{(n+1)}_{\mathrm{sum-stream}}\end{aligned}$$

where (a) and (c) follow by SINR duality and (b) and (d) follow since MMSE receive vectors maximize SINR. So the sum rate increases monotonically and therefore converges. ■

Although we know that $R^{(n)}_{\mathrm{sum-stream}}$ converges, since $R^{(n)}_{\mathrm{sum-stream}}$ is a non-convex function, we cannot show that $R^{(n)}_{\mathrm{sum-stream}}$ converges to a global maximum.

D. Using SIC

We can further modify the Modified Max SINR algorithm to make use of SIC. The algorithm is exactly the same except we compute MMSE-SIC receive vectors in place of MMSE receive vectors. With the use of SIC, we show that the sum stream rate and sum rate are identical, and so the sum rate also converges.

When we use SIC the definition of the sum stream rate is exactly the same with the corresponding definitions such as $\mathrm{SINR}_{k\ell}$ modified. For example, assuming cancellation in lexicographic order, the SINR expression would become

$$\mathrm{SINR}_{k\ell} = \frac{\boldsymbol{\rho}_{k\ell} G_{k\ell}^{k\ell}}{1 + \sum_{s=\ell+1}^{d_j} \boldsymbol{\rho}_{ks} G_{k\ell}^{ks} + \sum_{\substack{j=1 \\ j\neq k}}^{K} \sum_{s=1}^{d_j} \boldsymbol{\rho}_{js} G_{k\ell}^{js}}$$

with $G_{k\ell}^{js}$ defined in (7). As other modifications to definitions and channel models are straightforward, they are omitted here. As mentioned in III-B, the order of cancellation in the reverse channel must be the opposite of the order of the cancellation in the forward channel.

Now we show in Lemma 2 that the sum rate and the sum stream rate are identical:

Lemma 2. *In either direction of communication, for any $\boldsymbol{V}_k$ and any channel realization with MMSE-SIC receivers,*

$$R_{\mathrm{sum}} = \sum_{k=1}^{K} \sum_{\ell=1}^{d_k} \log\left(1 + \mathrm{SINR}_{k\ell}\right) = R_{\mathrm{sum-stream}}$$

Proof: This proof is just an application of the chain rule for mutual information and is almost the same as the one for the point-to-point MIMO channel in [22]. ■

Now it is clear that if we use the Max SINR algorithm with the same power control as Modified Max SINR and MMSE-SIC, then by Lemma 2 it follows that R_{sum} converges by using the same proof as that of Theorem 1.

IV. Performance of Modified Max SINR Algorithm

In this section, we provide simulation results that compare the performance of the Modified Max SINR algorithm to the original, and illustrate the performance of the distributed power control.

A. Sum Rate Performance

Fig. 3a shows a comparison of the Max SINR algorithm, the Modified Max SINR algorithm, and the algorithms from [5] and [6] for the case of $K = 4$ users each with $M_k = N_k = 4$ antennas and $d_k = 2$. As the plots shows, there is not much of a difference between the Max SINR and Modified Max SINR methods which outperform the other algorithms. So the Modified Max SINR algorithm has performance comparable to the Max SINR algorithm but with guaranteed convergence. Fig. 3b shows another comparison of the four algorithms with $K = 5$ users, each with $M_k = N_k = 3$ antennas and $d_k = 1$.

B. Convergence Behavior

For the case described in Fig. 3a, the convergence behavior is demonstrated in Fig.4a. For the case described in Fig. 3b, the convergence behavior is demonstrated in Fig. 4b. These two cases show that the Modified Max SINR algorithm and the Max SINR algorithm may have different convergence behavior. The Modified Max SINR algorithm may converge faster or slower than the Max SINR algorithm depending on the system parameters and channel coefficients. However, it should be emphasized that since the Max SINR algorithm has not been shown to be convergent, there is no guarantee that it will converge at all.

V. Conclusion

In this paper, a modification to the Max SINR algorithm is presented in which the sum rate converges. The key idea to make the Max SINR algorithm converge is to choose the power allocation appropriately every time we reverse the direction of communication to equate the SINR in both directions of communication. With this modification we show that the sum stream rate converges. If we also use SIC, then the sum rate converges.

Directions for future work include finding simpler modifications to guarantee that the Max SINR algorithm converges, and finding conditions under which the unaltered Max SINR algorithm converges. Also of interest is to develop other algorithms to find good interference alignment solutions. One final idea of interest is to extend these iterative algorithms to the case where multiple transmitters can cooperate to send a message to a given user. In contrast, for the algorithms considered in this paper, the transmitters can cooperate to design transmit and receive vectors, but do not cooperate in actually transmitting the messages.

Appendix A
Power Control for More General Networks

In this section we show that by selecting the power appropriately we can achieve the same SINRs on the forward and reciprocal channels, by extending the ideas from [18] to an interference network with SIC.

A. Forward Channel

Consider the power optimization problem (POP) given by

$$\begin{aligned} \text{minimize} \quad & \mathbf{1}^{\top} \overrightarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \overrightarrow{\mathrm{SINR}}_{k\ell} \geq \boldsymbol{\gamma}_{k\ell} \\ & \overrightarrow{\boldsymbol{\rho}} \geq \mathbf{0} \end{aligned}$$

which corresponds to transmitting with the least sum power to meet SINR constraints. The next two lemmas show that if this problem is feasible, then the SINR constraints uniquely determine the solution. Most of the proofs in this section are nearly identical to [18], so they are omitted.

Lemma 3. *For $\alpha > 1$,* $\mathrm{SINR}_{k\ell}(\overrightarrow{\boldsymbol{\rho}}) < \mathrm{SINR}_{k\ell}(\alpha\overrightarrow{\boldsymbol{\rho}})$.

Proof: See [18]. ■

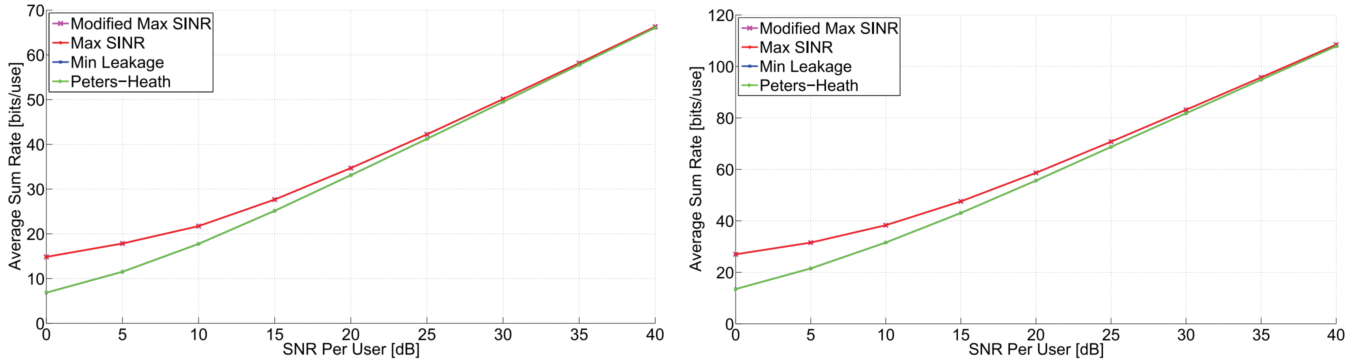
Lemma 4. *If POP is feasible, then all the inequality constraints are active and the solution is given by the unique solution to the SINR constraints with equality.*

Proof: See [18]. ■

We next consider the power maximization problem (PMP) given by

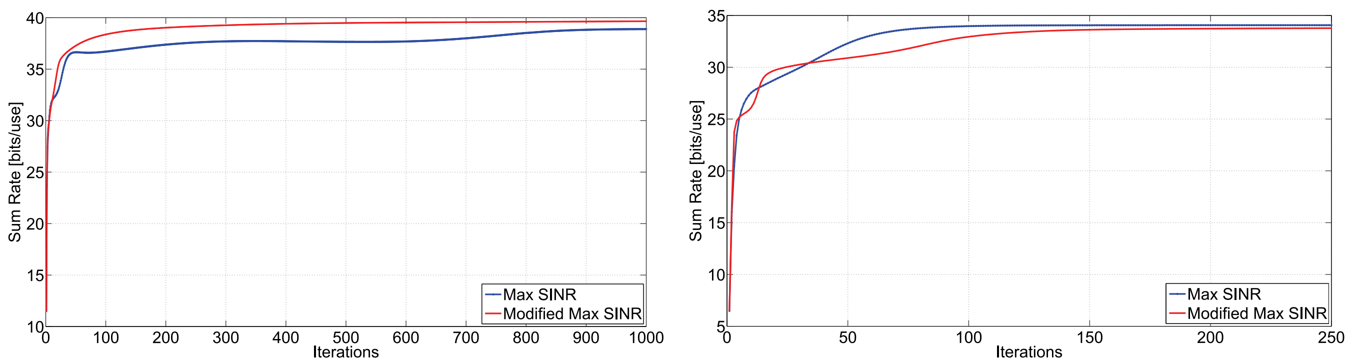
$$\begin{aligned} \text{maximize} \quad & \mathbf{1}^{\top} \overrightarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \overrightarrow{\mathrm{SINR}}_{k\ell} \leq \boldsymbol{\gamma}_{k\ell} \\ & \overrightarrow{\boldsymbol{\rho}} \geq \mathbf{0} \end{aligned}$$

Trying to use as much power as possible without exceeding an SINR constraint does not seem useful; however, we will be interested in the dual of POP which has the same form as



(a) $K = 4$, $M_k = N_k = 4$, and $d_k = 2$ (b) $K = 5$, $M_k = N_k = 3$, and $d_k = 1$

Fig. 3: Modified max SINR sum rate performance.



(a) $K = 4$, $M_k = N_k = 4$, and $d_k = 2$ (b) $K = 5$, $M_k = N_k = 3$, and $d_k = 1$

Fig. 4: Modified max SINR convergence behavior.

PMP. The following lemma shows that POP and PMP actually have the same solution.

Lemma 5. *If PMP is feasible, then all the inequality constraints are active and the solution is given by the unique solution to the SINR constraints with equality. Therefore, POP and PMP have the same solution.*

Proof: Follows same proof from Lemma 4. ■

B. Reciprocal Channel and Dual of POP

Next, we examine the dual of POP. It will turn out the dual of POP corresponds to reversing the direction of communication and switching the roles of $\boldsymbol{U}_k$ and $\boldsymbol{V}_k$. To find the dual of the POP problem, we need to put the SINR constraints in standard form. By expanding any given constraint, $\overrightarrow{\text{SINR}}_{k\ell} \geq \gamma_{k\ell}$ and simplifying, we get

$$\overrightarrow{\rho}_{k\ell} \frac{\overrightarrow{G}^{k\ell}_{k\ell}}{\gamma_{k\ell}} - \sum_{s=\ell+1}^{d_k} \overrightarrow{\rho}_{ks} \overrightarrow{G}^{ks}_{k\ell} - \sum_{j=1}^{K} \sum_{s=1}^{d_j} \overrightarrow{\rho}_{js} \overrightarrow{G}^{js}_{k\ell} \geq 1$$

For the case of SIC we introduce definitions analgous to (14) and (15). Define

$$\boldsymbol{D} = \text{diag}\{\frac{\gamma_{11}}{\overrightarrow{G}^{11}_{11}}, \ldots, \frac{\gamma_{Kd_K}}{\overrightarrow{G}^{Kd_K}_{Kd_K}}\}$$

and $\boldsymbol{G}$ such that

$$\boldsymbol{G}\left(\sum_{m=1}^{k-1} d_m + \ell, \sum_{n=1}^{j-1} d_n + s\right) = \begin{cases} 0, & \text{if } k = j \text{ and } \ell < s, \\ \overrightarrow{G}^{js}_{k\ell}, & \text{else} \end{cases}$$

Setting

$$\boldsymbol{A} = \boldsymbol{D}^{-1} - \boldsymbol{G}$$

allows us to express the SINR constraints as $\boldsymbol{A}\overrightarrow{\boldsymbol{\rho}} \geq \boldsymbol{1}$, so POP is given by:

$$\begin{aligned} \text{minimize} \quad & \boldsymbol{1}^\top \overrightarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \boldsymbol{A}\overrightarrow{\boldsymbol{\rho}} \geq \boldsymbol{1} \\ & \overrightarrow{\boldsymbol{\rho}} \geq \boldsymbol{0} \end{aligned}$$

In matrix form the solution to POP and PMP can be expressed as

$$\overrightarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-1}\boldsymbol{1} = \left(\boldsymbol{D} - \boldsymbol{G}^{-1}\right)\boldsymbol{1} \tag{18}$$

The dual of POP using [23] is

$$\begin{aligned} \text{maximize} \quad & \boldsymbol{1}^\top \overleftarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \boldsymbol{A}^\top \overleftarrow{\boldsymbol{\rho}} \leq \boldsymbol{1} \\ & \overleftarrow{\boldsymbol{\rho}} \geq \boldsymbol{0} \end{aligned}$$

which is of the same form as PMP and denoted DPMP. Now $\boldsymbol{A}^\top = \boldsymbol{D}^{-1} - \boldsymbol{G}^\top$, so for fixed (k, ℓ) the constraint

$\left(A^\top \overrightarrow{\boldsymbol{\rho}}\right)_{\sum_{i=1}^{k-1} d_i+\ell} \leq 1$ is given by

$$\overleftarrow{\boldsymbol{\rho}}_{k\ell} \frac{\overrightarrow{G}_{k\ell}^{k\ell}}{\boldsymbol{\gamma}_{k\ell}} - \sum_{s=1}^{\ell-1} \overleftarrow{\boldsymbol{\rho}}_{ks} \overleftarrow{G}_{k\ell}^{ks} - \sum_{j=1}^{K} \sum_{s=1}^{d_j} \overleftarrow{\boldsymbol{\rho}}_{js} \overrightarrow{G}_{js}^{k\ell} \leq 1$$

which can be expressed as $\overleftarrow{\mathrm{SINR}}_{k\ell} \leq \boldsymbol{\gamma}_{k\ell}$. Then DPMP is given by

$$\begin{aligned} \text{maximize} \quad & \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \overleftarrow{\mathrm{SINR}}_{k\ell} \leq \boldsymbol{\gamma}_{k\ell} \\ & \overleftarrow{\boldsymbol{p}} \geq \mathbf{0} \end{aligned}$$

Remark on Deriving the Dual: To derive the dual, $\boldsymbol{A}^\top$ must describe the dual network. Now $\boldsymbol{A}^\top = \boldsymbol{D}^{-1} - \boldsymbol{G}^\top$, so $\boldsymbol{G}^\top$ must describe the interference. This means that if we use SIC we must reverse the order of cancellation to ensure that the dual corresponds to the reciprocal network. As discussed in the section on link gains, since $\boldsymbol{G}^\top$ describes the dual network, the dual network corresponds to reversing the direction of communication and switching the roles of $\boldsymbol{V}_k$ and $\boldsymbol{U}_k$.

By a similar argument to Lemma 5, DPMP is equivalent to DPOP given by

$$\begin{aligned} \text{minimize} \quad & \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}} \\ \text{subject to} \quad & \overleftarrow{\mathrm{SINR}}_{k\ell} \geq \boldsymbol{\gamma}_{k\ell} \\ & \overleftarrow{\boldsymbol{p}} \geq \mathbf{0} \end{aligned}$$

Then the solution to DPOP and DPMP is given by

$$\overleftarrow{\boldsymbol{\rho}}^* = \boldsymbol{A}^{-\top}\mathbf{1} = \left(\boldsymbol{D} - \boldsymbol{G}^{-\top}\right)\mathbf{1} \tag{19}$$

The following theorem shows that it is possible to achieve the same SINRs on the forward and reciprocal channels:

Theorem 2. *POP is feasible iff DPOP is feasible. Furthermore,* $\mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}}^*$ *for the optimal power vectors and* $\overrightarrow{\mathrm{SINR}}_{k\ell} = \overleftarrow{\mathrm{SINR}}_{k\ell} = \boldsymbol{\gamma}_{k\ell}$.

Proof: The first part follows from strong duality. The SINR result follows from Lemma 4. ∎

When changing powers, we need to be sure we still meet our sum power constraint. The following lemma shows that as long as we meet the power constraint for some $\boldsymbol{\rho}$ we will meet it for $\boldsymbol{\rho}^*$ too.

Lemma 6. *Suppose we choose a power allocation* $\boldsymbol{\rho}$ *such that* $\mathbf{1}^\top \boldsymbol{\rho} \leq KP$ *and transmit and receive vectors* $\{\boldsymbol{V}_k\}$ *and* $\{\boldsymbol{U}_k\}$. *Let* $\boldsymbol{\gamma}_{k\ell}$ *be the resulting SINRs. Then for the optimal power allocation for POP and DPOP,* $\overrightarrow{\boldsymbol{\rho}}^*$ *and* $\overleftarrow{\boldsymbol{\rho}}^*$ *respectively,*

$$\mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}}^* \leq KP$$

Proof: Since $\boldsymbol{\rho}$ is feasible, we must have

$$\mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* \leq \mathbf{1}^\top \boldsymbol{\rho} \leq KP$$

By Theorem 2,

$$\mathbf{1}^\top \overleftarrow{\boldsymbol{\rho}}^* = \mathbf{1}^\top \overrightarrow{\boldsymbol{\rho}}^* \leq KP$$

∎

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